DETERMINATION OF THE REAL PART OF pp, $\bar{p}p$ FORWARD SCATTERING AMPLITUDE AND THE SLOPE OF DIFFERENTIAL CROSS SECTION USING ANALYTICITY IN COS θ PLANE

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Absolute values of the real part of the pp, $\bar{p}p$ forward scattering amplitude and the slope of the differential cross section have been determined on the basis of data on the elastic differential cross section outside the region of the Coulomb interference. The method is based on the optimal use of analyticity of the scattering amplitude in the cos θ plane and leads to significantly larger absolute values of the real part and slope in comparison with those obtained by means of the Bethe formula on the basis of data on differential cross sections in the region of the Coulomb interference.

1. *Introduction*

According to the commonly accepted point of view, the most precise way of determining the real part of the scattering amplitude of charged particles at high energies is the use of the Bethe formula [1]. For this purpose one must have data on differential cross section of elastic scattering at small angles where the interference between the nuclear and Coulomb interactions strongly manifests itself. Concrete methods of measuring differential cross sections at very small angles and analyses of experimental data by means of the Bethe formula in order to extract the real part of the pp forward scattering amplitude are described in detail in reviews [2, 3].

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In our paper we first dwell on difficulties (in our opinion serious) of use of the Bethe formula for obtaining the real part of the forward scattering amplitude. Then we describe another method (the idea of which was given by us in Ref. [4]) of determination of the real part of the forward scattering amplitude and the slope, based on the exploitation of analyticity of the scattering amplitude in the cos θ plane. Later we present the results of concrete calculations of these quantities in the case of the pp, $\bar{p}p$ forward scattering. At the end we discuss significant discrepancies in estimations of the real part of the amplitude and the slope obtained by means of two different methods and we consider a possibility of comparing the experimental data with the theoretical predictions.

2. *Analysis of data on differential cross section of elastic scattering of charged particles by means of the Bethe formula*

The Bethe formula has the following form:

$$\frac{d\sigma}{dt} = \xi\Big[A_C^2(t)+\mathrm{Re}^2 A(t)+\mathrm{Im}^2 A(t)-2A_C(t)\,\mathrm{Re}\,A(t)$$
$$-4A_C(t)\,\mathrm{Im}\,A(t)n\ln\frac{1.06}{vR\theta}\Big], \tag{1}$$

where ξ is the parameter of normalization[1], $A_C(t) = (\sqrt{\pi}/k)(2h^2nv/t)G(t)$ is the Coulomb amplitude, $G(t)$ is the nuclear formfactor of the target particle, v is the wave number, $n = 1/137\beta_{\mathrm{lab.s.}}$ is the Coulomb parameter, R is the radius of strong interactions, Re $A(t)$, Im $A(t)$ are the real and the imaginary part of the amplitude of nuclear interactions. The scattering angle θ in the c.m.s. is related to the momentum transfer in the usual way $t = -2k^2(1-\cos\theta)$.

We now want, by means of Eq. (1), on the basis of the measured values of $d\sigma/dt$ at small $|t|$ and the theoretical values of $A_C(t)$, to determine the quantity Re $A(0)$. One needs, for this purpose, to postulate in advance some functional forms for the real and the imaginary part of the amplitude, i.e., to parametrize somehow Re $A(t)$ and Im $A(t)$. Such functions are not known. Therefore, the first step which is made in order to apply practically the Bethe formula consists in postulating the equal (but unknown) dependence of Re A and Im A on t. Then Eq. (1) can be rewritten in the following form

$$\frac{d\sigma}{dt} = \xi\left[A_C^2(t)+\mathrm{Im}^2 A(t)\,(\alpha^2+1)-2A_C(t)\left(\alpha+2n\ln\frac{1.06}{vR\theta}\right)\right], \tag{1'}$$

where

$$\alpha \equiv \frac{\mathrm{Re}\,A(0)}{\mathrm{Im}\,A(0)} \tag{2}$$

does not depend on t.

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[1] This parameter is not related directly to the Bethe formula. However, it makes it possible to use the formula also in those cases when only the relative values of differential cross sections are available.

There are no arguments in favour of such an assumption, aside from hand-waving arguments that in the small interval of $|t|$, where fits are carried out, $\mathrm{Re}\, A(t)/\mathrm{Im}\, A(t)$ can hardly be very different from α. Recall, however, that just in the case of the pp, $\bar{\mathrm{p}}$p scattering $\mathrm{Re}\, A(t)$ has a pole at $t = m_\pi^2$ and $\mathrm{Im}\, A(t)$ does not have it. Therefore, their behaviour in the region of small $|t|$ may be rather different. There are also arguments (see e. g. [5, 6]) according to which the ratio of the real to the imaginary part of the amplitude for non-forward directions can reach large (small) values and not at all be a constant.

Equation (1′), where the unknown function $\alpha(t)$ has been replaced by an unknown constant α, still is not suitable for practical use. One has to make the next assumption: to postulate some functional form for $\mathrm{Im}\, A(t)$. It is common to make the following choice:

$$\mathrm{Im}\, A(t) = \left(\frac{d\sigma}{dt}\right)_{\mathrm{opt}}^{\frac{1}{2}} e^{\frac{1}{2}bt}, \tag{3}$$

where $(d\sigma/dt)_{\mathrm{opt}}$ is an optical point which can be determined from the data on the total cross section. The parameter b in this case coincides with the slope of the differential cross section defined as

$$\mathrm{slope} \equiv \frac{d}{dt}\left[\ln\left(\frac{d\sigma}{dt}\right)_{\mathrm{nuclear}}\right]. \tag{4}$$

Now Eq. (1) can be rewritten in the finite form

$$\frac{d\sigma}{dt} = \xi\left[A_C^2(t) + \left(\frac{d\sigma}{dt}\right)_{\mathrm{opt}} e^{bt}(\alpha^2+1)\right.$$
$$\left. -2A_C(t)\sqrt{\left(\frac{d\sigma}{dt}\right)_{\mathrm{opt}}}\, e^{\frac{1}{2}bt}\left(\alpha + 2n\ln\frac{1.06}{vR\theta}\right)\right]. \tag{1″}$$

One should note that the only argument in favour of the choice (3) is the assumption (see, e. g. [2]) that at small $|t|$ and high energies the scattering is of a diffractive nature. On the other hand, there are experimental as well as theoretical arguments against such a functional form. It has been observed that the slope inside the diffraction peak may have breaks (see, e. g. [7]). There are predictions (e. g. [8]) that the slope must grow continuously when $|t|$ tends to zero.

Equation (1″) which is finally used in practical calculations contains three unknown parameters: ξ, b and α. As a rule, it turns out to be impossible to determine all of them on the basis of measurements in the region of the Coulomb interference, i. e., due to correlations they are determined with large errors. Therefore, in practice sometimes the following procedure is carried out: The parameter b is determined on the basis of data outside the region of the Coulomb interference, it is assumed that b keeps the same value in the region of the Coulomb interference, one puts it fixed in Eq. (1″) and determines from the fit the parameters ξ and α with small errors.

What can be said about the reliability of the results obtained in such a way? Naturally, the results are based exclusively on the belief in validity of assumptions (2) and (3) in the region of measurements of $d\sigma/dt$, and also in the region of smaller $|t|$, up to $t = 0$. The latter circumstance is very important because the use of the Bethe formula does not, of course, remove the problem of extrapolation of nuclear amplitude from the region of measurements to the forward direction.

The above mentioned difficulties connected with the use of the Bethe formula are, of course, well known. We, however, would like to call attention to the following, somewhat strange, fact: There is no reflection of hypothesis of analyticity of the scattering amplitude in the t-plane in the above assumptions (2) and (3). On the other hand, this hypothesis is by no means less fundamental than the hypothesis of analyticity of the scattering amplitude in the s-plane, for testing of which, as a matter of fact, one tries to extract from the experiment the quantity α. Moreover, it is clear that neither Eq. (2) nor Eq. (3) is consistent with the hypothesis of analyticity in the whole t-plane. Are they consistent with analyticity in t in that interval where fits are carried out and through which the extrapolation to the forward direction is performed, is not known a priori.

3. Parametrization of differential cross section taking into account analyticity in the cos θ plane

Analytic properties of the scattering amplitude in the $\cos\theta$ plane were investigated rigorously for the first time in the paper of Lehmann [9]. He showed that the real and imaginary parts of the scattering amplitude are analytic functions in certain ellipses in the $\cos\theta$ plane. Martin [10] succeeded in proving that the scattering amplitude is analytic in significantly larger domains of the $\cos\theta$ plane. The detailed discussion of these questions is given in the review by Sommer [11].

In our problem the only measured quantity is the differential cross section. Therefore, we shall not distinguish between the analytic properties of the real and imaginary parts of the amplitude and shall identify the domain of analyticity of the differential cross section with the domain of analyticity of the real part which is smaller than that of the imaginary part.

We shall start with the assumption that the differential cross section is an analytic function in the $\cos\theta$ plane with the following analytic properties: in the case of the pp scattering there are symmetrically situated poles and cuts in the t- and u-channels

$$(\cos\theta)_{\text{pole}} = \pm\left(1 + \frac{m_\pi^2}{2k^2}\right), \tag{5}$$

$$(\cos\theta)_{\substack{\text{beginning}\\ \text{of the cut}}} = \pm\left(1 + \frac{2m_\pi^2}{k^2}\right); \tag{6}$$

in the case of the $\bar{\text{p}}$p scattering there are analogous pole and cut in the t-channel, in the u-channel there is no pole and the cut begins much further away

$$(\cos\theta)_{\substack{\text{beginning}\\ \text{of the cut}}} = -\left(1 + \frac{2m_p^2}{k^2}\right). \tag{7}$$

How to parametrize the differential cross section taking into account these analytic properties in the cos θ plane?

First of all one should introduce the pole terms in $d\sigma/dt$. Their explicit form is known [12]. Moreover, one can evaluate their magnitude, because they depend on the well known coupling constant $g^2_{\pi^0 pp}$, and subtract them from $d\sigma/dt$ enlarging thus the domain of analyticity. One can act also in another way: add to the experimental data an extra "experimental" point $\lim_{t\to m^2_\pi} (t-m_\pi^2)^2 \frac{d\sigma}{dt} \sim g^2_{\pi^0 pp}$. In such a case the problem of extrapolation of the differential cross section to $t = 0$ is replaced by the problem of interpolation of the differential cross section between the pole and the experimental data.

The functional form of $d\sigma/dt$ on the cuts is not known. There are two possible approaches. The first consists in writing $d\sigma/dt$ on the cuts in some parametric forms, the second consists in dispensing with an explicit form of the function and in accounting the mere existence of the cuts. The first approach inevitably leads to models (see, e. g. [8]) which, however, should be better than the "models" (2) and (3). The second approach (that of the present authors) can be formulated as follows: There exist measurements of $d\sigma/dt$ in a number of points in some interval of the physical region $\cos\theta_1 \leqslant \cos\theta \leqslant \cos\theta_2$, the positions of the cuts of the function $d\sigma/dt$ in the cos θ plane are known. How to represent this function in the domain of analyticity, without introducing some explicit functional form? Obviously, only one possibility exists; to write the function in a form of a series and to find the corresponding coefficients by fitting to experimental data. Such a procedure is mathematically well developed and is a subject for optimization. It was described in detail in our previous paper[2] [4]. The essence of it is the conformal mapping of the cos θ plane into the unifocal ellipse in the z-plane so as the region $\cos\theta_1 \leqslant \cos\theta \leqslant \cos\theta_2$ gets mapped into the interval $-1 \leqslant z \leqslant 1$ and the cuts onto the ellipse; $d\sigma/dt$ is then expanded in a series of the Chebyshev polynomials $T_m(z)$[3]:

$$\frac{d\sigma[z(t)]}{dt} = \sum_{m=1}^{M} A_m B_m T_m[z(t)], \tag{8}$$

where B_m are quantities related to dimensions of the ellipse, A_m are coefficients to be found from a fit. On the basis of these coefficients it is possible to construct certain function Φ[4] which, when added to χ^2, gives the quantity X. The minimal value of the latter corresponds to that number M of terms in the expansion (8) when one should truncate it.

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[2] In a broader aspect such a problem belongs to the so-called inverse problems. The detailed discussion of them in elementary particle physics and of the corresponding mathematical techniques is given in reviews [13–18].

[3] It turns out that such a series is converging maximally fast and leads to the smallest possible errors in the process of extrapolation. In the present paper we consider the region outside the Coulomb interference, therefore the expansion (8) is simpler than the expansion (18) of Ref. [4].

[4] Due to the difference of the expansion (8) of this paper and the expansion (18) of Ref. [4] one must replace Eq. (19) of Ref. [4] by $h_m = A^2_{m+1}/\sum_{k=1}^{m} A_k^2$, $(1 \leqslant m \leqslant M-1)$.

After fitting the series (8) with the optimal number of terms, one carries out the extrapolation, i. e., evaluates the series at $z = z\ (0)$, obtaining thus $(d\sigma/dt)_0$. Further, one can calculate the quantity $|\alpha|$:

$$|\alpha| = \sqrt{\frac{(d\sigma/dt)_0}{(d\sigma/dt)_{\text{opt}}} - 1}\,. \tag{9}$$

The slope is evaluated according to the definition (4) where $(d\sigma/dt)_{\text{nuclear}}$ is substituted by the series (8).

4. *Analysis of experimental data*

We have reanalyzed, according to the above scheme, experimental data on $d\sigma/dt$ outside the region of Coulomb interference ($|t| \gtrsim 0.05\ (\text{GeV}/c)^2$) for the pp scattering from 1.349 GeV/c to 21.88 GeV/c [19–26] and for the $\bar{\text{p}}$p scattering from 1.11 GeV/c to 40.1 GeV/c [27–39]. In those cases when data on $d\sigma/dt$ were not presented in the form of

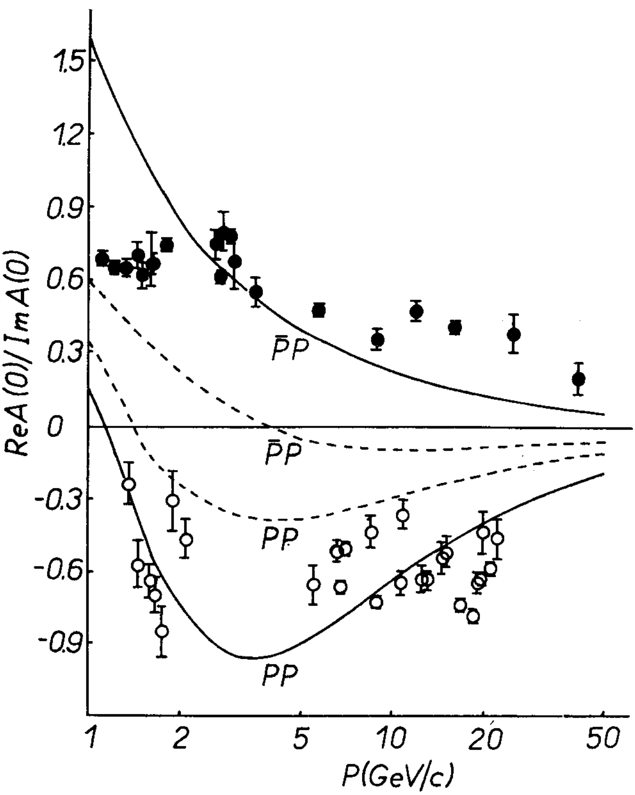

Fig. 1. Ratio of the real to the imaginary part of the pp (○) and $\bar{\text{p}}$p (●) forward scattering amplitude obtained in present work (signs of α have been chosen arbitrarily) as a function of momenta of the initial particle. Solid curves correspond to dispersion calculations of Ref. [45, 46], dashed ones to calculations of Ref. [43]. Errors originating from the truncation of the series (8) are not shown

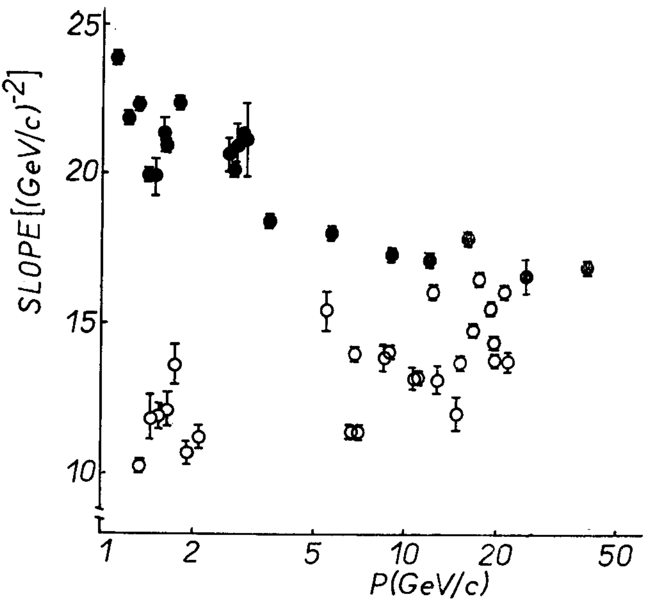

Fig. 2. Slopes of the differential cross section of the pp (○) and $\bar{p}p$ (●) scattering at $t = 0$ obtained in the present work as a function of momenta of the initial particle. Errors originating from the truncation of the series (8) are not shown

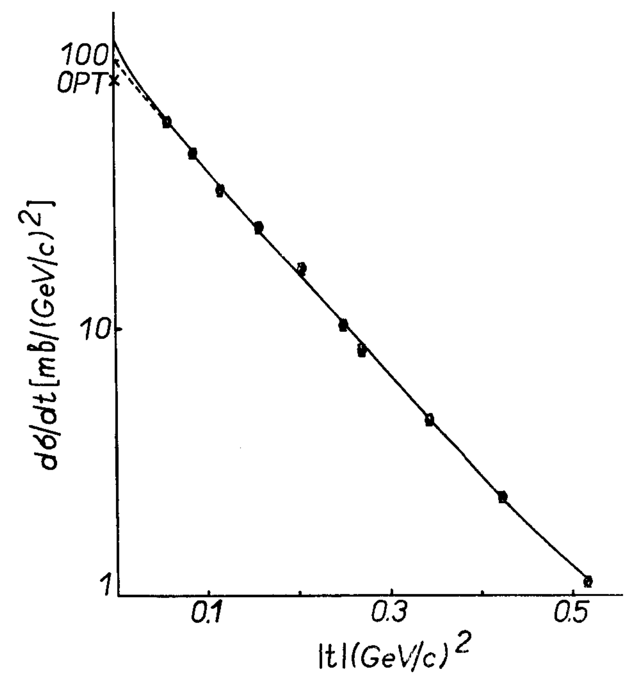

Fig. 3. Illustration of the fit and of the extrapolation of the differential cross section of the pp scattering at 10.8 GeV/c [24] using the exponent (10) (dashed curve, $(d\sigma/dt)_0 = 102.7$ mb/(GeV/c)^2) and the expansion (8) (solid curve, $(d\sigma/dt)_0 = 115.8$ mb/(GeV/c)^2). In the region of measurements both fits are practically indistinguishable. The optical point (80.3 mb/(GeV/c)^2) is marked by ×

tables we used compilations [40, 41] where numerical values of $d\sigma/dt$ were extracted from original papers.

It should be noted that in our analysis we have not used data on $d\sigma/dt$ outside the region of the Coulomb interference from a number of experimental works, e. g. measurements performed at Serpukhov [42]. The point is that we need absolute values of $d\sigma/dt$ because in our metod (see Eq. (9)) $|\alpha|$ is determined as the excess of $(d\sigma/dt)_0$ over the optical point. In some papers, hovever, among them in Ref. [42], even outside the region of the Coulomb interference $d\sigma/dt$ was measured in relative units only.

It should be stressed that when determining $|\alpha|$ by the above procedure, it is very important to take into account also systematic error of the differential cross section, because even a small rise or fall of the curve of $d\sigma/dt$ as a whole can lead sometimes to large changes in the obtained values of $|\alpha|$. Aside from that, it is crucial to draw oneself back sufficiently far from small $|t|$ to be sure that the data used are not contaminated by electromagnetic interactions.

The results of calculations of the quantity $|\alpha|$ and of the slope at $t = 0$ are presented in Figs 1 and 2. In addition, as an illustration in Fig. 3 we show the difference between extrapolations to the forward direction by means of the exponent

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_0 e^{bt + ct^2} \tag{10}$$

and according to the expansion (8).

It should be noted that our "parametrization" (8), contrary to Eq. (10), covers the whole region of measurements of $d\sigma/dt$ including also the diffraction minimum and the tail of the differential cross section in those cases when measurements there exist. Therefore, it is natural that the number of exploited terms in the expansion (8) is different for different experiments. In Table we give one example of concrete calculations.

5. *Discussion and conclusions*

The values of $|\alpha|$ and the slopes obtained by means of standard analyses, i. e., those on the basis of the data on $d\sigma/dt$ in the region of the Coulomb interference with the help of formula (1″), are not shown in Figs 1, 2 because they are well known (see e. g. reviews [2, 3]). We note only that the values of α_{pp} agree well with the curve (dashed one in Fig. 1) obtained in Ref. [43] by evaluating dispersion relations. On the other hand, the slope for the pp scattering is logarithmically rising from ~ 5.3 $(\mathrm{GeV}/c)^{-2}$ to ~ 13.1 $(\mathrm{GeV}/c)^{-2}$ in the region from 1.7 GeV/c to 2070 GeV/c, and logarithmically falling from ~ 13 $(\mathrm{GeV}/c)^{-2}$ to ~ 11 $(\mathrm{GeV}/c)^{-2}$ for the $\bar{p}p$ scattering in the region from 7 GeV/c to 70 GeV/c (see review [3]).

Hence, our results differ from those commonly accepted. Our values of $|\alpha|$ and the slope at $t = 0$ are significantly larger.

What can we say with respect to this discrepancy?

The drawbacks and possible mistakes, when using the Bethe formula, were discussed in detail in Sect. 2. Concerning our method one can hardly find arguments against it from the general physical point of view because it is free of any assumptions, aside from the fundamental hypothesis about analyticity of the scattering amplitude in the cos θ plane. The problem reduces to stability and reliability of our model-independent numerical extrapolation. Here again from the general mathematical point of view one should have no objections, because our extrapolation is of the I $\rightarrow$ I type (see review [18]), i. e. the extrapolation from a segment inside the domain of analyticity to a point ($t = 0$) also inside it. Such an extrapolation is stable in the sense that if the errors of $d\sigma/dt$ in the region of measurements tend to zero then the error of $(d\sigma/dt)_0$ also tends to zero. However, in realistic cases when the errors of $d\sigma/dt$ are not equal to zero, the estimation of the error of $(d\sigma/dt)_0$ is a rather complicated task [13–18]. The errors shown in Figs 1, 2 essentially reflect only the errors of $d\sigma/dt$ in the region of measurements and are not connected with errors originating from the truncation of the series (8).

Both methods can also be compared as follows. Using the Bethe formula (digressing from the poorly founded assumptions (2) and (3)) one extrapolates through a short distance on the basis of data in a very small (but close to cos $\theta = 1$) region of the cos θ plane. In our method one uses the data from a much larger region of the cos θ plane but one has to extrapolate significantly further. We take the risk of saying that from the point of view of analyticity the more reliable information can be obtained if one extrapolates through the larger distance on the basis of significantly richer analytic information (data in a wide region of cos θ plane together with the knowledge of analytic properties in the cos θ plane), than when one extrapolates through a shorter distance on the basis of experimental information in a tiny region and ignoring the global analytic properties of the amplitude in the cos θ plane. The sense of the analytical approach consists just of the fact that the behaviour of the function "at the head" ($t \approx 0$) may be closely connected with its behaviour at other values of t, among them also in the "tail"!

Unfortunately we do not see any way of using in our approach the data in the region of the Coulomb interference, and of combining them with data outside the region of interference. The point is that as soon as one enters the region of interference one has to distinguish between the behaviour of the real and the imaginary part of the amplitude as a function of t. If we refuse to do this, as in our previous paper [4], when obtaining the results presented in Table III of that paper, we immediately must believe in the assumption (2) and the difference between our approach and the commonly accepted one consists merely in replacing the parametrization (3) by the general expansion (8). Formally one could use two separate conformal mappings and two separate expansions: one for the real part of the amplitude, another for the imaginary part. However, at energies under consideration, the domains of analyticity of the real and imaginary parts practically coincide, and having at the disposal only the data on $d\sigma/dt$ one can hardly hope to separate the parameters of both expansions.

Unfortunately, we also do not see a simple way of comparing the values of $|\alpha|$ and the slope obtained by both methods with the theoretical predictions.

Due to the presence of a large unphysical region and the unknown low-energy region of the $\bar{p}p$ scattering the calculations based on dispersion relations do not permit us, according to our understanding, to answer the question of whether, for example, at

TABLE I

Illustration of the method of fitting and extrapolation of the pp scattering at 10 GeV/c [24], $(d\sigma/dt)_{opt} = 80.3$ mb/(GeV/c)^2, N is the number of experimental points, M is the number of terms of the expansion (8) used in the fitting. The quantities FI and X are defined in Ref. [4]. The series (8) should be truncated at the minimal value of X. The value of slope is given also at t_{max} which corresponds to the experimental point with the minimal $|t|$

Region of fit $\|t\|$ (GeV/c)^2	N	$d\sigma/dt$ ($t = 0$) mb/(GeV/c)^2	M	χ^2	FI	X	$\|\alpha\|$	slope (GeV/c)$^{-2}$ $t = t_{max}$	slope (GeV/c)$^{-2}$ $t = 0$
.058—.824	13								
		73.0 ± .6	3	376.66	4.55	381.21	—	—	—
		123.9 ± 1.6	4	13.89	8.20	22.09	.74 ± .01	11.0 ± .0	14.4 ± .1
		115.8 ± 4.0	5	12.18	8.43	20.62	.66 ± .04	10.5 ± .1	13.1 ± .4
		99.7 ± 10.7	6	11.26	18.34	29.59	.49 ± .14	9.5 ± .1	8.2 ± 1.7
		128.3 ± 29.1	7	10.83	32.04	42.86	.77 ± .23	10.6 ± .2	18.7 ± 3.6

$\sim$ 10 GeV/c $|\alpha_{pp}| \approx 0.6$, as we have obtained, or $|\alpha_{pp}| \approx 0.3$, as it has been found in analyses of the data in the region of the Coulomb interference with help of the Bethe formula.[5]

As for the slope, it should be mentioned that the Regge theory gives only the energy dependence and does not predict the value of the slope at fixed energy. The energy dependence of the slope at $t = 0$ obtained by us qualitatively agrees with the commonly accepted one. The large values of the slope are directly connected with large values of $|\alpha|$. It was observed in all cases that in the region of extrapolation the slope continuously increased with the decrease of $|t|$.

The belief in our results (at least in the qualitative sense: values of $|\alpha|$ and the slope at forward direction are larger than commonly accepted), according to our mind, inspires the fact that a similar extrapolation procedure has been applied for many years with good results (see review [18]) to extract the coupling constants from the data on differential cross sections for different processes. Moreover, in many cases the extrapolation to the corresponding poles is carried out on the basis of significantly poorer experimental data on $d\sigma/dt$ and always over much larger distances than those concerned in this paper.

Perhaps, it would be interesting to carry out similar investigation in the case of the $\pi^{\pm}p$ scattering where, on the one hand, the values of α have been precisely evaluated on the basis of dispersion relations and, on the other hand, in the forward direction there are no spin effects which are present and ignored in the case of the pp, $\bar{p}$p forward scattering, though, in principle, they can distort all the effects and conclusions.

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[5] More detailed discussion on this is given in Ref. [44].

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