ISOSPIN CONSERVATION AND THE INDEPENDENT EMISSION OF CHARGED AND NEUTRAL CLUSTERS IN HIGH ENERGY PROTON-PROTON COLLISIONS

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Isospin-zero and isospin-one clusters are constructed out of pions and used to describe final states with the correct value of the total isospin in high energy proton-proton collisions. General formulas for averages and correlation functions are given. A model for uncorrelated production and decay of clusters is studied in detail and shown to give good agreement with experiment.

1. Introduction

In a previous paper [1] we showed that models, which describe the production of pions in high energy collisions in terms of a wave function which is symmetric with respect to the interchange of two pion momenta and include exact conservation of isospin, will necessarily lead to a negative $\pi^- - \pi^0$ correlation, contradicting the experiments. We also showed briefly that clusterproduction can reproduce the experimental data.

In this paper we construct isospin-zero and isospin-one clusters out of pions and use these clusters to describe final states with the correct value of the total isospin. We will show that an uncorrelated jet model of clusters in which isospin is conserved and where the clusters decay isotropically into pions, can accurately reproduce multiplicities and correlations as well as topological cross sections.

2. Description of the model

We start from the most general state with isospin $I = L$ and $I_3 = M$ composed of isospin-one objects (clusters)

$$|LM\rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\vec{\tau}\, Y_{LM}(\vec{\tau}) \int d_4k_1 \dots d_4k_n \psi_L(p_1, p_2, q_1, q_2, k_1 \dots k_n)$$

$$\times \delta_4(p_1+p_2-q_1-q_2-k_1- \dots -k_n) \prod_{i=1}^{n} \sqrt{2k_{0i}}\, \delta(k_i^2-M^2)\theta(k_{i0})\, (\vec{\tau} \cdot \vec{A}^*(k_i))\, |0\rangle, \tag{1}$$

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where $\vec{p}_1$ and $\vec{p}_2$ are the momenta of the incoming nucleons and $\vec{q}_1$ and $\vec{q}_2$ of the outgoing ones.

$A_+^*(k)$, $A_-^*(k)$ and $A_0^*(k)$ are the creation operators for an isospin-one triplet. We assume that they obey the usual commutation relations. It turned out that only $I = 1$ clusters were not sufficient to reproduce the data. Therefore we introduce $I = 0$ clusters from which we can build a state in a way completely analogous to (1)

$$|\psi_0\rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \int d_4 q_1 \dots d_4 q_n \varphi(q_1 \dots q_n) \delta_4 \left(P - \sum q_1 \right)$$

$$\times \prod_{i=1}^{n} \sqrt{2q_{0i}}\, \delta(q_i^2 - M^2) \theta(q_{0i}) A^*(q_i)\, |0\rangle, \tag{2}$$

where $A^*(q_i)$ is the creation operator for an $I = 0$ cluster. By replacing the vacuum state $|0\rangle$ in (1) by $|\psi_0\rangle$ and changing the conservation of momentum accordingly we have the most general state with isospin $I = L$ and $I_3 = M$ built out of $I = 0$ and $I = 1$ clusters.

For a proton-proton collision we can now construct the final state with the same isospin value as the initial state [2].

$$|\text{final}\rangle = \int d\vec{q}_1 \int d\vec{q}_2 \left[A(\text{pp})\,|00\rangle + B \left\{ \frac{1}{\sqrt{2}} (\text{pp})\,|10\rangle - \tfrac{1}{2} (\text{pn}+\text{np})\,|11\rangle \right\} \right.$$

$$\left. + C \left\{ \frac{1}{\sqrt{10}} (\text{pp})\,|20\rangle - \sqrt{\tfrac{3}{20}}\, (\text{pn}+\text{np})\,|21\rangle + \sqrt{\tfrac{3}{5}}\, (\text{nn})\,|22\rangle \right\} + \frac{D}{\sqrt{2}} (\text{pn}-\text{np})\,|11\rangle \right], \tag{3}$$

where the normalization condition requires

$$|A|^2 + |B|^2 + |C|^2 + |D|^2 = 1, \tag{4}$$

(pp) is a state of two protons with momenta $\vec{q}_1$ and $\vec{q}_2$ and $\dfrac{1}{\sqrt{2}}$ (pn$\pm$np) of a proton and a neutron with momenta $\vec{q}_1$ and $\vec{q}_2$. Every $I = 1$ cluster decays into pions according to

$$A_m^*(k) \to \sum_{n=0}^{\infty} \frac{1}{n!} \int d\vec{\tau}\, Y_{1m}(\vec{\tau}) \int dq_1 \dots dq_n\, \phi\, (q_1, \dots, q_n)$$

$$\times\, \delta_4(k - \sum q_i) \prod_{i=1}^{n} \sqrt{2q_{0i}}\, \delta(q_i^2 - m^2) \theta(q_{i0}) \vec{\tau} \cdot \vec{a}^*(q_i) \tag{5}$$

with $a_+^*(q)$, $a_-^*(q)$, $a_0^*(q)$ pion creation operators.

A completely analogous formula holds for the isospin-zero operator $A^*(k)$.

Although formulas (1), (2) and (5) are symmetric under the interchange of two momenta, the symmetry of the total wave function is not defined for the interchange of two pion momenta from different clusters. So the model here is not covered by the proof of Ref. [1].

Following the same reasoning as in Ref. [1] we can put all energy and momentum dependence in the coefficients $N_l^{-1}|C_n(l)|^2$ and $N_l^{-1}|d_n(l)|^2$ and work with the states:

$$|LM\rangle = N_L^{-1/2} \sum_{n=0}^{\infty} \frac{C_n(L)}{n!} \int d\vec{\tau}\, Y_{LM}(\vec{\tau})\, (\vec{\tau}\cdot\vec{A}^*)^n |\psi_0\rangle,$$

$$|\psi_0\rangle = N_0^{-1/2} \sum_{n=0}^{\infty} \frac{C_n(0)}{n!} (A^*)^n |0\rangle,$$

$$A_m^* \to N_1^{-1/2} \sum_{n=0}^{\infty} \frac{d_n(1)}{n!} \int d\vec{\tau}\, Y_{1m}(\vec{\tau})\, (\vec{\tau}\cdot\vec{a}^*)^n,$$

$$A^* \to N_0^{-1/2} \sum_{n=0}^{\infty} \frac{d_n(0)}{n!} \int d\tau\, Y_{00}(\vec{\tau})\, (\vec{\tau}\cdot\vec{a}^*)^n. \tag{6}$$

Because we are only interested in multiplicity distributions and correlations these states will give the same results as (1), (2) and (5). Further on we will define uncorrelated production and uncorrelated decay by choosing the coefficients $C_n(L)$, $d_n(1)$ and $d_n(0)$ respectively g_1^n, g_2^n and g_3^n. This leads to Poisson like distributions for the $I = 1$ clusters and for the pions inside the clusters. For the $I = 0$ clusters we get a pure Poisson distribution.

All multiplicity distributions and correlations for charged and for neutral particles can now be expressed in terms of the coefficients A, B, C and D and the multiplicity distributions for clusters and pions. A consequence of (6) is that the distributions for the pions inside the clusters are special cases of the distributions for the $I = 1$ clusters. Explicit expressions for these distributions were calculated in Ref. [3].

3. *Averages and correlations*

The averages and correlations can be calculated very easily from the generating function [16]. Because of the property

$$G(x_+, x_-, x_0) = G_{I=0}(x_+, x_-, x_0) G_{I=1}(x_+, x_-, x_0), \tag{7}$$

the contributions to any average from the $I = 0$ and the $I = 1$ clusters can just be added.

Because of the Poisson distribution of the $I = 0$ clusters we obtain the simple formula

$$G_{I=0}(x_+, x_-, x_0) = \exp\left[\langle N_0\rangle \left(g(x_+, x_-, x_0) - 1\right)\right], \tag{8}$$

with $\langle N_0\rangle$ the average number of $I = 0$ clusters and $g(x_+, x_-, x_0)$ the generating function for the pions inside the $I = 0$ clusters. For the $I = 1$ clusters we get

$$G_{I=1}(x_+, x_-, x_0) = \sum_{\substack{N_+ N_- N_0 \\ N = N_+ + N_- + N_0}} (g_+(x_+, x_-, x_0))^{N_+} (g_-(x_+, x_-, x_0))^{N_-} \times (g_0(x_+, x_-, x_0))^{N_0} P(N, N_0), \tag{9}$$

where g_+, g_- and g_0 are the generating functions for the pions inside respectively positive, negative and neutral $I = 1$ clusters. $P(N, N_0)$ is the probability distribution of the $I = 1$ clusters.

The generating functions g_+, g_-, g_0 and g are constructed in the normal way from the pion distributions, which are special cases of the cluster distributions $P(N, N_0) \cdot P(N, N_0)$ on its turn is equal to the pion distribution in references [1–3]. Explicit expressions can be found there.

Wihout specifying the distributions one can derive the following general formulas for the average number of neutral and charged pions in the $I = 1$ clusters

$$\langle n_0 \rangle = \langle N_+ \rangle \langle n_0 \rangle_{c_+} + \langle N_- \rangle \langle n_0 \rangle_{c_-} + \langle N_0 \rangle \langle n_0 \rangle_{c0},$$

$$\langle n_\pm \rangle = \langle N_+ \rangle \langle n_\pm \rangle_{c_+} + \langle N_- \rangle \langle n_\pm \rangle_{c_-} + \langle N_0 \rangle \langle n_\pm \rangle_{c0}, \tag{10}$$

where the subscripts c_+, c_- and c0 refer to positive respectively negative and neutral clusters, the capital N's to clusters and the small n's to pions.

For the correlation functions $f_{2c} = \langle n_c(n_c - 1) \rangle - \langle n_c \rangle^2$ and $f_{c0} = \langle n_c n_0 \rangle - \langle n_c \rangle \langle n_0 \rangle$ we get similarly

$$f_{2c} = \langle N_c \rangle \langle n_c(n_c - 1) \rangle_c + \langle N_0 \rangle \langle n_c(n_c - 1) \rangle_{c0}$$
$$+ F_{2c} \langle n_c \rangle_c^2 + 2F_{c0} \langle n_c \rangle_c \langle n_c \rangle_{c0} + F_{20} \langle n_c \rangle_{c0}^2 \tag{11}$$

and

$$f_{c0} = \langle N_c \rangle \langle n_c n_0 \rangle_c + \langle N_0 \rangle \langle n_c n_0 \rangle_{c0} + F_{2c} \langle n_c \rangle_c \langle n_0 \rangle_c$$
$$+ F_{20} \langle n_c \rangle_{c0} \langle n_0 \rangle_{c0} + F_{c0} \langle n_c \rangle_{c0} \langle n_0 \rangle_c + F_{c0} \langle n_c \rangle_c \langle n_0 \rangle_{c0}, \tag{12}$$

where F_{2c}, F_{c0} and F_{20} are correlation functions for clusters and the subscript c refers to charged clusters. To get the total averages and correlations we have to add the contribution from the $I = 0$ clusters, which follows from (8). Explicit expressions for cluster correlations and averages as well as for the averages and correlations for the pions inside the clusters can again be found in references [1–3].

One of the major shortcomings of earlier models was the negative sign of the correlation function f_{c0}. The first two terms in (12) however and the contribution from the $I = 0$ clusters are always positive, so it is clear that f_{c0} will be positive if we can make these terms large enough to cancel the possibly negative terms in (12).

4. *Results and conclusions*

Table I gives the results of a fit to the topological cross sections for the energies 102, 205, 303 and 405 GeV. The data were taken from Ref. [4, 5 and 6]. The theoretical topological cross sections were calculated in a straight forward way from the generating functions (8) and (9). For the coefficients c_n and d_n in (6) we have chosen $c_n = g_1^n$, $d_n(1) = g_2^n$ and $d_n(0) = g_3^n$ to get uncorrelated production and decay as mentioned earlier. The parameters to be fitted were the average number of $I = 0$ clusters $\langle N_0 \rangle$, the aver-

TABLE I

Results of the fit for various energies

	$\langle N_0\rangle$	$\langle N_1\rangle$	$\langle n\rangle$	α	χ^2/D.O.F.	$\frac{3}{2}\langle n_c\rangle$ exp
102	$2.86\pm.07$	$1.10\pm.02$	$7.61\pm\ .27$	1.70	2.0	$7.48\pm.10$
205	$3.14\pm.08$	$1.12\pm.06$	$9.67\pm\ .48$	1.71	2.0	$9.52\pm.10$
303	$3.20\pm.09$	$1.14\pm.08$	$11.03\pm\ .66$	1.72	3.1	$10.75\pm.18$
405	$2.72\pm.15$	$1.14\pm.04$	11.69 ± 1.05	1.72	2.5	$11.49\pm.21$

age number of $I = 1$ clusters $\langle N_1\rangle$, the average of the total number of pions $\langle n\rangle$ and $\alpha = |A|^2+2|B|^2-|D|^2$.

We see from Table I that $\langle N_0\rangle$, $\langle N_1\rangle$ and α are approximately constant with energy and that $\langle n\rangle$ is very close to the experimental value (calculated from $\langle n_c\rangle = \frac{2}{3}\langle n\rangle$). Figures 1–4 show plots of the experimental and theoretical topological cross sections.

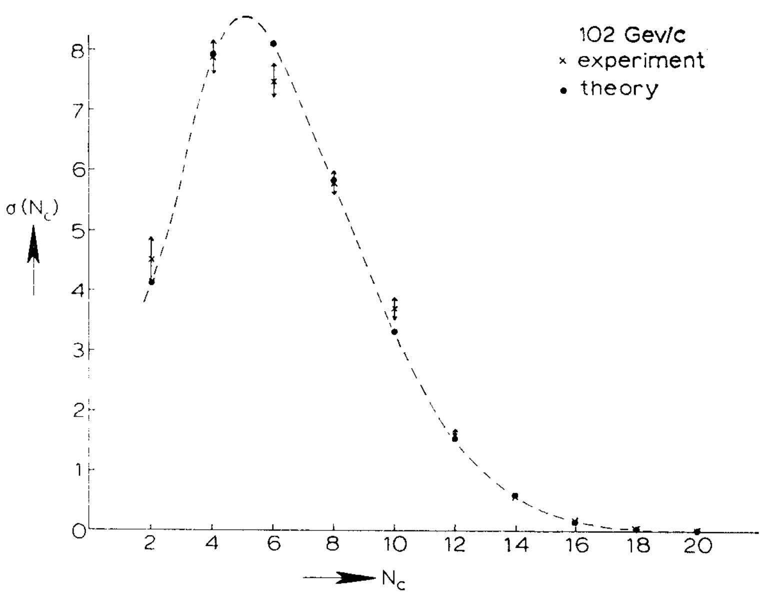

Fig. 1. The experimental and theoretical charged prong topological cross-sections for $P_{\text{lab}} = 102$ GeV/c

A fit made to these four energies together gave a χ^2 of 71.5 for 40 data points, where we took the parameters $\langle N_0\rangle$ and $\langle N_1\rangle$ independent of energy and assumed a logarithmic growth with energy for $\langle n\rangle$, $\langle n\rangle = x+y^{10}\log P_{\text{lab}}$. For the values of the parameters we found

$$\langle N_0\rangle = 2.96\pm0.04; \quad \langle N_1\rangle = 1.08\pm0.02; \quad x = -6.59\pm0.28; \quad y = 7.04\pm0.16. \tag{13}$$

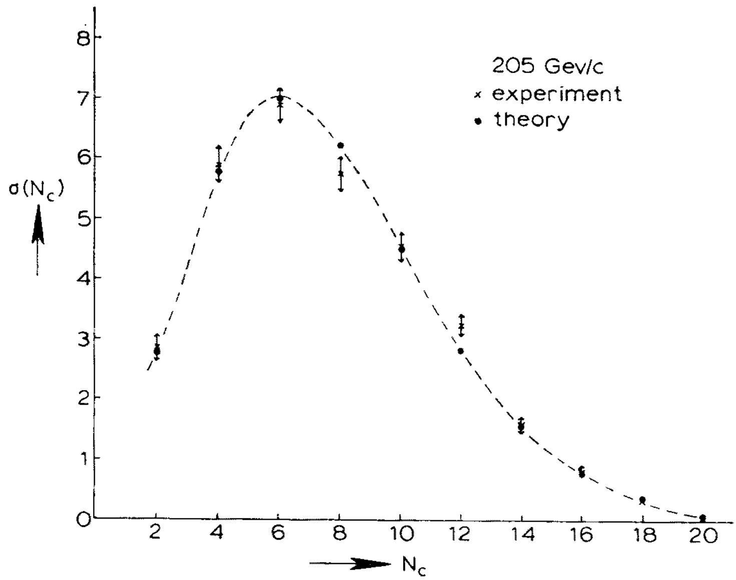

Fig. 2. The experimental and theoretical charged prong topological cross-sections for $P_{lab} = 205$ GeV/c

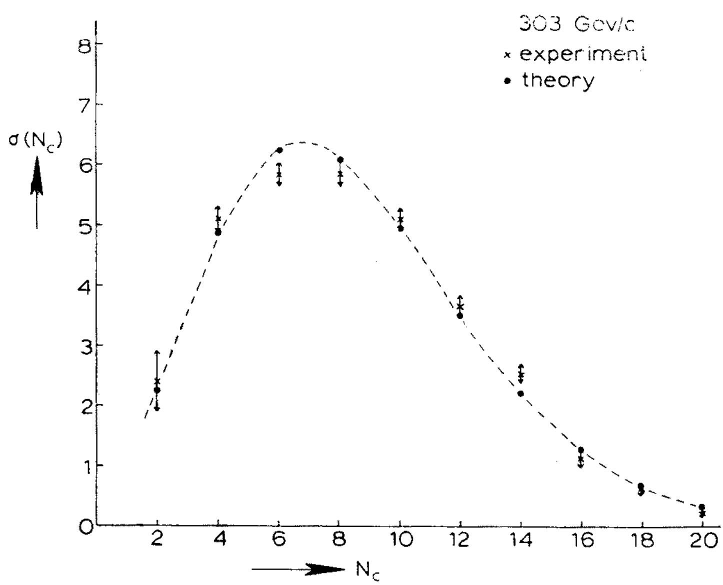

Fig. 3. The experimental and theoretical charged prong topological cross-sections for $P_{lab} = 303$ GeV/c

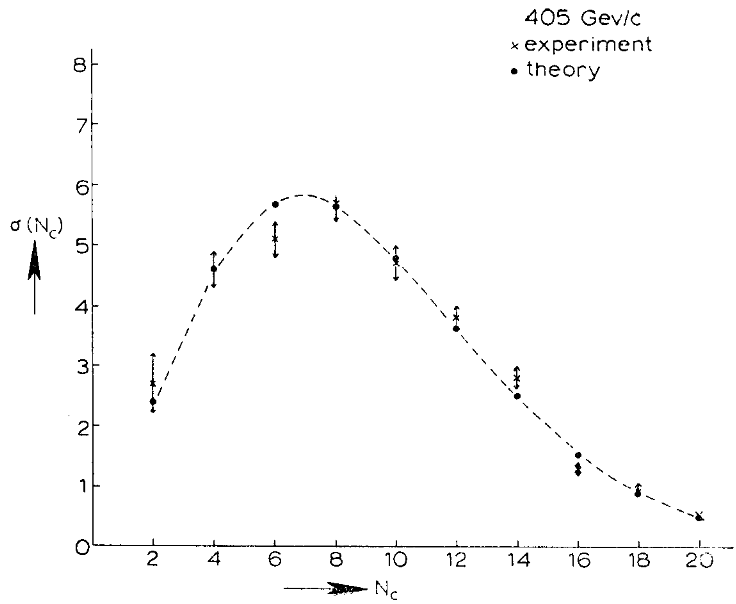

Fig. 4. The experimental and theoretical charged prong topological cross-sections for $P_{lab} = 405$ GeV/c

In Table II and III we show a comparison of the experimental values of various averages and correlation functions with those given by the model. The data were mostly taken from Ref. [7]; updated experimental results were found in references [8–12].

From the tables can be seen that we have excellent agreement with experiment for averages and correlation functions for charged particles. For the dispersion $D_c = (\langle N_c^2\rangle - \langle N_c\rangle^2)^{\frac{1}{2}}$ with $\langle N_c\rangle$ the total number of charged particles including the protons we found,

$$D_c = \beta(\langle N_c\rangle - 1) \text{ with } \beta = 0.579 \pm 0.015 \tag{14}$$

(0.015 gives the variation of β for the various energies). This result is in excellent agreement with various compilations [14, 15].

TABLE II

P_{lab}	$\langle N_c\rangle$		$\langle n_-\rangle$		$\langle n_{\pi 0}\rangle$	
	exp	model	exp	model	exp*	model
102	6.32 ± 0.07	6.34 ± 0.17	2.16 ± 0.04	2.17 ± 0.09	2.62 ± 0.25	2.66 ± 0.09
205	7.68 ± 0.07	7.67 ± 0.32	2.84 ± 0.04	2.84 ± 0.15	3.34 ± 0.24	3.37 ± 0.17
303	8.50 ± 0.12	8.55 ± 0.43	3.25 ± 0.05	3.27 ± 0.22	3.95 ± 0.38	3.84 ± 0.23
405	8.99 ± 0.14	8.96 ± 0.69	3.50 ± 0.07	3.48 ± 0.35	—	4.09 ± 0.36

* Data from Ref. [8, 9] and [10].

TABLE III

Correlation parameters f_2

P_{lab}	any two charged		$(-,-)$		$(-,0)$	
	exp	model	exp	model	exp*	model
102	3.45±0.25	3.26±0.36	0.28±0.07	0.25±0.06	—	0.12±0.02
205	6.89±0.35	6.91±0.84	0.80±0.09	0.82±0.15	0.9±0.7	0.39±0.05
303	9.45±0.51	9.96±1.36	1.24±0.13	1.35±0.25	2.3±0.6	0.63±0.10
405	13.5 ±0.9	13.40±2.5	2.1 ±0.2	2.11±0.44	—	0.90±0.19

P_{lab}	$(+,-)$		(0, 0) model	any two model
	exp	model		
102	2.53±0.12	2.42±0.13	1.57±0.16	5.23±0.62
205	3.76±0.16	3.65±0.29	3.15±0.38	11.57±1.45
303	4.80±0.23	4.63±0.44	4.45±0.60	16.91±2.37
405	—	5.59±0.79	5.91±1.07	22.89±4.3

* Data from Ref. [11] and [12].

More interesting is the $\pi^- - \pi^0$ correlation which, although positive, still seems to be too small. The latest data [12], however, give smaller values than earlier measurements [7], which makes at least the value for 205 GeV/c in agreement with experiment.

With this result we have explicitly shown that the introduction of clusters can remove the shortcomings of other models [1], which necessarily gave a negative sign for this correlation because of iso-spin conservation. Figures 5–7 show $\langle n_0(N_c)\rangle$ for 102, 205

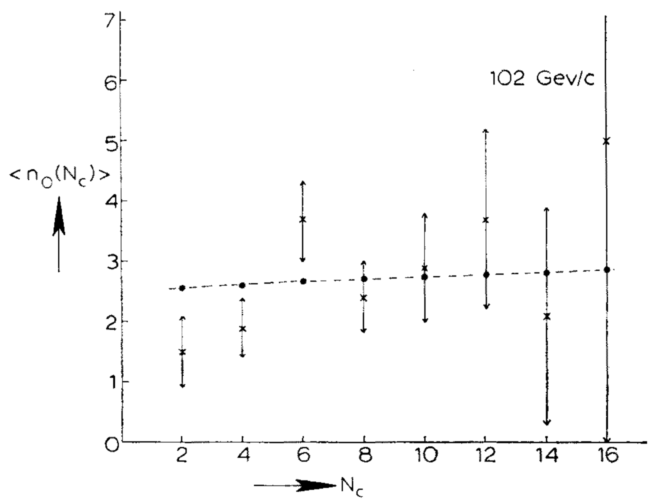

Fig. 5. The average number of neutral pions for a fixed number of charged particles at 102 GeV/c. The dots are the theoretical points

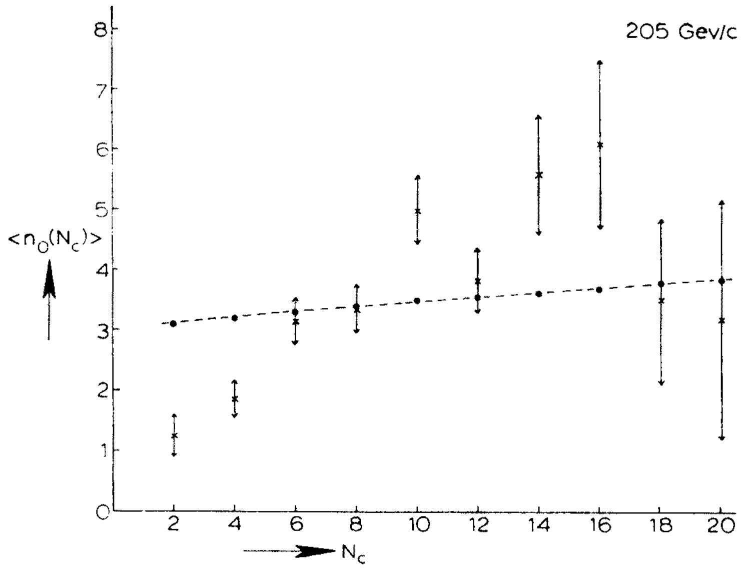

Fig. 6. The average number of neutral pions for a fixed number of charged particles at 205 GeV/c. The dots are the theoretical points

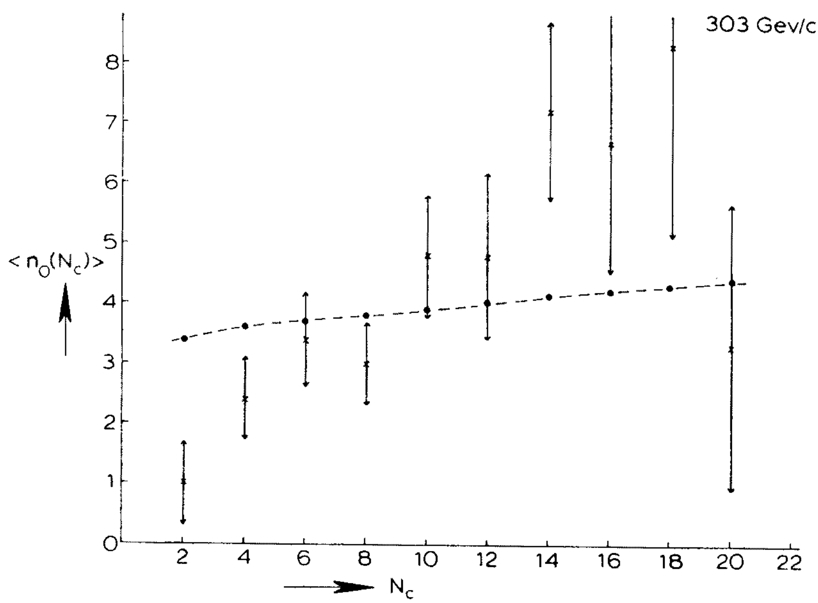

Fig. 7. The average number of neutral pions for a fixed number of charged particles at 303 GeV/c. The dots are the theoretical points

and 303 GeV/c. The slope in $\langle n_0(N_c)\rangle$ found in the model seems to be smaller than the experimental one (the large error bars do not allow a definite conclusion with respect to this point). There is however a direct relationship [13] between the slope s in $\langle n_0(N_c)\rangle$ and f_{c0}, viz.

$$f_{c0} = sD_c^2 \tag{15}$$

with D_c the dispersion for the charged particles. Therefore smaller values for f_{c0} as mentioned in Ref. [12] imply a smaller slope for $\langle n_0(N_c)\rangle$ too.

A further result of our model is an approximately constant number of clusters in the energy range between 100 and 400 GeV/c, which seems to be in contradiction with rapidity correlation measurements. Using more or less model dependent assumptions several authors [17] extract from these measurements the information that the average number of particles per cluster must be constant with energy. This implies a growth of the average number of clusters with energy.

Our results with respect to the energy dependence of the number of clusters are a consequence of the uncorrelated production of the clusters and of isospin conservation. Assuming a growing number of clusters with energy, one can show from equation (11), adopting the expressions for F_{2c}, F_{c0} and F_{20} from references [1–3], that at high energies Wróblewski's relation for the dispersion of the charged particles cannot be satisfied if the cluster distribution is Poisson.

Finally we have to mention that the first model in which a constant number of clusters was proposed, has been constructed by Levy [18], who was able to reproduce the multiplicity distribution for the charged particles. No attempt was made, however, to fit the charged-neutral correlation, nor has isospin conservation been brought into account, which is one of the basic ingredients of our approach.

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