PHASE CONTOURS OF FORWARD COMPTON SCATTERING AMPLITUDE IN THE COMPLEX ENERGY PLANE

By O. Dumbrajs*

Institute of Theoretical Physics, Warsaw University**

and M. Staszel

Department of Mathematics, Warsaw University

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The forward photon-proton scattering amplitude has been evaluated for complex energies. The question of the number and locations of zeros of this amplitude is considered.

We shall consider the photon-proton forward scattering amplitude, which has the well-known spin structure [1]:

$$f(\omega) = f_1(\omega)\boldsymbol{e}_1\boldsymbol{e}_2 + i\boldsymbol{\sigma}(\boldsymbol{e}_2 \times \boldsymbol{e}_1)f_2, \tag{1}$$

where $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are the polarization vectors of the initial and final photons, respectively and ω is the photon lab energy. The pion production threshold is $\omega_0 = m_\pi + m_\pi^2/2m_N$. The corresponding total and forward differential cross sections are defined by the expressions:

$$\sigma_{\mathrm{tot}}(\omega) = \frac{4\pi}{\omega} \operatorname{Im} f_1(\omega), \tag{2}$$

$$\left.\frac{d\sigma}{dt}\right|_{t=0} = \frac{\pi}{\omega^2}\left[|f_1(\omega)|^2 + |f_2(\omega)|^2\right]. \tag{3}$$

In this paper we confine ourselves to the consideration of the amplitude $f_1(\omega)$ as the most interesting one because its imaginary part is directly connected via Eq. (2) with an easily measurable quantity σ_{tot}: total cross section of unpolarized photons on unpolarized protons.

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* On leave of absence from the Institute of Nuclear Physics, Moscow State University, USSR.

** Address: Instytut Fizyki Teoretycznej UW, Hoża 69, 00-681 Warszawa, Poland.

The amplitude $f_1(\omega)$ in the lowest order with respect to the electromagnetic interactions has the following general properties:

(*i*) it is analytic in the complex ω-plane with cuts $(-\infty, -\omega_0)$, $(\omega_0, +\infty)$ along the real axis;

(*ii*) it satisfies the crossing relation

$$f_1^*(\omega) = f_1(-\omega); \tag{4}$$

(*iii*) it satisfies unitarity in the following sense:

$$\operatorname{Im} f_1(\omega) > 0. \tag{5}$$

We assume that

$$|f_1(\omega)| < \text{const}\, \omega^2, \quad \omega \to \pm \infty. \tag{6}$$

These properties allow one to formulate and to investigate various sum rules and inequalities which should be satisfied by the amplitude $f_1(\omega)$ (see e. g. [2–5] and references therein). Moreover, they allow one to write down [1] subtracted dispersion relations and to calculate [6] the real part of the amplitude $f_1(\omega)$ on the basis of experimental data on total cross sections. All the knowledge obtained from these investigations so far refers to the behaviour of $f_1(\omega)$ at real values of ω. No information exists on the forward Compton scattering for complex values of ω, contrary to the present knowledge of the $\pi^{\pm}$p forward scattering [7], $K^{\pm}$p forward scattering [8] and pp, $\bar{\text{p}}$p forward scattering amplitudes [9] in the complex energy plane.

Of special interest for complex values of energy are the curves of constant phase or phase contours [10–12]. In addition to indicating the location of zeros and poles of the amplitude such contours are valuable for studying the general properties of the amplitude. They can be used, for example, to set bounds on high-energy behaviour, test the predictions of various models and indicate the location of resonances.

In this paper we evaluate $f_1(\omega)$ at complex values of energy on the basis of the once-subtracted (at $\omega = 0$) dispersion relation:

$$\operatorname{Re} f_1(\omega) = f_1(0) + \frac{2\omega^2}{\pi} \int_{\omega_0}^{\infty} \frac{\operatorname{Im} f_1(\omega') d\omega'}{\omega'(\omega'^2 - \omega^2)}, \tag{7}$$

where the subtraction constant $f_1(0)$ is well known. According to the low-energy theorem [13, 14] one has

$$f_1(0) = -\alpha/m_N, \tag{8}$$

where $\alpha = 1/137$. We use exactly the same input on $\operatorname{Im} f_1(\omega)$ as used in Ref. [6].

The results (see Fig. 1) are presented in the form of curves of constant phase

$$\Phi \equiv \operatorname{arctg} \frac{\operatorname{Im} f_1(\omega)}{\operatorname{Re} f_1(\omega)} = \text{const}, \tag{9}$$

and constant modulus

$$|f_1(\omega)| = \text{const}, \tag{10}$$

which are mutually orthogonal. These curves which describe the model-independent behaviour of the forward Compton scattering amplitude in the complex energy plane may be useful when studying some models of this amplitude.

In general, phase contours do not intercept except in the points corresponding to the locations of poles or zeros of the amplitude. The amplitude $f_1(\omega)$ has no poles. If one assumes that $\text{Re}\, f_1(\omega_0) < 0$, then one can show [15] that the amplitude $f_1(\omega)$ has no zeros,

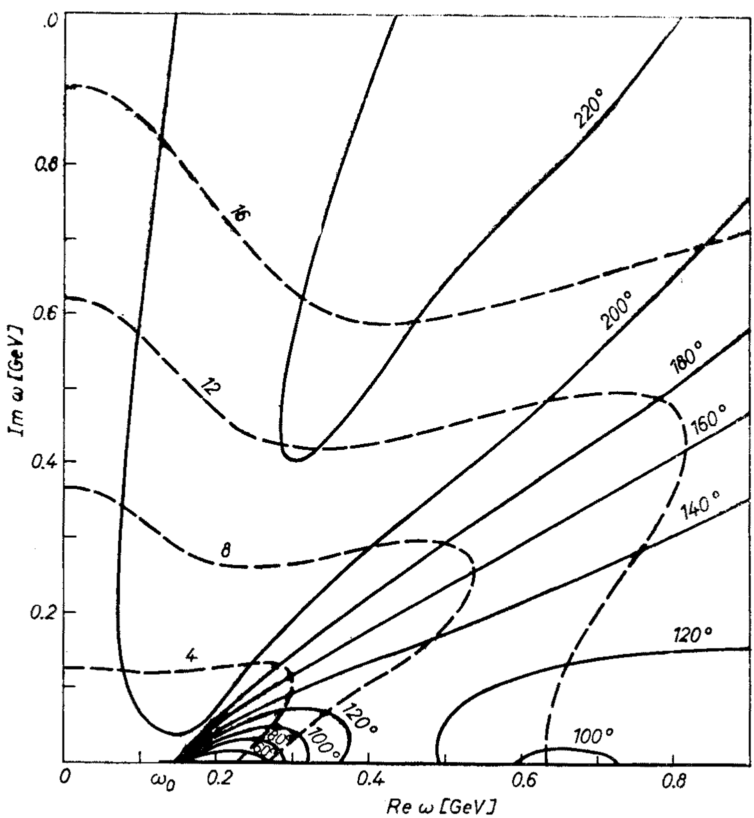

Fig. 1. Curves of constant phase (solid lines) and of constant modulus (dashed lines, fermi units) in the complex energy plane

either, in the complex ω-plane. On the other hand, if we assume that $\text{Re}\, f_1(\omega_0) > 0$, then similar arguments lead to the conclusion that the amplitude $f_1(\omega)$ has two zeros which are located symmetrically on the real axis between the two cuts. Thus the number of zeros in our case is determined solely by the sign of the real part at the threshold. The only information about the sign of $\text{Re}\, f_1(\omega_0)$ nowadays comes from the dispersion relation calculations, because no phase-shift analyses are available at low energies that could determine the sign of $\mathrm{Re}\, f_1(\omega_0)$, i. e. the sign of the scattering length. The dispersion relation calculations [6] predict the negative sign for this quantity and a very small absolute value, which is the consequence of the fact that $\mathrm{Re}\, f_1(\omega)$ goes from positive to negative values, i. e. has a zero just slightly above the threshold.

Hence, the amplitude $f_1(\omega)$ has no poles and no zeros. This, of course, is in agreement with the results shown in Fig. 1: the phase contours do not intercept. The point on the real axis, $\omega \approx 0.15$ GeV, where the phase contours accumulate (they do not cross there, only our scale is too small) is not really a zero but corresponds to the point where only the real part of the amplitude has a zero. If in the future more accurate experimental data on σ_{tot} will be available at low energies and if this will change the dispersion relation prediction for the sign of $\mathrm{Re}\, f_1(\omega_0)$ then, as mentioned, two zeros of the amplitude $f_1(\omega)$ will appear on the real axis.

Knowledge about the number and locations of zeros of the amplitude is very important if one wants to use sum rules and dispersion relations for the inverse amplitude or for the logarithm of the amplitude (see e. g. [2, 16–20]). For example, provided that the amplitude $f_1(\omega)$ has no poles and no zeros the dispersion relation for its logarithm can be written as follows (see e. g. [18]):

$$\Phi(\omega) = \frac{2\omega}{\pi} \mathrm{P} \int_{\omega_0}^{\infty} \frac{\ln |f_1(\omega')| d\omega'}{\sqrt{\omega'^2 - \omega_0^2}\, (\omega'^2 - \omega^2)} . \tag{11}$$

The dispersion relation (11) might be useful for investigating spin effects in γp forward scattering. From Eq. (3) one can see that

$$|f_1(\omega)| = \frac{\omega}{\sqrt{\pi}} \left(\left. \frac{d\sigma}{dt} \right|_{t=0} \right)^{1/2} , \tag{12}$$

if $f_2(\omega)$ is negligible, i. e. if spin effects are negligible. If the phase of $f_1(\omega)$ calculated by means of Eq. (11) would coincide with the phase calculated on the basis of ordinary dispersion relation (7), it would mean absence of spin effects. A discrepancy between the two calculations would indicate the importance of spin effects. Of course, a direct test of the presence of spin effects at each specific energy can be carried out by simply comparing an ordinary dispersion relation prediction for $\left. \frac{d\sigma}{dt} \right|_{t=0}$ with its experimental values [21–23]. However, the use of Eq. (11) has the advantage due to its integral form which smoothes the accidental errors. It would be very interesting to carry out this test practically. For this purpose, one needs the experimental values of $\left. \frac{d\sigma}{dt} \right|_{t=0}$ at many energies which, unfortunately, are not available at present. More experimental data on $\frac{d\sigma}{dt}$ for small momentum transfers would be welcome.

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REFERENCES

[1] M. Gell-Mann, M. L. Goldberger, W. Thirring, *Phys. Rev.* **95**, 1612 (1954).
[2] T. N. Truong, *Phys. Lett.* **31B**, 461 (1970).
[3] M. Creutz, *Phys. Rev.* **D6**, 3533 (1972).
[4] V. Baluni, O. Dumbrajs, *Nucl. Phys.* **B51**, 289 (1973).
[5] I. Guiasu, E. E. Radescu, *Phys. Rev.* **D10**, 357 (1974).
[6] M. Damashek, F. Gilman, *Phys. Rev.* **D1**, 1319 (1970).
[7] S. Jorna, J. A. McClure, *Nucl. Phys.* **B13**, 68 (1969).
[8] O. Dumbrajs, *Nucl. Phys.* **B38**, 600 (1972).
[9] O. Dumbrajs, *Nucl. Phys.* **B46**, 164 (1972).
[10] C. B. Chiu, R. J. Eden, C. I. Tan, *Phys. Rev.* **170**, 1490 (1968).
[11] R. J. Eden, C. I. Tan, *Phys. Rev.* **170**, 1516 (1968).
[12] R. J. Eden, C. I. Tan, *Phys. Rev.* **172**, 1583 (1968).
[13] M. Gell-Mann, M. L. Goldberger, *Phys. Rev.* **96**, 1433 (1954).
[14] F. Low, *Phys. Rev.* **96**, 1428 (1954).
[15] M. Sugawara, A. Tubis, *Phys. Rev.* **130**, 2127 (1963); Yu. S. Vernov, *Teor. Mat. Fiz.* **4**, 3 (1970).
[16] R. Wit, *Acta Phys. Pol.* **28**, 865 (1965).
[17] R. Wit, *Acta Phys. Pol.* **29**, 563 (1966).
[18] R. Odorico, *Nuovo Cimento* **54A**, 96 (1968).
[19] J. A. McClure, L. E. Pitts, *Phys. Rev.* **D5**, 109 (1972).
[20] O. Dumbrajs, M. Staszel, *J. Phys.* **G1**, 172 (1975).
[21] R. L. Anderson, D. Gustavson, J. Johnson, I. Overman, D. Ritson, B. H. Wiik, *Phys. Rev. Lett.* **25**, 1218 (1970).
[22] G. Buschhorn, L. Griegee, L. Dubal, G. Frankl, C. Geweniger, P. Heide, R. Kotthaus, G. Poelz, U. Timm, K. Wegener, H. Werner, M. Wong, W. Zimmermann, *Phys. Lett.* **33B**, 241 (1970).
[23] A. M. Boyarski, D. H. Howard, S. Ecklund, B. Richter, D. Sherden, R. Siemann, C. Sinclair, *Phys. Rev. Lett.* **26**, 1600 (1970).