ANGULAR CORRELATIONS IN MUON CAPTURE INCLUDING LINEAR POLARIZATION OF THE DE-EXCITATION GAMMA--QUANTA

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Angular correlations coefficients are calculated for the nuclear cascade process $j_0(\text{mezoatom}) \overset{\mu^-}{\to} j_1 \overset{\gamma}{\to} j_2$. When introducing explicitly a photon polarization density matrix, a new quantity is distinguished. It is the imaginary part of the muon capture partial amplitudes, providing another check of the T-invariance in the μ-capture. The new quantity could be measured only by means of the linear polarization of the emitted γ-quanta.

The progress of experimental techniques in the nuclear muon capture (cf. Miller's et al. experiment or the Grenacs's et al. measurements [1]) preambles the search for new quantities and kinematical relations between them [2]. Because of the rotational invariance the measurable quantities expressed in terms of the independent weak amplitudes are subject to kinematical restrictions. The information on the muon interaction with the nuclear matter follows from studying of the recoil nuclear polarization states by looking at the angular and polarization correlations of the decay particles. In the present paper we restrict ourselves to the case of the gamma-neutrino angular correlations. Considering μ-capture by the final nucleus in the ground state one may measure only the recoil linear polarization. The reason is that the β-particles from the decay of nucleus have spin 1/2, while the angular distribution of the γ-rays from the excited final nuclear state provides a new information on the alignment.

Our aim is at giving the results of the Popov calculations [3] modified by using a polarization density matrix for the emitted γ-ray in the transition

$$j_0(\text{mezoatom}) \overset{\mu^-}{\to} j_1 \overset{\gamma}{\to} j_2. \tag{1}$$

In our notation j_0(mezoatom) is an initial nucleus of a spin j_0 to which muon is coupled before the capture. The excited (after the muon capture and the subsequent neutrino emission) nuclear state with a spin j_1 comes to the final state with a spin j_2 through the

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γ-emission. The process (1) is discussed with no assumptions on the dynamics of the μ-capture. The convenient multipole amplitude decomposition of the matrix element of the transition operator $\hat{H}_\mu$ for the first step of the cascade (1), $j_0 \xrightarrow{\mu} j_1$, is available [4]

$$\langle h; j_1 m_1 | \hat{H}_\mu | j_0 m_0; \hat{\vec{v}} m\rangle = \frac{1}{\sqrt{3\pi}} \sum_{LM} C^{j_1 m_1}_{j_0 m_0 LM} \hat{L} i^{-L} D^L_{Mm-h}(\Omega_\nu) T^{m-h}_L. \tag{2}$$

In this transition amplitude the spin components m and h of the leptons are along the direction of the neutrino linear momentum $\hat{\vec{v}}$, while for the nuclei spin components the z-axis is the direction of quantization.

The gamma-transition matrix element H_γ is taken such as in [3]

$$H_\gamma = \sum_m C^{j_1 m_1}_{Lm j_2 m_2} b_{L\eta} A_{L\pi} D^L_{m\eta}(\varphi, \vartheta, 0), \tag{3}$$

$$b^{(\mathrm{el})}_{L\eta} = \hat{L}, \quad b^{(\mathrm{mag})}_{L\eta} = \eta \hat{L}, \quad \hat{L} = \sqrt{2L+1}, \quad L = |j_1 - j_2|.$$

Only pure radiation of the multipolarity 2^L is considered.

The angular correlations probability is calculated as the trace of the final polarization density matrix, normalized with due regard to an integration over emitted radiations.

To this end, the amplitudes (2) and (3) have been used and the initial density matrix has been taken from [3] with the quantization axes for the muonic states rotated such as those in (2).

So, the probability W was found to be[1]

$$W = \frac{\sqrt{2}\,(2j_1+1)^2}{3\sqrt{\pi}\,\hat{j}_0} \sum (-1)^{1/2+h-p-f} \hat{f} \hat{p} \hat{r}\, W(sJrg; pf) \begin{Bmatrix} j_0 & j_1 & I \\ j_0 & j_1 & I' \\ p & s & J \end{Bmatrix} B^{gfJ}_{II'h} Q_{pgr} B_{rfs},$$

$$A B^{gfJ}_{II'h} = (-i)^{I+I'} \sum_m \big(C^{J0}_{g0f0} C^{g0J}_{1/2-m\,1/2m} C^{J0}_{Im-hI'h-m} T^{m-h}_I \overset{*}{T}{}^{m-h}_{I'}$$

$$+ C^{J2m}_{g2mf0} C^{g-2m}_{1/2-m\,1/2-m} C^{J2m}_{Im-hI'h+m} T^{m-h}_I \overset{*}{T}{}^{-m-h}_{I'} \big),$$

$$A = \tfrac{2}{3} \frac{2j_1+1}{2j_0+1} \sum_{I,m=\pm h} |T^{m-h}_I|^2,$$

$$Q_{pgr} = 2\sqrt{2\pi}\,\hat{j}_0 \hat{p} \hat{g} \hat{r} \sum_F (-1)^{p+g-r} p_F \hat{F} \begin{Bmatrix} j_0 & F & \frac{1}{2} \\ j_0 & F & \frac{1}{2} \\ p & r & g \end{Bmatrix} \left\{ \delta^0_r + \delta^1_r \frac{\sqrt{F(F+1)}}{3\sqrt{3}}\, a_F \right\}. \tag{4}$$

Here p_F is the probability of occupation of a hf level in the muonic atom with total spin $F = j_0 \pm \frac{1}{2}$,

$$a_{j_0 \pm 1/2} = \pm \frac{3|\vec{P}_\pm|}{j_0 \pm 1 + \frac{1}{2}},$$

$\vec{P}_\pm$ is the residual muon polarization in the hf states.

[1] Henceforth, evident summation over "blind" variables is not indicated.

The only new quantity B_{rfs} appeared in (4), and the others correspond with those introduced in [3, 5]. For the neutrino azimuthal angle in the plane perpendicular to the γ-quanta momentum direction $\hat{\vec{k}}$ is not observed because of experimental obstacles, we have integrated over this angle so that the correlation between neutrino and muon polarization vanishes. Therefore, B_{rfs} looks as follows

$$B_{0ss} = \frac{\hat{s}}{4\pi}\left[\sigma_{s+}^{-1}P_s + \frac{\sigma_{s+}^{1}}{\sqrt{(s+2)(s+1)s(s-1)}}(2\hat{\vec{k}}\cdot\hat{\vec{v}}P'_s - s(s+1)P_s)\right], \tag{5}$$

$$B_{1fs} = \frac{\sqrt{3}}{4\pi}(\hat{\vec{k}}\cdot\vec{P})\left\{\left(\frac{\delta^f_{s+1}}{\sqrt{s+1}} + \frac{\delta^f_{s-1}}{\sqrt{s}}\right)\left[\sigma_{s+}^{-1}(P'_s - \hat{\vec{k}}\cdot\hat{\vec{v}}P'_f)\right.\right.$$

$$\left. + \frac{\sigma_{s+}^{1}}{\sqrt{(s+2)(s+1)s(s-1)}}(4\hat{\vec{k}}\cdot\hat{\vec{v}}P''_s - (s+2)(s-1)P'_s - \hat{\vec{k}}\cdot\hat{\vec{v}}(4\hat{\vec{k}}\cdot\hat{\vec{v}}P''_f - (f+2)(f-1)P'_f))\right]$$

$$\left. + \frac{\delta^f_s 2\hat{s}\sigma^1_{s-}}{s(s+1)\sqrt{(s+2)(s-1)}}[1-(\hat{\vec{k}}\cdot\hat{\vec{v}})^2]P''_s\right\},$$

$$\sigma^k_{s\pm} = (-1)^L(2L+1)W(sLj_1j_2; Lj_1)C^{s1+k}_{L1Lk}(\sigma_{1-k}\pm(-1)\,\sigma_{-1k}).$$

Here $\sigma_{k'k}$ is the polarization density matrix for the photon. It was convenient to put $P_l := P_l(\hat{\vec{k}}\cdot\hat{\vec{v}})$ for the Legendre polynomials and their derivatives.

When considering the capture by spin zero targets, $0 \xrightarrow{\mu} I \xrightarrow{\gamma} j$, the correlations formula reduces to the form

$$W = \frac{\hat{I}}{4\pi}\sum_{s=0}^{2I}\left\{\sigma_{s+}^{-1}[a_s^+P_s + (-1)^{1/2-h}\hat{\vec{k}}\cdot\vec{P}(\hat{\vec{k}}\cdot\hat{\vec{v}}a_s^-P_s + 2\,\mathrm{Re}\,b_s s(P_{s-1} - \hat{\vec{k}}\cdot\hat{\vec{v}}P_s))]\right.$$

$$+ \frac{\sigma^1_{s+}}{\sqrt{(s+2)(s+1)s(s-1)}}[a_s^+(2\hat{\vec{k}}\cdot\hat{\vec{v}}P'_s - s(s+1)P_s) + (-1)^{1/2-h}\hat{\vec{k}}\cdot\vec{P}(a_s^-(2\hat{\vec{k}}\cdot\hat{\vec{v}}P'_s$$

$$-s(s+1)P_s) + 2\,\mathrm{Re}\,b_s(1-(\hat{\vec{k}}\cdot\hat{\vec{v}})^2)(2\hat{\vec{k}}\cdot\hat{\vec{v}}P''_s - (s+2)(s-1)P'_s))]$$

$$\left. + \frac{(-1)^{1/2+h}\sigma^1_{s-}4\,\mathrm{Im}\,b_s}{\sqrt{(s+2)(s+1)s(s-1)}}[1-(\hat{\vec{k}}\cdot\hat{\vec{v}})^2]P''_s\right\},$$

$$a_s^{\pm} = \hat{s}\,\frac{C^{I0}_{I0s0}Ix^2 \pm (I+1)C^{I-2h}_{I-2hs0}}{Ix^2+I+1},$$

$$b_s = (-1)^s\hat{s}\sqrt{\frac{I(I+1)}{s(s+1)}}\,C^{I0}_{I2hs-2h}\frac{xe^{i\varphi}}{Ix^2+I+1},$$

$$\frac{T_I^0}{T_I^{-2h}} = \sqrt{\frac{I}{I+1}}\,xe^{i\varphi}, \qquad x = \sqrt{\frac{I+1}{I}}\left|\frac{T_I^0}{T_I^{-2h}}\right|. \tag{6}$$

The coefficients a_s^+, a_s^- and $2\text{Re}b_s$ for even s are the a_s, α_s and β_s defined in [4], respectively. The new coefficient $2\text{Im}b_s$ contains the imaginary part of the ratio of the multipole amplitudes T_I^0/T_I^{-2h}. This ratio was earlier found in the case of the $0 \xrightarrow{\mu} 1 \xrightarrow{\gamma} 0$ transition for the $h = -1/2$ neutrino [6].

Next we proceed to determine, in general, what are experimental circumstances in which $2\text{Im}b_s$ might be measured. In order to do it one should consider the coefficient σ_{s-}^1. Under the stipulation that the emitted γ-quanta are in the pure polarization states, σ_{s-}^1 differs from zero just for the linearly polarized γ-radiation.

As to the T-invariance in the muon capture, the qualitative conclusion, irrespective of the dynamics, has been arrived at that the T-violation appears for the relative phase $\varphi \neq 0$ in the amplitudes ratio. In other words, if the T-invariance in the muon capture by the nucleus is broken, the quantity $\text{Im}b_s$ will be manifestly observed in angular correlations with linearly polarized γ-quanta.

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