SPECTRAL REPRESENTATION FOR ONE-LOOP DIAGRAM WITH ARBITRARY SPIN

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In this note we apply the results of our previous paper to the investigation of spectral representation of one-loop diagram with arbitrary spin for a class of derivative couplings.

1. Introduction

The program of this work consists of two steps:

1. In the second section we give the spectral representation of the coefficients a_J, a_{J-1} and a_{J-2} defined by (1.1)

$$A_J(s,t) = (-1)^J g^4(J)\delta(p+q-p'-q')\,\{a_J P_J(\cos\theta)+a_{J-1}P_{J-1}(\cos\theta)$$
$$+a_{J-2}P_{J-2}(\cos\theta)+\text{terms with spin} \leqslant J-3\}, \tag{1.1}$$

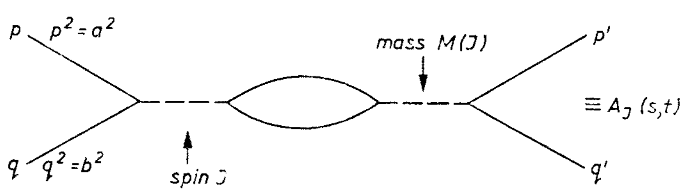

Fig. 1. The Born term for the exchange of the spin J propagator with the self-energy loop

where the scattering amplitude A_J is presented in Fig. 1. We assume the following form of the Hamiltonian

$$\mathscr{H}_J(x) = g(J)\colon \varphi_2(x)\psi^{\mu_1\ldots\mu_J}\partial_{\mu_1}\ldots\partial_{\mu_J}\varphi_1(x)\colon +\text{h.c.}, \tag{1.2}$$

where φ_1, φ_2 are the scalar fields corresponding to particles with the masses a and b, and $\psi^{\mu_1\ldots\mu_J}$, which belongs to the carrier space of the $\left(\frac{J}{2},\frac{J}{2}\right)$ Lorentz group representa-

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tion, describes the particle with the spin J. $\mathscr{H}_J$ given by (1.2) belongs to the family of Hamiltonians used to investigate the Van Hove-type of models of the reggeon exchange [1, 2].

2. In the third section we shall use another coupling for the calculation of the amplitude from Fig. 1. It is given by

$$\mathscr{H}_J^{(n)}(x) = g(J)\colon [\partial_{\mu_1} \dots \partial_{\mu_n}\varphi_2(x)]\,[\partial_{\mu_{n+1}} \dots \partial_{\mu_J}\varphi_1(x)]\psi^{\mu_1 \dots \mu_J}(x)\colon + \text{h.c.},$$
$$n = 0, 1, \dots, J \tag{1.3}$$

and this is a simple generalization of (1.2).

Remarks:

1. We treat all the distributions formally, however, all the expressions, especially spectral representations, can be understood in the framework of the distribution theory in a rigorous way and we follow here the approach of Pfaffelhuber [3].

2. Having calculated the amplitude A_J, one is able to obtain the Van Hove-type model as in [1] or [2], simply by summing it over J with help of the Sommerfeld–Watson transformation.

3. Only the case of an integer spin is considered.

2. *Calculation of the amplitude A_J for coupling (1.2)*

From [4] it follows that

$$\Delta_1(x-y; a)\partial_{\mu_1} \dots \partial_{\mu_J}\partial_{\nu_1} \dots \partial_{\nu_J}\Delta_F(x-y; b)$$
$$= \frac{i}{(2\pi)^3}\sum_{k=0}^{J} [g; -i\partial; \mu_1 \dots \mu_J, \nu_1 \dots \nu_J; 2k] \int\limits_{(a+b)^2}^{\infty} ds C_k^{2J}(s) f(s)\Delta_F(x-y; s), \tag{2.1}$$

where the tensor $[g; p; \mu_1 \dots \mu_J; k]$, which is built up of the $g^{\mu\nu}$ tensor and p^μ-four-vector, the coefficients $C_k^{(N)}$, and the function $f(s)$ are given in [4]. So we have

$$A_J = (-1)^J g^4(J)\delta(p+q-p'-q')p^{\mu_1} \dots p^{\mu_J} \frac{\Gamma^J_{\mu_1 \dots \mu_J;\nu_1 \dots \nu_J}(M^2)}{\mathscr{P}^2 - M^2(J)}$$
$$\times \sum_{k=0}^{J} \int\limits_{(a+b)^2}^{\infty} ds[g; \mathscr{P}; \nu_1 \dots \nu_{2J}; 2k] \frac{C_k^{2J}(s)}{\mathscr{P}^2 - s} \frac{\Gamma^J_{\nu_{J+1} \dots \nu_{2J};\sigma_1 \dots \sigma_J}(M^2)}{\mathscr{P}^2 - M^2(J)} p'^{\sigma_1} \dots p'^{\sigma_J}, \tag{2.2}$$

where $\mathscr{P} = p+q$ and $\Gamma^J_{\mu;\nu}(M^2)$ is an off-mass-shell propagator. A_J is a scalar function depending on p^2, p'^2 and $p_\mu p'^\mu$. We can expand it in the basis of the Legendre polynomials $P_l(\cos\theta)$ where θ is an angle between p and p' in the c.m.f.

$$A_J = (-1)^J g^4(J)\delta(p+q-p'-q') \frac{\sum\limits_{l=0}^{J} P_l(\cos\theta)a_l}{\mathscr{P}^2 - M^2(J)}\,.$$

Here we shall calculate only a_J, a_{J-1}, a_{J-2} coefficients of this expansion which correspond to the parent and the first two daughter trajectories if an application mentioned in remark 2 is made.

To calculate (2.2) we expand the off-mass-shell propagator $\Gamma^J_{\mu;\nu}(M^2)$ with respect to the on-mass-shell propagators [2].

$$\Gamma^J_{\mu_1\ldots\mu_J;\nu_1\ldots\nu_J}(M^2) = \Gamma^J_{\mu_1\ldots\mu_J;\nu_1\ldots\nu_J}(\mathscr{P}^2)$$
$$+c_1\sum_{i,j=1}^{J}\mathscr{P}_{\mu_i}\mathscr{P}_{\mu_j}\Gamma^{J-1}_{\mu_1\ldots\hat{\mu}_i\ldots\mu_J;\nu_1\ldots\hat{\nu}_j\ldots\nu_J}(\mathscr{P}^2)$$
$$+\sum_{\substack{i<j\\ m<n}}^{J}\left\{c_2\left(\frac{\mathscr{P}_{\mu_i}\mathscr{P}_{\mu_j}}{\mathscr{P}^2}g_{\mu_m\nu_n}(\mathscr{P}^2)+\frac{\mathscr{P}_{\nu_m}\mathscr{P}_{\nu_n}}{\mathscr{P}^2}g_{\mu_i\mu_j}(\mathscr{P}^2)+c_3\frac{\mathscr{P}_{\mu_i}\mathscr{P}_{\mu_j}\mathscr{P}_{\nu_m}\mathscr{P}_{\nu_n}}{\mathscr{P}^4}\right.\right.$$
$$\times\Gamma^{J-2}_{\mu_1\ldots\hat{\mu}_i\hat{\mu}_j\ldots\mu_J;\nu_1\ldots\hat{\nu}_m\hat{\nu}_n\ldots\nu_J}(\mathscr{P}^2)+(\text{terms with spin} \leqslant J-3), \tag{2.3}$$

where

$$c_1 = -\frac{1}{J}\,\frac{\mathscr{P}^2-M^2(J)}{M^2(J)\mathscr{P}^2}, \quad c_2 = -\frac{2}{J(J-1)\,(2J-1)}c_1, \quad c_3 = -\frac{4}{J(2J-1)}c_1,$$

and $\hat{\mu}$ means "no μ".

$$g_{\mu\nu}(\mathscr{P}^2) \equiv g_{\mu\nu}-\frac{\mathscr{P}_\mu\mathscr{P}_\nu}{\mathscr{P}^2}.$$

Substituting (2.3) into (2.2) and performing all the necessary contractions we obtain as a result

$$A_J = (-1)^J g^4(J)\delta(p+q-p'-q')\left[\frac{1}{\mathscr{P}^2-M^2(J)}\right]^2$$
$$\times\{a_J P_J(\cos\theta)+a_{J-1}P_{J-1}(\cos\theta)+a_{J-2}P_{J-2}(\cos\theta)$$
$$+\text{ terms with spin} \leqslant J-3\}, \tag{2.4}$$

where

$$a_J = (-1)^J D_J J!|p|^J|p'|^J\int\limits_{(a+b)^2}^{\infty}ds\,\frac{C_0^{2J}(s)}{\mathscr{P}^2-s}f(s),$$

$$a_{J-1} = c_1\mathscr{P}^2J''(J-1)!(\mathscr{P}p)\,(\mathscr{P}p')\,(-1)^{J-1}D_{J-1}|p|^{J-1}|p'|^{J-1}$$
$$\times\int\limits_{(a+b)^2}^{\infty}ds\,\frac{C_0^{2J}(s)+C_1^{2J}(s)}{\mathscr{P}^2-s}f(s),$$

$$a_{J-2} = (-1)^{J-2} D_{J-2} |p|^{J-2} |p'|^{J-2} (J-2)! \left[\frac{J(J-1)}{2}\right]^4$$

$$\times \int_{(a+b)^2}^{\infty} ds \, \frac{d_0 C_0^{(2J)} + d_1 C_1^{(2J)} + d_2 C_2^{(2J)}}{\mathscr{P}^2 - s} f(s).$$

Coefficients D_J, d_0, d_1, d_2 are given below

$$D_J = \frac{2^J (J!)^3}{(2J)!},$$

$$d_0 = 2c_2 |p|^2 |p'|^2 \frac{(p\mathscr{P})^2 (p'\mathscr{P})^2}{\mathscr{P}^4} (15c_2^2 + 2c_2 c_3 + 3c_3^2) + 3(c_2^2 + c_2 c_3) d_3,$$

$$d_1 = \mathscr{P}^2 \left\{ 4c_2^2 |p|^2 |p'|^2 + 3(c_2^2 + 2c_2 c_3) d_3 + 6(c_2 c_3 + c_3^2) \frac{(\mathscr{P}p)^2 (\mathscr{P}p')^2}{\mathscr{P}^4} \right\},$$

$$d_2 = c_2^2 (2 + 4\mathscr{P}^2 + \mathscr{P}^4) + c_2 c_3 \mathscr{P}^4 d_3 + c_3^2 (\mathscr{P}p)^2 (\mathscr{P}p')^2,$$

$$d_3 = \frac{(p\mathscr{P})^2}{\mathscr{P}^2} \left[p'^2 - \frac{(p'\mathscr{P})}{\mathscr{P}^2} \right] + \frac{(p'\mathscr{P})^2}{\mathscr{P}^2} \left[p^2 - \frac{(p\mathscr{P})}{\mathscr{P}^2} \right].$$

3. Calculation of the amplitude A_J for the coupling (1.3)

In full analogy with the previous case we get for A_J presented in Fig. 1

$$A_J + g^4(J) p_{\varrho_1} \dots p_{\varrho_n} q_{\varrho_{n+1}} \dots q_{\varrho_J} \frac{\Gamma^J_{\varrho_1 \dots \varrho_J; \nu_1 \dots \nu_J}(M^2)}{[\mathscr{P}^2 - M^2(J)]^2}$$

$$\times \mathscr{F}[\partial_{\mu_1} \dots \partial_{\mu_n} \partial_{\nu_1} \dots \partial_{\nu_n} \Delta_F(x; a) \partial_{\mu_{n+1}} \dots \partial_{\mu_J} \partial_{\nu_{n+1}} \dots \partial_{\nu_J} \Delta_F(x; b)]$$

$$\times \Gamma^J_{\mu_1 \dots \mu_J; \sigma_1 \dots \sigma_J}(M^2) p'_{\sigma_1} \dots p'_{\sigma_n} q'_{\sigma_{n+1}} \dots q'_{\sigma_J}, \tag{3.1}$$

where $\mathscr{F}$ denotes the Fourier transform and according to [4] the Fourier transform of the expression in brackets on the right-hand side of equation (3.1) is given by

$$\mathscr{F}[\partial_{\mu_1} \dots \partial_{\mu_n} \partial_{\nu_1} \dots \partial_{\nu_n} \Delta_F(x; a) \partial_{\mu_{n+1}} \dots \partial_{\mu_J} \partial_{\nu_{n+1}} \dots \partial_{\nu_J} \Delta_F(x; b)]$$

$$= \sum_{l=0}^{2n} \frac{1}{l!} \sum_{i_1 \dots i_l = 1}^{2n} \mathscr{P}_{\tau_{i_1}} \dots \mathscr{P}_{\tau_{i_l}}$$

$$\times \left\{ \delta^E(2J-l) \sum_{k=0}^{J-\frac{k}{2}} [g; \mathscr{P}; \mu_1 \nu_1 \dots \hat{\tau}_{i_1} \dots \hat{\tau}_{i_l} \dots \mu_J \nu_J; 2k] \int_{(a+b)^2}^{\infty} ds f(s) \frac{C_k^{2J-k}(s)}{\mathscr{P}^2 - s} \right.$$

$$\left. + \delta^O(2J-l) \sum_{k=0}^{J-\frac{l-1}{2}} [g; \mathscr{P}; \mu_1 \nu_1 \dots \hat{\tau}_{i_1} \dots \hat{\tau}_{i_l} \dots \mu_J \nu_J; 2k+1] \int_{(a+b)^2}^{\infty} ds f(s) \frac{C_k^{2J-k}(s)}{\mathscr{P}^2 - s}, \right. \tag{3.2}$$

where

$$\hat{\tau}_{i_1} \ldots \hat{\tau}_{i_l} = \begin{cases} \nu_{i_1-n} \ldots \nu_{i_l-n} & \text{for } i_1, \ldots, i_l > n, \\ \mu_{i_1} \ldots \mu_{i_l} & \text{for } i_1, \ldots, i_l \leqslant n. \end{cases}$$

Substituting (2.3) into (3.1) and after performing some tedious contractions we obtain as a result

$$A_J = (-1)^J g^4(J)\delta(p+q-p'-q')\,[\mathscr{P}^2 - M^2(J)]^{-2}$$
$$\times \{b_J P_J(\cos\theta) + b_{J-1}P_{J-1}(\cos\theta) + (\text{terms with spin} \leqslant J-2)\}, \tag{3.3}$$

where

$$b_{J-1} = -c_1^2 D_{J-1}(J-1)!\mathscr{P}^4$$
$$\times \Bigg\{ \int\limits_{(a+b)^2}^{\infty} dsf(s) \frac{(Jn^3 - n^2)C_0^{2J-2} + n^2 C_1^{2J}}{\mathscr{P}^2 - s} (\mathscr{P}p)(\mathscr{P}p')(\hat{\partial}_\mu q^\mu)^{J-n}(\hat{\partial}'_\nu q'^\nu)^{J-n}$$
$$+ \int\limits_{(a+b)^2}^{\infty} dsf(s) \frac{[(J-n)n^2 - n(n-1)]C_0^{2J-2} + n(n-1)C_1^{2J-2}}{\mathscr{P}^2 - s}$$
$$\times [(\mathscr{P}q)(\mathscr{P}q')(\hat{\partial}_\mu q^\mu)^{J-n-1}(\partial'_\nu q'^\nu)^{J-n-1} + (\mathscr{P}q)(\mathscr{P}p')(\hat{\partial}_\mu q^\mu)^{J-n-1}(\partial'_\nu q'^\nu)^{J-n}$$
$$+ (\mathscr{P}q')(\mathscr{P}p)(\hat{\partial}_\mu q^\mu)^{J-n}(\hat{\partial}'_\nu q'^\nu)^{J-n-1}]\Bigg\} |p|^{J-1}|p'|^{J-1},$$

and

$$b_J = \int\limits_{(a+b)^2}^{\infty} dsf(s) \frac{C_0^{2J}}{\mathscr{P}^2 - s} D_J(J-n)^{-2}(\hat{\partial}_\mu q^\mu)^{J-n}(\hat{\partial}'_\nu q'^\nu)^{J-n}|p|^J|p'|^J,$$

$$|p|^2 \equiv p_\mu p^\mu, \quad (\mathscr{P}p) \equiv \mathscr{P}_\mu p^\mu.$$

$(\hat{\partial}_\mu q^\mu)^J$ means $\hat{\partial}_{\mu_1} q^{\mu_1} \hat{\partial}_{\mu_2} q^{\mu_2} \ldots \hat{\partial}_{\mu_J} q^{\mu_J}$ and $\hat{\partial}_\mu(\hat{\partial}'_\nu)$ is a derivative with respect to $p_\nu(p'_\nu)$. θ is an angle between $\bar{p}$ and $\bar{p}'$ in the c.m.f.

4. Final remark

The amplitudes calculated here could be used to investigate, in more detail, daughter trajectories and for the broader class of derivative couplings than the ones encountered in the existing literature [1, 2].

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