THE UNIVERSAL IMPACT PARAMETER HYPOTHESIS AND THE QUARK MODEL

By J. Szwed

Institute of Physics, Jagellonian University, Cracow*

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The introduction of b-universality into the additive quark model reduces the number of nonvanishing helicity amplitudes and offers an exact form of t-dependence for all amplitudes. Strong constraints on the additivity frame are obtained. Comparison with the data is made for the reaction $\pi^+p \to \pi^0\Delta^{++}$. The obtained results rule out the Gottfried-Jackson frame as the additivity frame.

1. *Introduction*

Recently a scheme describing the momentum transfer dependence of two-body helicity amplitudes has been proposed and shown to agree very well with the data [1, 2]. It states that the s-channel helicity amplitudes for two-body reactions with fixed impact parameter b are linearly related to each other (the b-universality). The assumed structure in the impact parameter space implies derivative relations between helicity amplitudes with different net helicity flip $n = |\lambda_i - \lambda_f|$, so that one is left with only one independent amlitude. It is interesting to investigate the connections of this hypothesis with the additive quark model, which also provides us in many cases with relations between the helicity amplitudes.

In this paper we combine the b-universality with the additive quark model and apply them to the reaction

$$\pi^+ p \to \pi^0 \Delta^{++}. \tag{1.1}$$

We solve the resulting equations for the helicity amplitudes and check if they are consistent with the data. Our conclusions can be formulated as follows:

a) the constraints following from the quark model and the derivative relations are consistent with each other and with the existing data,

b) the s-channel helicity system is a possible additivity system for the reaction (1.1),

c) the Gottfried-Jackson frame is ruled out as the additivity frame for the reaction (1.1) except possibly for very small momentum transfer.

* Address: Instytut Fizyki UJ, Reymonta 4, 30-059 Kraków, Poland.

The plan of the paper is as follows. In Section 2 and 3 we shortly discuss the universal impact parameter hypothesis and the additivity assumption in the quark model. The predictions are given and compared with data in Section 4. Our conclusions and summary close the paper.

2. *Universal impact parameter hypothesis*

The amplitude analyses of the two-body reactions [5] suggest that the s-channel helicity amplitudes are the most suitable for studying the t-dependence of a process. The shape of the amplitudes can be classified according to the total net helicity flip $n = |\lambda_a - \lambda_b - \lambda_c + \lambda_d|$ [6]. One of the prominent features of the amplitudes is that the position of the zeros is independent on t and s. This strongly suggests the geometrical origin of the observed structure and the impact parameter representation as the most convenient description. In fact, most of the zeros coincide with the first zeros of the Bessel function $J_n(R\sqrt{-t'})$ with R equal to 1 fermi.

The above mentioned properties of the amplitude can be derived from the following assumption [1]

$$\tilde{M}_n(s, b) = g_n(s) b^n f(s, b), \tag{2.1}$$

where $\tilde{M}_n(s, b)$ is the profile function.

It means that all the amplitudes have the behaviour defined by the same function $f(s, b)$. In formula (2.1) b^n is a kinematical factor and $g_n(s)$ is a complex function of the incident energy. By expressing the amplitude in terms of the profile function

$$M_n(s, t) = 2q^2 \int_0^\infty b db J_n(b\sqrt{-t}) \tilde{M}_n(s, b) \tag{2.2}$$

($t' = t - t_{\min}$, q — c. m. momentum), we get another form of the b-universality hypothesis, namely the derivative relations

$$M_n(s, t) = C_{n-1}(s) \sqrt{-t}^{\,n} \left(\frac{1}{\sqrt{-t}} \frac{\partial}{\partial \sqrt{-t}} \right)^n M_{n=0}(s, t). \tag{2.3}$$

These relations have been obtained from the dual peripheral model by Schrempp and Schrempp [2]. The dual peripheral model suggests that this hypothesis is valid for $t \neq 0$ and $s \gg 0$. It should be noted that the derivative relations lead to a wrong analytic structure in the amplitudes. If $M_n(s, t)$ had a simple pole, all amplitudes with $n' > n$ would get poles of a higher order. This is the reason why relations (2.3) disagree with the data of the π exchange reactions. A way to avoid these troubles is to formulate the hypothesis in terms of singularity-free amplitudes [7] for the π exchange reactions

$$M'_n(s, t) = (t - \mu^2) M_n(s, t), \tag{2.4}$$

where μ is the mass of the exchanged object.

The b-universality was checked in model independent tests [2] and applied to particular models [1, 7, 8]. In both cases good agreement with the data was found.

3. Additive quark model and additivity frame

The additivity assumption in the quark model tells us that there is only one quark in the particle which interacts during the scattering. The remaining quarks are called spectators and do not change their spin states. Obviously, this statement depends strongly on the reference frame, in particular on the direction of the spin quantization axes of the quarks or particles. The choice of this frame which is called the additivity frame is of a great importance in the quark model computations of resonance decays and in the amplitude analysis of the few body reactions. We recall the exact definition of the additivity frame and at the same time the formulation of the additivity assumption after Ref. [3]:

There exists a spin reference frame (called the additivity frame) in which the spin states of the spectators quarks do not change during the collision.

Assuming that the spin quantization axis are n the reaction plane, the additive amplitudes for a two-body process can be expressed in terms of s-channel helicity amplitudes

$$M^{\text{add}}_{\lambda_c\lambda_d\lambda_a\lambda_b}(s,t) = \sum_{\substack{\lambda'_a\lambda'_b\\ \lambda'_c\lambda'_d}} d^{s_a}_{\lambda_a\lambda'_a}(\vartheta_a)d^{s_b}_{\lambda_b\lambda'_b}(\vartheta_b)$$

$$d^{s_a}_{\lambda_c\lambda'_c}(\vartheta_c)d^{s_d}_{\lambda_d\lambda'_d}(\vartheta_d)M^{s}_{\lambda'_c\lambda'_d\lambda'_a\lambda'_b}(s,t). \tag{3.1}$$

The angles ϑ_a, ϑ_b, ϑ_c and ϑ_d are called additivity angles.

Heuristic arguments were given [3, 4] that the additivity frame is identical with the Gottfried-Jackson frame.

In the following we limit ourselves to the reaction

$$0^-\tfrac{1}{2}^+ \to 0^-\tfrac{3}{2}^+, \tag{3.2}$$

where the quark model gives strong predictions. In particular we consider the process

$$\pi^+\text{p} \to \pi^0\Delta^{++}, \tag{3.3}$$

where the derivative relations and the quark model were tested and showed good agreement with data [8, 9].

4. Quark model and derivative reactions for the process $\pi^+p \to \pi^0\Delta^{++}$

We write explicitly the derivative relations for the reaction $\pi^+\text{p} \to \pi^0\Delta^{++}$:

$$D_0(s)M^s_1(s,t) = \partial/\partial\sqrt{-t}\, M^s_0(s,t), \tag{4.1}$$

$$M^s_2(s,t) = D_1(s)\sqrt{-t}\,\partial/\partial\sqrt{-t}\,(M^s_1(s,t)/\sqrt{-t}), \tag{4.2a}$$

$$M^s_2(s,t) = D_2(s)\,(\sqrt{-t})^2(1/\sqrt{-t}\,\partial/\partial\sqrt{-t})^2M^s_0(s,t). \tag{4.2b}$$

Thus we are left with only one independent amplitude, say $M_0^s(s, t)$, the remaining ones being related to it by Eq. (4.1) and (4.2). We encounter a similar situation in the additive quark model. The relations in the additivity frame are as follows:

$$\sqrt{3}\, M^{\text{add}}_{1/2\,-1/2}(s, t) = M^{\text{add}}_{3/2\;1/2}(s, t), \tag{4.3}$$

$$M^{\text{add}}_{3/2\,-1/2}(s, t) = 0, \tag{4.4}$$

$$M^{\text{add}}_{1/2\;1/2}(s, t) = 0. \tag{4.5}$$

Transforming them to the s-channel helicity basis we obtain

$$\sqrt{3}\, M^{s}_{1/2\,-1/2}(s, t) = M^{s}_{3/2\;1/2}(s, t), \tag{4.6}$$

$$-\sqrt{3}\, M^{s}_{1/2\;1/2}(s, t) = M^{s}_{3/2\,-1/2}(s, t), \tag{4.7}$$

$$\sin\left(\frac{\vartheta_{\mathrm{p}}-\vartheta_{\Delta}}{2}\right) M^{s}_{1/2\,-1/2}(s, t) = -\cos\left(\frac{\vartheta_{\mathrm{p}}-\vartheta_{\Delta}}{2}\right) M^{s}_{1/2\;1/2}(s, t), \tag{4.8}$$

where ϑ_p, ϑ_Δ are the additivity angles for proton and Δ respectively (if the additivity frame is the Gottfried-Jackson frame, these angles are well known [10]). We see in Eq. (4.6) that the amplitudes with equal net helicity flip have indeed the same t-dependence, as postulated by the derivative relations.

Let us now discuss the consequences of the conditions (4.1), (4.2) and (4.6)–(4.8). One can check that they admit a degenerate solution when some of the helicity amplitudes identicaly vanish in the whole t-region. There are two such cases and we consider them below

Ia.

$$M_0^s(s, t) \equiv 0 \quad \text{or} \quad M_2^s(s, t) \equiv 0. \tag{4.9}$$

From Eq. (4.7) it follows that they have to vanish simultaneously

$$M_0^s(s, t) = M_2^s(s, t) \equiv 0. \tag{4.10}$$

The remaining amplitude, i. e. $M_1^s(s, t)$, has to be different from zero. Consequently we have (see Eqs (4.1), (4.2) and (4.8))

$$D_0(s) = D_1(s) = 0, \tag{4.11}$$

$$\vartheta_{\mathrm{p}}-\vartheta_{\Delta} = 0, 2\pi, 4\pi, \dots . \tag{4.12}$$

The last relations imply that $M_1^s(s, t)$ is not bounded by any conditions. Thus we can obtain agreement with the data. It is worth noting that the additivity frame, being restricted by Eq. (4.12), could be identical with the s-channel helicity frame (where $\vartheta_p-\vartheta_\Delta = 0$).

Ib. Another degenerate solution is

$$M_1^s(s, t) \equiv 0. \tag{4.13}$$

Similarly we obtain

$$\frac{1}{D_0(s)} = 0, \quad \frac{1}{D_1(s)} = 0, \tag{4.14}$$

$$\vartheta_p - \vartheta_\Delta = \pi, 3\pi, 5\pi, \ldots .$$

Combining (4.2b) and (4.7) we determine the shape of the non-vanishing amplitudes $M_0^s(s, t)$ and $M_2^s(s, t) = -\sqrt{3}\, M_2^s(s, t)$ from the equation

$$-\sqrt{3}\, M_0^s(s, t) = D_2(s)\partial^2 M_0^s(s, t)/\partial \sqrt{-t}^{\,2} - \frac{D_2(s)}{\sqrt{-t}}\, \partial M_0(s, t)/\partial \sqrt{-t}\,. \tag{4.15}$$

The solution of the above equation is

$$M_0^s(s, t) = M_0^s(s, t_0)\sqrt{-t}\, l_1\left(\frac{\sqrt[4]{3}}{\sqrt{D_2}}\sqrt{-t}\right), \tag{4.16}$$

where $l_k(x)$ is any linear combination of the Bessel functions $J_k(x)$ and $Y_k(x)$.

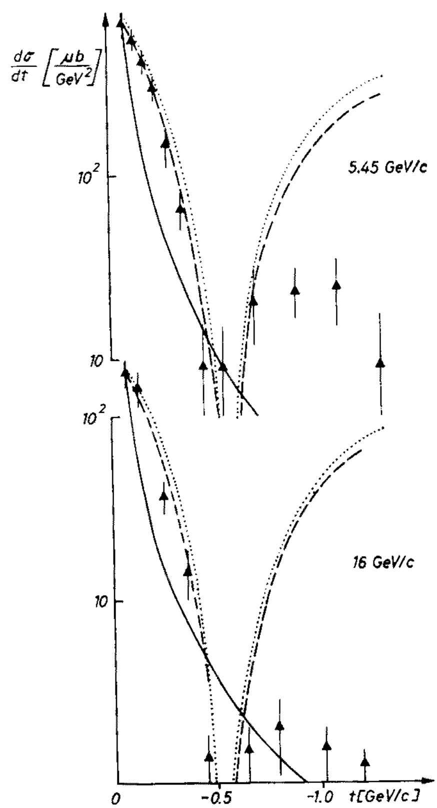

Fig. 1. Differential cross section $d\sigma/dt$ for $\pi^+ p \to \pi^0 \Delta^{++}$ at 16 and 5.45 GeV/c. Solid line — the fit when Jackson angles are used. Dashed line — the fit when the solution of Eq. (5.3) is used. Dotted line — the fit with $M_1^s(s, t)$ vanishing

To see if the obtained amplitudes describe correctly the data we have compared in Fig. 1 the differential cross-section $d\sigma/dt$ with the predicted curve. The free parameters of the fit are $D_2(s)$, one complex parameter α defined by

$$l_k(x) = J_k(x)+\alpha Y_k(x)$$

and $M_0^s(s, t_0)$ — the value of $M_0^s(s, t)$ at a given t_0. We see that agreement is obtained only for small t.

II. In the search for other solutions we assume that none of the amplitudes vanishes in the whole t-region. In addition to (4.16) we get from (4.1) and (4.16)

$$M_0^s(s, t) = M_0^s(s, t_0)\sqrt{-t}\, l_1\left(\frac{\sqrt[4]{3}}{\sqrt{D_2}}\sqrt{-t}\right),$$

$$|M_0^s(s, t)| = |M_0^s(s, t_0)| \exp\left\{-D_0 \int\limits_{\sqrt{-t_0}}^{\sqrt{-t}} \cot\left(\frac{\vartheta_p-\vartheta_\Delta}{2}\right) d\sqrt{-t'}\right\}$$

$$M_1^s(s, t) = M_0^s(s, t_0)\frac{\sqrt[4]{3}\sqrt{-t}}{D_0\sqrt{D_2}}\, l_0\left(\frac{\sqrt[4]{3}}{\sqrt{D_2}}\sqrt{-t}\right), \tag{4.17}$$

$$|M_1(s, t)| = |M_0(s, t_0)| \left|\cot\left(\frac{\vartheta_p-\vartheta_\Delta}{2}\right)\right| \exp\left\{-D_0 \int\limits_{\sqrt{-t_0}}^{\sqrt{-t}} \cot\left(\frac{\vartheta_p-\vartheta_\Delta}{2}\right) d\sqrt{-t'}\right\}. \tag{4.18}$$

The additivity angles are now expressed through the Bessel functions

$$\cot\left(\frac{\vartheta_p-\vartheta_\Delta}{2}\right) = -\frac{\sqrt[4]{3}}{\sqrt{D_2}\,D_0}\,\frac{l_0\left(\frac{\sqrt[4]{3}}{\sqrt{D_2}}\sqrt{-t}\right)}{l_1\left(\frac{\sqrt[4]{3}}{\sqrt{D_2}}\sqrt{-t}\right)}. \tag{4.19}$$

From (4.8) and (4.2) we obtain an important conclusion that the unknown functions of c. m. energy $D_0(s)$, $D_1(s)$ and $D_2(s)$ are real. This is shown to be well satisfied also in other two-body reactions [2].

It is seen from Eq. (4.17) and (4.18) that the t-dependence of the amplitudes for the process $\pi p \to \pi\Delta$ is uniquely determined by the t-dependence of the additivity angles $\vartheta_p-\vartheta_\Delta$. Thus we obtain again a possibility of testing different hypothesis concerning the additivity frame.

The first observation one can make in this context is that the Jackson angles do not satisfy Eq. (4.19). To see how important is this effect in the data, we have plotted in Fig. 1 the predictions for differential cross-sections, assuming that the additivity frame coincides with the Gottfried-Jackson frame. In this case all the amplitudes can be obtained in terms of two free parameters: $D_0(s)$ — defined in Eq. (4.1) and $M_0^s(s, t_0)$ — the value of the non-spin flip amplitude at a given t_0.

It is seen in Fig. 1 that the obtained curve does not describe the data. In particular, it does not follow a characteristic dip-dump structure at $t = -0.6$ GeV^2.

The next problem is if one can find the correct t-dependence of the amplitudes $M_0^s(s, t)$, $M_1^s(s, t)$ and $M_2^s(s, t)$ using Eqs (4.17) and (4.18) under the condition (4.19) i. e. if the very existence of the additivity frame (different from that pointed out in case Ia where $M_0^s(s, t)$ and $M_2^s(s, t)$ vanish) is compatible with the b-universality. Thus we attempt to find the solution which would be compatible with the existing data. Unfortunately we do not have results of amplitude analysis in the high energy process $\pi^+ p - \pi^0 \Delta^{++}$. Thus, we use the differential cross-section $d\sigma/dt$ at $p_{lab} = 5.45$ and 16 GeV/c [11, 12]. The expression for $d\sigma/dt$ contains four free parameters: $M_0^s(s, t_0)$ — the value of the non-spin flip amplitude at a given t_0, $D_0(s)$ and $D_2(s)$ — the constants relating different helicity

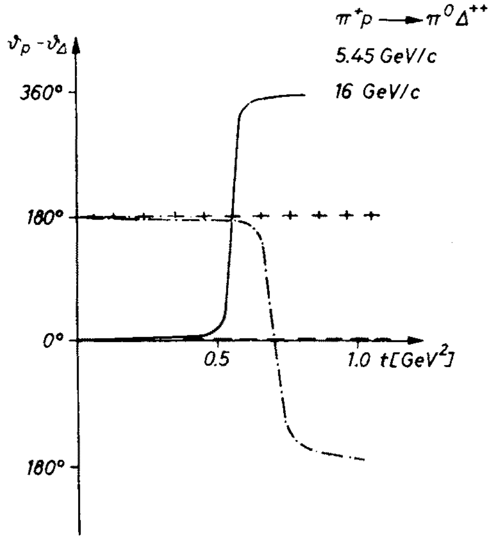

Fig. 2. The additivity angles obtained from Eq. (4.19) and the fit to the data for 5.45 and 16 GeV/c. Dashed lines are the degenerate solution Ia and Ib. (——— $M_1^s(s, t)$ dominating, —·—·— $M_0^s(s, t)$ and $M_2^s(s, t)$ dominating)

amplitudes and α — a real constant from the linear combination of the Bessel functions. The results (Fig. 1) show satisfactory agreement only in the range $|t| < 0.7$ GeV^2 $\left(\text{including the minimum which is caused by the change of sign of } \cot\left(\frac{\vartheta_p - \vartheta_\Delta}{2}\right)\right)$. We obtain nearly the same curve for two sets of parameters. The first one (D_0 small) is near the limiting case where $M_1^s(s, t)$ dominates, in the second (D_0 large) $M_0^s(s, t)$ and $M_2^s(s, t)$ dominate. Although in the first case we are near the limit where one amplitude is unbounded, the fits are wrong for $|t| > 0.7$ GeV^2. This proves that the solution with $M_0^s(s, t)$ and $M_2^s(s, t)$ vanishing is very unstable.

Fig. 2 displays the t-dependence of the additivity angles as obtained from the fit (there is no dependence on the incident energy). We have plotted both above-mentioned cases with their limiting values.

Concluding, the results prove that using Eq. (4.17) – (4.18) and Eq. (4.19) as the constraint for the additivity angles it is impossible to obtain good agreement in a wide momentum transfer range.

5. *Conclusions*

In this paper we have combined two different models of two-body processes: the b-universality and the additive quark model. Used together, they provide a powerful tool to analyze their assumptions, especially that of the additivity frame in the quark model. Our conclusions in the case of the reaction $\pi^+p - \pi^0\Delta^{++}$ could be summarized as follows:

1) Agreement with the data could be obtained when two amplitudes vanish. In this case we make exact predictions for the additivity frame. In particular, the s-channel helicity frame is an acceptable one. The contributions to the vanishing amplitudes should thus belong to the nonadditive component.

2) Assuming that none of the amplitudes vanishes in the whole t-range we get an explicit form of all helicity amplitudes. They turn out to depend on the momentum transfer only through the difference of the additivity angles, a formula for the additivity angles (Eq. (4.19)).

3) The arbitrary functions of c. m. energy $D_0(s)$, $D_1(s)$ and $D_2(s)$ relating the helicity amplitudes in the b-universality picture have to be real.

4) The Jackson angles do not fulfill Eq. (4.19) and do not fit the data. This eliminates the Gottfried-Jackson frame as the additivity frame.

5) The agreement with the data cannot be obtained when applying Eqs (4.17 – (4.19)) except for a narrow t-range. In other words, in the case where none of the amplitudes vanishes there is no additivity frame which ensures agreement with the data in the region of the second maximum.

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