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PHASE RELATIONS IN (d, p) REACTIONS

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The angular distributions in (d, p) reactions with the target nucleus ^{12}C at energies of the incident deuterons up to 20 MeV are studied. The distributions are compared with predictions of the standard DWBA, the sudden approximation, and model calculations within the latter. The study supports the notion to subject the sudden approximation to some changes in its treatment of the deuteron break-up.

1. *Introduction*

While in the old days the PWBA [1] enjoyed considerable success, in the course of time the DWBA [2] took the lead. In spite of the improvements it brought about, some changes seemed to be necessary, at least as far as the description of (d, p) reactions was concerned [3–6]. The case considered here is the application of the sudden approximation [5, 6] by Butler et al. [6] (referred to as BHMM). It seems to be capable of giving a slightly better account of experiment than the standard DWBA, when reactions involving light targets, high Q-values and higher energies of the incident projectile are viewed [6–10]. Its predictions are very poor for energies below the Coulomb barrier [11]. In recent contributions it has been suggested to subject the treatment of the deuteron break-up in the BHMM to some amendments in order to provide a better correspondence to experiment [12].

To supplement our knowledge of the BHMM the present study of the phase relations of the angular distributions for the (d, p) cross-sections has been performed. To a certain

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extent its spirit is in line with an early work of Hamburger [13] considering (d, p) reactions with the target nuclei ^{12}C and ^{16}O, covering an energy range from about 2 MeV to 20 MeV. The model discussed there was the PWBA. We restrict ourselves to the nucleus ^{12}C and the same energy range, considering the conventional DWBA, the sudden approximation, as applied by Butler et al., and model calculations within the latter.

Our intention is not to obtain good fits to experiment, but to observe the changes in the structure of $d\sigma/d\Omega$ as the energy varies. Thus, it is hoped to get a better feeling of the reaction mechanism and the reasons why the BHMM does not work better. As far as the model calculations are concerned it is only intended to get a zeroth order estimate of whether the recommended alterations [12] really can bring about the expected changes. After recalling the formulae employed, the optical model parameters used are introduced, followed by a discussion of the angular distributions, which is succeeded by a short summary.

2. *Formalism*

As in the original BHMM the starting point for our considerations is the matrix element

$$T_{fi} \simeq \langle F_n S^{1/2} \psi_p^- | V_{np} | \psi_d^+ \rangle \tag{1}$$

which corresponds to what is used in the standard DWBA. S is the usual spectroscopic amplitude, F_n — the bound-state wavefunction for the transferred neutron in the residual nucleus, ψ_p^- denotes the wavefunctions for the proton scattered by the residual nucleus while ψ^+ is taken for the incident deuteron. V_{np} stands for the interaction between the neutron and proton in the deuteron. In the simple DWBA distorted waves are to be inserted for the deuteron and proton wavefunctions while the PWBA requires plane waves instead. The BHMM will be seen to be intermediate between the two.

A set of wavefunctions $\Psi^+ \sim F_n^+(S)^{1/2}\psi_p^+$, which Butler et al. assert to be complete is inserted into the matrix element to yield after some algebra and a few approximations believed to be of minor importance [6, 9, 12]

$$T_{fi} \simeq \int dk_n^+ \int dk_p^+ \langle F_n | \psi_n^+ \rangle \langle \psi_p^- | \psi_p^+ \rangle t_{np,d} \tag{2}$$

with

$$t_{np,d} = \langle \psi_n^+ \psi_p^+ | V_{np} | \psi^+ \rangle. \tag{3}$$

Into the first two brackets of Eq. (2) the proponents of the BHMM insert distorted waves for the continuum wavefunctions as done in the DWBA. In $t_{np,d}$ plane waves are used what is said to be equivalent to the application of the sudden or impulse approximation [6]. Thus a δ-function in the neutron momenta is achieved reducing the dimension of the integration by three. This procedure seems to be very suggestive though there are some doubts about its validity to the extent claimed by Butler et al., [9, 12]. Since distorted and plane waves are employed, it is understood that the model should display features inherent to both of them.

The form of $t_{\mathrm{np,d}}$ chosen by Butler et al. implies a loss of spatial information manifesting itself in the spatially constant term achieved for this expression. As criticized earlier [10, 12], this is not quite in accordance with the physical picture of a surface reaction . To heal this deficiency, at least partially, an appropriately normalized weighting factor f has to be inserted into the radial integral resulting from the neutron overlap [12].

$$J_{\mathrm{n}} = \int dr\, u_{\mathrm{n}} v_{\mathrm{n}} \rightarrow \int dr\, u_{\mathrm{n}} v_{\mathrm{n}} f(R, \theta, E_{\mathrm{d}}, E_{\mathrm{p}}). \tag{4}$$

u_{n} and v_{n} stand for the free and bound neutron wavefunctions, respectively. To obtain a first estimate of the order of magnitude of the introduced corrections, the Schrödinger equation was solved for a particle with energy E moving in a constant complex potential $V = V_0 + iW_0$. The probability of finding the particle somewhere within the range of the potential decreases exponentially due to the absorption caused by the imaginary part of the potential.

$$f_0(R) \simeq \exp\left[-R\frac{2}{h}\left(M\sqrt{|W_0|^2+(E+|V_0|)^2} - M(E+|V_0|)\right)^{1/2}\right]. \tag{5}$$

R, V_0 and W_0 were taken to correspond to the optical model parameters used in the entrance channel.

3. *Discussion of the cross-sections*

The optical model parameters required by the conventional DWBA are the ones for the deuteron and proton, while the proton and neutron parameters are needed for the BHMM. Four our purpose the energy dependent average parameters suggested by Rosen et al. [15] and by Satchler [16] were found to be sufficient. In order to observe the structure of the angular distributions distinctly, the DWBA calculations were performed with $V_{\mathrm{so}} = 0$ MeV, and the BHMM was applied using a simple form neglecting the (proton) spin-dependent distortions

$$d\sigma/d\Omega \sim \sum_{m=-l_n}^{l_n} \left|\sum_{l=0}^{\infty} \eta_l Y_{lm}(\theta) I_l^m\right|^2, \tag{6}$$

where l_{n} is the angular momentum of the transferred neutron, η_l results from the combined nuclear and Coulomb phase shifts, while the other terms contain the angular and radial dependences of the nuclear matrix element [6]. The use of the appropriate V_{so} and of the complete form of $d\sigma/d\Omega$ has basically the effect of smoothing out the structure, having little influence on the overall behaviour of the curves. The optical model parameters were designed for such an energy range that they cannot be expected to yield a reasonable correspondence to experiment for energies of the incident projectile below 3 MeV. In addition to that, because of the nature of the sudden approximation, it is not reasonable to apply it to reactions at energies of the order of the deuteron binding energy. In spite of these obvious restrictions the lowest energy chosen is $E = 1.86$ MeV, where the influence of compound nucleus formation is quite significant.

Unfortunately the experimental data at low energies do not extend to large angles while for higher energies there are not always sufficient data in the vicinity of the stripping peak, cf. Fig. 1. Extrapolation from experiments performed at higher energies [14] indicates that the maxima and minima are gradually shifted towards smaller angles, the curves getting as well steeper in descend, a trend which is already apparent in the limited energy range considered here. In the intermediate angular region the changes are of a different nature; for low energies there is only one minimum while for intermediate energies a pronounced structure is observed which becomes smoother for higher energies.

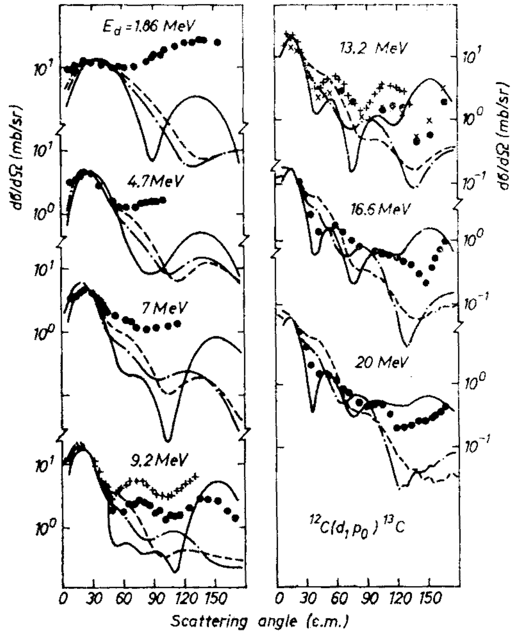

Fig. 1. Angular distributions. Full curves correspond to the standard DWBA, dashed and dashed-dotted curves to the original BHMM and to the model calculations, respectively. The experimental data — some of which had to be renormalized in magnitude — stem from the following references given in the order of increasing energy [17]; [18]; [18]; [18, 19]; [18, 19, 20]; [20]; [20]

It is the first maximum or — if not measured — at least its slope towards larger angles which is chosen for the normalization of the theory to the experiment. The underlying philosophy is that in this region we have the best knowledge of what is going on in the reaction. Another fix-point which lends itself to closer inspection is the last maximum/minimum which, besides being shifted towards smaller angles, is gradually decreasing and getting less pronounced with increasing energy.

As it was expected, the correspondence of all the three theoretical curves to experiment is very poor for 1.86 MeV. The sudden approximation gives only an indication of the location of the stripping peak, while the DWBA provides as well a qualitatively correct estimate for the maximum in the backward hemisphere. For the next energy the situation is very similar, the sudden approximation doing at least some attempt to reproduce the last maximum. At 7 MeV all four curves are in phase with each other the sudden approximation doing not worse than the DWBA. For $E = 9$ MeV the last maximum of the sudden approximation is out of phase while the DWBA is in correspondence to the experiment for large and small angles, the situation being worse for intermediate angles. In the next two cases the use of distorted waves yields reasonable results for small and intermediate angles, this time loosing phase in the very backward region; the BHMM gives the appropriate phase for small and large angles failing in the intermediate region. For the highest energy considered, 20 MeV, these relations are almost unchanged, the DWBA improving slightly for large angles.

In the energy range considered, all the models reproduce the stripping peak at least qualitatively (which in the case of the conventional DWBA is not necessarily true for higher energies [8, 10]). Because the BHMM ignores the second maximum, it is not always capable of predicting the proper intermediate structure. The DWBA, on the other hand, seems to be able to account for the intermediate structure while the location of the last maximum remains almost unchanged in the energy range investigated. For higher energies it broadens, making this shortcoming less obvious, cf. the case with 20 MeV.

It would be very optimistic, indeed, to say that the model calculations within the BHMM yield better results than the original theory. However, it is seen that the changes are very small in the vicinity of the stripping peak where the original version does not do bad either. For the very backward angles the alterations are more obvious, almost healing the loss of phase at 9 MeV. In the intermediate region the changes are quite strong considering the crude picture put in.

4. *Summary*

From the foregoing it is seen that the conventional DWBA and the BHMM fail in different ways making it necessary to reconsider their original forms [3, 12]. As indicated in the second reference the deviations of the BHMM from the DWBA are attributed mainly to the use of plane waves in the evaluation of $t_{\mathrm{np,d}}$, Eq. (3). Following the interpretation that the DWBA underestimates the break-up of the deuterons, while the sudden approximation overestimates it [8, 12], a consideration of the above discussion and the results of a PWBA analysis [13] suggests that the plane waves help the BHMM to reproduce the phases (but for the second peak — a feature not found in the other works, e.g. [6, 9]). That it does not account for the fine structure in the intermediate angular region is readily understood since plane waves correspond to high energies, where this structure is less pronounced, or to spatial distances far away from the scatterer where its structure cannot yet be "observed" properly.

The model calculations within the BHMM show that even a zeroth order estimate induces considerable changes which cannot be said to worsen the correspondence to experiment. At least for 9 MeV they give rise to a definite improvement. Thus some support can be given for the idea that the suggested modifications of the BHMM [12] really can bring about significant improvements. Naturally, a final verdict has to be left to a comparison of the appropriately modified sudden approximation to experiment and to the standard DWBA.

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REFERENCES

[1] S. T. Butler, *Proc. Roy. Soc.* (London) **A208**, 559 (1951).

[2] W. Tobocman, *Phys. Rev.* **115**, 99 (1959).

[3] L. J. B. Goldfarb, *Proc. Third Intern. Symp. on Pol. Phen.*, eds. H. H. Barschall and W. Haeberli, University of Wisconsin Press, Madison 1971, p. 205.

[4] C. A. Pearson, M. Coz, *Nucl. Phys.* **82**, 533, 545 (1966); S. T. Rihan, *Phys. Rev.* **164**, 1247 (1967); R. C. Johnson, P. J. R. Soper, *Phys. Rev.* **C1**, 976 (1970); J. D. Harvey, R. C. Johnson, *Phys. Rev.* **C3**, 636 (1971).

[5] M. Tanifuji, *Nucl. Phys.* **58**, 81 (1964).

[6] S. T. Butler, R. G. L. Hewitt, B. H. J. McKellar, R. M. May, *Ann. Phys.* **43**, 282 (1967).

[7] S. T. Butler, R. G. L. Hewitt, J. S. Truelove, *Phys. Rev.* **162**, 1062 (1967); K. King, B. H. J. McKellar, *Aust. J. Phys.* **23**, 453, 641 (1970); K. King, B. H. J. McKellar, *Phys. Rev.* **C9**, 1309 (1974); G. M. Hudson, G. B. Crinean, D. T. Kelly, B. M. Spicer, *Nucl. Phys.* **A184**, 175 (1972); G. B. Crinean, G. M. Hudson, L. W. J. Wild, B. M. Spicer, *Nucl. Phys.* **A244**, 77 (1975).

[8] M. Tanifuji, H. Noya, *Prog. Theor. Phys.* **50**, 515 (1973).

[9] T. F. Baker, L. J. B. Goldfarb, *Nucl. Phys.* **A146**, 577 (1970).

[10] L. J. B. Goldfarb, E. F. Hefter, *Phys. Lett.* **38B**, 379 (1972).

[11] T. F. Baker, P. J. A. Buttle, L. J. B. Goldfarb, *Phys. Lett.* **27B**, 348 (1968).

[12] K. A. Gridnev, E. F. Hefter, *Prog. Theor. Phys.* **52**, 1707 (1974); K. A. Gridnev, E. F. Hefter, *Z. Phys.* **A273**, 99 (1975).

[13] E. W. Hamburger, *Phys. Rev.* **123**, 619 (1961).

[14] K. Sattler, Ch. Weddigen, priv. communication; K. Sattler, thesis, Universität and Kernforschungszentrum Karlsruhe, 1971.

[15] L. Rosen, J. G. Beery, A. S. Goldhaber, *Ann. Phys.* **34**, 96 (1965).

[16] G. R. Satchler, *Nucl. Phys.* **85**, 273 (1966).

[17] M. T. McEllistrem, K. W. Jones, K. Chiba, R.-A. Douglas, D. F. Herring, E. A. Silverstein, *Phys. Rev.* **104**, 1008 (1956).

[18] N. I. Zaika, O. F. Nemets, M. A. Zerino, *Zh. Eksp. Teor. Fiz.* **39**, 3 (1960).

[19] F. Baldeweg, V. Bredel, H. Guratzsch, K.-H. Hermann, R. Klabes, B. Kühn, G. Stiller, S. Tesch, Report, ZFK-173 (1969).

[20] S. Morita, N. Kawai, N. Takano, Y. Gotô, R. Hanada, Y. Nakajima, S. Takemoto, Y. Yaegashi, *J. Phys. Soc. Jap.* **15**, 550 (1960).