UNITARIZATION OF INDEPENDENT CLUSTER EMISSION MODEL WITH ENERGY AND LONGITUDINAL MOMENTUM CONSERVATION

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The independent cluster emission model is unitarized by means of the Lippman-Schwinger equation. We study two possible generalizations of this model to processes involving the absorption of clusters. The first one (a) is of the type discussed by Auerbach, Aviv, Sugar and Blankenbecler, the second (b) is based on the generalization of Białas and Czyż. In case (a) we show that the introduction of energy and longitudinal momentum conservation does not change the general conclusions of the AASB paper — the leading singularity lies at $J = 1-\lambda/2$, λ being the rapidity density of clusters produced by the input potential. In case (b) we show that diffractive production exists and is a direct consequence of energy and longitudinal momentum conservation.

1. *Introduction*

It is well known that the independent cluster emission model (ICEM) describes the general properties of nondiffractive multiparticle production processes. However, it does not take inelastic diffraction into account. The inelastic diffraction is believed to be a shadow effect, due to the absorption of incident and outgoing hadron waves (Good and Walker, 1960) which can be calculated by a unitarization procedure. One is tempted to find out if ICEM is able to describe this type of hadronic processes when properly unitarized. Such an attempt has been made by Białas and Kotański (1973) who have calculated only the first unitarity correction and neglected all higher order terms (that is, shadow of shadow and so on...). They found that the diffractive production amplitude appears and its energy dependence is similar to that of elastic scattering. However, in view of the large coupling constant, it is natural to expect that many higher order corrections are important and can affect the s-dependence of amplitudes. We are thus facing the problem of exact unitarization of ICEM. So far, known examples of unitarized ICEM do not guarantee energy and momentum conservation (see Białas, Jurkiewicz, Zalewski 1976).

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In this paper we consider two generalizations of the ICEM which after unitarizing explicitly take into account energy and longitudinal momentum conservation. The first one is of the type which was discussed first by Auerbach, Aviv, Sugar and Blankenbecler (1972) (AASB). Our conclusion is that the elastic scattering amplitude has a leading singularity at $J = 1-\lambda/2$. This is identical to the AASB result. The second generalization was proposed by Białas and Czyż (1974). We have found that the elastic scattering amplitude has in this case a leading singularity at $J = 1$. Inelastic diffractive scattering is caused by energy and longitudinal momentum conservation in a way analogous to that found by Białas and Kotański. After switching off the energy and longitudinal momentum conservation, inelastic diffraction vanishes. Our treatment is based on the Lippman-Schwinger equation which after a suitable choice of the imaginary part of the propagator automatically guarantees the unitarity of the whole S-matrix.

Basic definitions, normalization and potentials are given in Section 2. In Section 3 we study the solution of the Lippman-Schwinger equation for the AASB-type potential. The energy dependence of amplitudes for a BC-type potential is analysed in Section 4. Section 5 contains our conclusions.

2. The Lippman-Schwinger equation and the potentials

Let V be a Hermitian interaction potential to be specified later and M — the transition matrix. We write the Lippman-Schwinger equation in the operator form

$$M = V+VLM, \tag{1}$$

where L is the propagator of an intermediate many particle state.

De Groot and Ruijgrok (1975) have shown that the unitarity demands L to fulfill the condition

$$\mathrm{Im}\, L = \tfrac{1}{2}\, \delta^4(\hat{P}-P_{\mathrm{in}}), \tag{2}$$

where P_{in} is the initial four-momentum and $\hat{P}$ is the operator of the four-momentum of the intermediate state. We now require L to satisfy a dispersion relation which can be written in the form

$$L = \int_0^\infty d\eta f(\eta)\delta^4(\hat{P}-\eta P_{\mathrm{in}}). \tag{3}$$

In the work of de Groot and Ruijgrok $f(\eta)$ was assumed in such a form which corresponds to the conservation of total three-momentum in the total c. m. s. In our treatment the exact form of $f(\eta)$ is left unspecified as we shall never need it. The linearity of the Lippman-Schwinger equation (1) is its great advantage when compared to the nonlinear unitarity relation.

The standard form of the operator creating the coherent state in the ICEM is

$$S_\pi = \exp\left(i \int \tilde{d}k\, \varrho(\vec{k})a^+(\vec{k})\right). \tag{4}$$

Here $đk$ denotes d^3k/k_0 and the commutation relations of the annihilation and creation operators are

$$[a(\vec{k}), a^+(\vec{k}')] = k_0\delta^3(\vec{k}-\vec{k}'). \tag{5}$$

The possible dependence of S_π on spin, isospin and other quantum numbers is ignored here as we are merely interested in the effects of imposing the energy and momentum conservation. We have to choose now the generalized form for the operator creating the coherent state. The possibility of cluster annihilation should also be incorporated. As V should be Hermitian we take

$$V = \int đp_a đp_b đp_c đp_d \Psi(p_a, p_b; p_c, p_d) b_c^+ b_d^+ b_a b_b W, \tag{6}$$

where b, b^+ are annihilation and creation operators of nucleons with commutation relations $[b(\vec{k}), b^+(\vec{k})] = k_0\delta^3(\vec{k}-\vec{k}')$, $[a(\vec{k}), b(\vec{k}')] = [a(\vec{k}), b^+(\vec{k}')] = 0$. The hermiticity of V requires Ψ to fulfill the condition

$$\Psi(p_a, p_b; p_c, p_d) = \Psi^*(p_c, p_d; p_a, p_b).$$

Further, W is an operator which describes cluster production. Because of the factorizable form of Eq. (6) at this stage there are no correlations between nucleons and clusters. The AASB-type choice of Ψ and W is

$$\Psi = G\sqrt{E_aE_bE_cE_d}\, s^{-\beta}\theta(p_{a\|})\theta(-p_{b\|})\theta(p_{c\|})\theta(-p_{d\|})w((p_{a\perp}-p_{c\perp})^2)w((p_{b\perp}-p_{d\perp})^2), \tag{7a}$$

$$W = \exp\left[i\int đk\varrho(\vec{k})a^+(\vec{k})\right]\exp\left[-i\int đk\varrho^*(\vec{k})a(\vec{k})\right] = W_{\mathrm{AASB}}.$$

The function $w(p_\perp^2)$ is normalized by $\int d^2p_\perp w(p_\perp^2) = 1$. We see that the Born term of nucleon-nucleon elastic scattering behaves like $s^{1-\beta}$.

For the BC-type potential we have

$$\Psi = G\sqrt{E_aE_bE_cE_d}\,\theta(p_{a\|})\theta(p_{c\|})\theta(-p_{b\|})\theta(-p_{d\|})w((p_{a\perp}-p_{c\perp})^2)w((p_{b\perp}-p_{d\perp})^2),$$

$$W = S_c + S_c^+ = W_{\mathrm{BC}}, \tag{7b}$$

$$S_c = \exp\left(i\int đk\varrho(\vec{k})a^+(\vec{k}) + i\int đk\varrho^*(\vec{k})a(\vec{k})\right).$$

We assume that $\varrho(\vec{k})$ depends only on $k_\perp$. The Born term for the BC-potential behaves now like $s^{1-\lambda/2}$ where λ is given by Eq. (14). As it will turn out, despite the highly complicated mathematical form for the full amplitude, it is still possible to analyse its energy dependence. We limit ourselves to the case without transverse momentum conservation. Thus we may neglect the additional dependence of Ψ on the transverse momenta of the nucleons by assuming that G is an effective coupling constant. We also choose the c. m. s. ($P_{\mathrm{in}} = [P_0, 0, 0, 0]$). In order to analyse the full transition matrix M we will investigate the formal solution of the Lippman-Schwinger equation given by the perturbation series

$$M = V\sum_{n=0}^{\infty}(LV)^n. \tag{8}$$

We follow rather closely the treatment of Białas and Kotański and write the propagator L in the form

$$L = \int_0^\infty d\eta f(\eta) \frac{1}{(2\pi i)^2} \int_{\varepsilon - i\infty}^{\varepsilon + i\infty} d^2\bar{y} \exp\left(\eta \bar{P}_{\text{in}}\bar{y} - \bar{y} \int đk\bar{k}a^+(\vec{k})a(\vec{k}) - \bar{y} \int đp\bar{p}b^+(\vec{p})b(\vec{p})\right), \tag{9}$$

where $\bar{k} \equiv (k_0, k_{\|})$, $\bar{y} \equiv (y_0, y_{\|})$.
Then the LV operator has the form

$$LV = \int_0^\infty d\eta f(\eta) \frac{1}{(2\pi i)^2} \int_{\varepsilon - i\infty}^{\varepsilon + i\infty} d^2\bar{y} \exp\left(\eta \bar{P}_{\text{in}}\bar{y} - \bar{y} \int đp\bar{p}b^+(\vec{p})b(\vec{p})\right)$$

$$\times \int đp_a đp_b đp_c đp_d \Psi(p_a, p_b; p_c, p_d) b_c^+ b_d^+ b_a b_b D(\bar{y}),$$

where

$$D(\bar{y}) = \exp\left(-\bar{y} \int đk\bar{k}a^+(\vec{k})a(\vec{k})\right)W. \tag{11}$$

The Eqs (8), (10) and (11) are our basic formulae used in further calculations.

3. *The full amplitude for the AASB-type potential*

In this section we examine the AASB potential (7a). Let us define the coherent state

$$|\{h(\vec{k})\}\rangle_{\text{coh}} = \exp\left(i \int đkh(\vec{k})a^+(\vec{k})\right)|0\rangle. \tag{12}$$

Here $|0\rangle$ denotes the state of cluster vacuum. It is straight-forward procedure to show that

$$W_{\text{AASB}} D_1 D_2 \dots D_m|0\rangle = \prod_{i=1}^{m} F(\bar{y}_i) \prod_{i=1}^{m-1} F(\bar{y}_i + \bar{y}_{i+1}) \dots \prod_{i=1}^{m-k} F(\bar{y}_i + \dots + \bar{y}_k) \dots F\left(\sum_1^m \bar{y}_i\right)$$

$$\times |\{\varrho(\vec{k})(1 + (1 + (1 + \dots (1 + e^{-\bar{k}\bar{y}_m})e^{-\bar{k}\bar{y}_{m-1}}) \dots)e^{-\bar{k}\bar{y}_1})\}\rangle_{\text{coh}}. \tag{13}$$

Here

$$D_i \equiv D(\bar{y}_i), \qquad F(\bar{y}) \equiv \exp\left(\int đk|\varrho(k_\perp)|^2 e^{-\bar{k}\bar{y}}\right).$$

In deriving formula (13) we have used the relation

$$\exp\left(-\bar{y} \int đk\bar{k}a^+(\vec{k})a(\vec{k})\right) \exp\left(i \int đk\varrho(\vec{k})a^+(\vec{k})\right)$$

$$= \exp\left(i \int đk\varrho(\vec{k})e^{-\bar{y}\bar{k}}a^+(\vec{k})\right) \exp\left(-\bar{y} \int đk\bar{k}a^+(\vec{k})a(\vec{k})\right).$$

In the work of de Groot it is shown that in the high energy limit it is sufficient to know the explicit form of $F(\bar{y})$ only for small values of $\bar{y} \cdot \bar{y}$ which is

$$F(\bar{y}) \underset{\bar{y} \cdot \bar{y} \sim 0}{\simeq} \left(\frac{\bar{\mu}}{2}\right)^{-2\lambda} (\sqrt{\bar{y} \cdot \bar{y}})^{-2\lambda}, \tag{14}$$

where $\lambda \ln \frac{\bar{\mu}}{\mu} = \int d^2k_\perp |\varrho(k_\perp)|^2 \ln \frac{e^\gamma \sqrt{k_\perp^2+\mu^2}}{\mu}$, γ = Euler constant, $\lambda = \int d^2k_\perp |\varrho(k_\perp)|^2$, μ = mass of the produced cluster.

Projecting the coherent state from Eq. (13) onto the state of cluster vacuum, then integrating over the momenta of the leading nucleons and over the total energies of the intermediate states; after some rearrangements one obtains

$$\langle \vec{p}_3\vec{p}_4| \langle 0|V(LV)^n|0\rangle |\vec{p}_1\vec{p}_2\rangle = G^{m+1}\sqrt{E_1E_2E_3E_4}\, s^{-\beta(m+1)}$$

$$\times \frac{1}{(2\pi i)^{2m}} \int \prod_1^m d\eta_i f(\eta_i) \int \prod_1^m dx_i'\theta(x_i')dx_i''\theta(x_i'') \int \prod_1^m d^2\bar{y}_i$$

$$\times \exp\left(\sum_1^m \chi_i\bar{y}_i\right) \prod_1^m F\left(\frac{\bar{y}_i}{\sqrt{s}}\right) \prod_1^{m-1} F\left(\frac{\bar{y}_i+\bar{y}_{i+1}}{\sqrt{s}}\right) \dots F\left(\frac{\sum_1^m \bar{y}_i}{\sqrt{s}}\right), \tag{15}$$

where $E_1, E_2(E_3, E_4)$; $\vec{p}_1, \vec{p}_2(\vec{p}_3, \vec{p}_4)$ are the energies and momenta of the initial and final nucleons, p_i', p_i'' are the momenta of nucleons in the i-th intermediate state and $\chi_i = \frac{\eta_i}{\sqrt{s}} \times (P_{\text{in}}-p_i'-p_i'')$ $x_i' = \frac{P_{i\parallel}'}{\sqrt{s}}$, $x_i'' = -\frac{P_{i\parallel}''}{\sqrt{s}}$. From the explicit form of $F(\bar{y})$ we can find the energy dependence of the n-th term in the elastic scattering amplitude and after summing over n we get

$$\langle \vec{p}_3\vec{p}_4| \langle 0|M|0\rangle |\vec{p}_1\vec{p}_2\rangle = G\sqrt{E_1E_2E_3E_4}\, s^{-\beta} \sum_{n=0}^{\infty} (2Gs^{-\beta})^n \left(\frac{s}{4}\left(\frac{2}{\bar{\mu}}\right)^2\right)^{\frac{\lambda}{2}m(m+1)} T_m(\lambda), \tag{16}$$

where $T_m(\lambda)$ does not depend on s and is given by

$$T_m(\lambda) = \int_0^\infty \prod_1^m d\eta_i f(\eta_i)\theta_{m+}(\{\eta_i\})\theta_{m-}(\{\eta_i\}),$$

$$\theta_{m+}(\{\eta_i\}) = \frac{1}{(2\pi i)^m} \int_0^{1/2} \prod_1^m dx_i' \int_{\varepsilon-i\infty}^{\varepsilon+i\infty} \prod_1^m dy_{i+} \exp\left(\sum_{i=1}^m y_{i+}\eta_i(1-2x_i')\right)$$

$$\times \prod_1^m y_{i+}^{-\lambda} \prod_1^{m-1} (y_{i+}+y_{i+1\,+})^{-\lambda} \dots \left(\sum_{i=1}^m y_{i+}\right)^{-\lambda}.$$

Here $\theta_{m_-}(\{\eta_i\})$ is obtained from $\theta_{m_+}(\{\eta_i\})$ by $x_i' \to x_i''$, $y_{i_+} \to y_{i_-}$, $y_{i_\pm} = \frac{y_0 \pm y_\parallel}{2}$. We thus see that the perturbation series (8) is divergent and behaves like $s^{1-\beta} \sum_{m|0}^{\infty} c_m s^{\frac{\lambda}{2}m^2+m\left(\frac{\lambda}{2}-\beta\right)}$. If we take the contribution from $m = 0, 1$ we recover for $\beta = \lambda/2$ the result obtained by Białas and Kotański, i. e. the elastic scattering amplitude is proportional to s. However, $m = 0, 1$ terms are only the first two terms in a very complicated sum of all checkerboard graphs, whose presence is required by unitarity. The series (16) obviously violates Froissart bound. The mechanism by which Froissart bound is enforced is identical to the one found by Auerbach, Aviv, Sugar and Blankenbecler. A generalization of the AASB result was given by Schwimmer and Veneziano. They showed that any model in which amplitude can be written in the form (16) has a cut in J-plane whose position is fixed by the positive solution of the quadratic equation

$$\frac{\lambda}{2}m^2+m\left(\frac{\lambda}{2}-\beta\right)+1-\beta = J, \tag{17}$$

that is $m_{\text{cut}} = \frac{1}{\lambda}\left(\beta-\frac{\lambda}{2}+\sqrt{\left(\frac{\lambda}{2}+\beta\right)^2+2\lambda(J-1)}\right)$. The only poles on the physical sheet are those for which $m \leqslant \frac{1}{\lambda}\left(\beta-\frac{\lambda}{2}\right)$. For ICEM we have $\lambda = 2\beta$ and the leading singularity is $J = 1-\lambda/2$. This singularity is present only if one takes into account the exchange of an infinite number of chains. Comparing our result with that of Blankenbecler we see that the introduction of energy and longitudinal momentum conservation did not change the s-behaviour of the elastic amplitude. For $\beta = \lambda/2$ we find that

$$\langle \vec{p}_3\vec{p}_4| \langle \vec{q}|M|0\rangle |\vec{p}_1\vec{p}_2\rangle = i\varrho(q_\perp)\sqrt{E_1E_2E_3E_4}\, s^{-\frac{\lambda}{2}}$$

$$\times \sum_{m=0}^{\infty}\left(2G\left(\frac{2}{\mu}\right)^{\lambda}\right)^m \left(\frac{s}{4}\left(\frac{2}{\mu}\right)^2\right)^{\frac{\lambda m^2}{2}} \sum_{n=0}^{m} T_{m,n}(\lambda, x) \tag{18}$$

where $T_{m,n}(\lambda, x)$ for $x = \frac{q_\parallel}{\sqrt{s}} > 0$ is

$$T_{m,n}(\lambda, x) = \int_0^\infty \prod_1^m d\eta_i f(\eta_i) \theta_{m_+}(\{\eta_i\})\theta_{m_-,n}(\{\eta_i\}, x)$$

$$\theta_{m_-,n}(\{\eta_i\}, x) = \frac{1}{(2\pi i)^m}\int \prod_1^m dx_i'' \int \prod_1^m dy_{i_-} \exp\left(\sum_1^m y_{i_-}\eta_i(1-2x\theta(n-i)-2x_i'')\right)$$

$$\times \prod_1^m y_{i_-}^{-d} \prod_1^{m-1} (y_{i_-}+y_{i+1_-})^{-\lambda} \dots \left(\sum_1^m y_{i_-}\right)^{-\lambda}.$$

From (18) we see once again that the leading singularity lies at $J = 1-\lambda/2$ also for particle production processes. This result can be generalized to many particle production amplitudes. We conclude that for the AASB-type choice of the potential the energy dependence of the inelastic exclusive channel amplitudes is similar to that of elastic scattering both being unfortunately wrong.

4. The BC-type choice of the potential

The second possible generalization of ICEM is the BC-type choice of the potential (7b). Hence from now on $W = W_{BC}$ and

$$D(\bar{y}_j) \to \tilde{D}(\bar{y}_j) \equiv \tilde{D}_j = \exp\left(-\bar{y}_j \int d\!\!{}^-\! k\bar{k}a^+(\vec{k})a(\vec{k})\right) \times \sum_{\varepsilon_j=\pm 1} (i\varepsilon_j[\int d\!\!{}^-\! k\varrho(\vec{k})a^+(\vec{k}) + \int d\!\!{}^-\! k\varrho^*(\vec{k})a(\vec{k})]). \tag{19}$$

After some algebra we obtain

$$W_{BC}\tilde{D}_n \ldots \tilde{D}_2\tilde{D}_1 |0\rangle = \exp\left(-\frac{n+1}{2}\int d\!\!{}^-\! k|\varrho(k_\perp)|^2\right) \times \sum_{\varepsilon_1,\ldots,\varepsilon_{n+1}=\pm 1} F_{-\varepsilon_1\varepsilon_2}(\bar{y}_1)F_{-\varepsilon_2\varepsilon_3}(\bar{y}_2)\ldots F_{-\varepsilon_n\varepsilon_{n+1}}(\bar{y}_n)F_{-\varepsilon_1\varepsilon_3}(\bar{y}_1+\bar{y}_2)$$
$$\times F_{-\varepsilon_2\varepsilon_3}(\bar{y}_2+\bar{y}_3)\ldots F_{-\varepsilon_{n-1}\varepsilon_{n+1}}(\bar{y}_{n-1}+\bar{y}_n)F_{-\varepsilon_1\varepsilon_4}(\bar{y}_1+\bar{y}_2+\bar{y}_3)\ldots F_{-\varepsilon_n\varepsilon_{n+1}}\left(\sum_1^n \bar{y}_i\right)$$
$$\times |\{\varrho(k)\,(\varepsilon_{n+1}+\varepsilon_n e^{-\bar{y}_n\bar{k}}+\varepsilon_{n-1}e^{-(\bar{y}_n+\bar{y}_{n-1})\bar{k}}+\ldots+\varepsilon_1 e^{-(\sum_1^n \bar{y}_i)\bar{k}})\}\rangle_{coh}. \tag{20}$$

Here

$$F_\kappa(\bar{y}) \equiv \exp\left(\kappa\int d\!\!{}^-\! k|\varrho(k_\perp)|^2 e^{-\bar{y}\bar{k}}\right) = [F(\bar{y})]^\kappa.$$

As only small $\bar{y}\cdot\bar{y}$ values contribute in the high energy limit, we replace $F(\bar{y})$ by Eq. (14) and after scaling the variables we have

$$\langle\vec{p}_3\vec{p}_4|\,\langle 0|M|0\rangle\,|\vec{p}_1\vec{p}_2\rangle = G(m^2)^{\lambda/2}\sqrt{E_1E_2E_3E_4}\sum_{n=0}^{\infty}\sum_{\{\varepsilon_i=\pm 1\}} s^{-\frac{\lambda}{2}(\sum_1^{n+1}\varepsilon_i)^2}\gamma_n(\{\varepsilon_i\}), \tag{21}$$

where $\gamma_n(\{\varepsilon_i\})$ does not depend on s and is

$$\gamma_n(\{\varepsilon_i\}) = [G(m^2)^{\lambda/2}]^n \int \prod_1^n d\eta_i f(\eta_i) \int dx_i' dx_i'' \theta(x_i')\theta(x_i'')$$
$$\times \frac{1}{(2\pi i)^{2n}} \int \prod_1^n d^2 y_i \exp\left(\sum_1^n \bar{y}_j \frac{\bar{Q}_j}{\sqrt{s}}\right) F_{-\varepsilon_1\varepsilon_2}(\bar{y}_1)F_{-\varepsilon_2\varepsilon_3}(\bar{y}_2)\ldots F_{-\varepsilon_1\varepsilon_{n+1}}\left(\sum_1^n \bar{y}_i\right).$$

Thus we observe that the leading singularity is a pole at $J = 1$. The terms which contribute to this pole fulfill $\sum \varepsilon_i = 0$. This is the case when from the whole product $\prod (S_c + S_c^+)$ we consider only the contribution from diagrams involving an equal number of S_c and S_c^+ operators. The first nonleading singularity lies at $J = 1 - \lambda/2$ and is obtained from all these terms $\prod S_c \prod S_c^+$ where one and only one operator S_c (or S_c^+) is not cancelled by S_c^+ (or S_c). It is also possible to find one particle production amplitude

$$\langle \vec{p}_3 \vec{p}_4 | \langle \vec{q} | M | 0 \rangle | \vec{p}_1 \vec{p}_2 \rangle = i\varrho(q_\perp) \sqrt{E_1 E_2 E_3 E_4}\, G(m^2)^{\lambda/2}$$

$$\times \sum_{n=0}^{\infty} \sum_{\{\varepsilon_i = \pm 1\}} s^{-\frac{\lambda}{2}\left(\sum_1^{n+1} \varepsilon_i\right)^2} \tilde{\gamma}_n(\{\varepsilon_i\}), \tag{22}$$

where $\tilde{\gamma}_n(\{\varepsilon_i\})$ does not depend on s and can be obtained from $\gamma_n(\{\varepsilon_i\})$ by multiplying the integrand in $\gamma_n(\{\varepsilon_i\})$ by

$$\Delta_n \equiv \varepsilon_{n+1} + \varepsilon_n e^{-\bar{y}_n \frac{\bar{q}}{\sqrt{s}}} + \varepsilon_{n-1} e^{-(\bar{y}_n + \bar{y}_{n-1}) \frac{\bar{q}}{\sqrt{s}}} + \ldots + \varepsilon_1 e^{-\left(\sum_1^n \bar{y}_i\right) \frac{\bar{q}}{\sqrt{s}}}. \tag{23}$$

In the high energy limit the whole amplitude behaves like s and this behaviour comes from the terms fulfilling the condition $\sum \varepsilon_i = 0$. We would like to emphasize that this behaviour of one particle production amplitude is implied by energy and longitudinal momentum conservation. Indeed, negligence of energy and momentum conservation is equivalent to putting $\vec{q}$ equal zero. Then $\Delta_n = \sum \varepsilon_i = 0$ and the term proportional to s vanishes. Thus in such a case there are no diffractively produced particles: The next singularity is at $J = 1 - \lambda/2$. One can also generalize this result to the production of a greater number of particles.

The fact that energy and longitudinal momentum conservation force the diffractive production of fast clusters was first discovered by Białas and Kotański. They used however the technique of the overlap matrix which does not guarantee the unitarity of the whole S-matrix and, furthermore, solved the problem only in the first approximation. In our approach both diffractive and nondiffractive amplitudes are calculated from the known Born terms and the exact unitarity of the whole S-matrix is assured.

5. *Summary and conclusions*

We have studied the influence of energy and longitudinal momentum conservation on two different versions of the unitarized independent cluster emission model. Our results could be summarized as follows:

(*i*) Energy and longitudinal momentum conservation incorporated into the Auerbach, Aviv, Sugar and Blankenbecler model of multiparticle production processes does not change the AASB result.

(*ii*) In the Białas-Czyż-Kotański model energy and longitudinal momentum conservation implies the existence of inelastic diffractive scattering. Thus we extend the results of Białas and Kotański to all orders in the coupling constant. In particular, we have shown that

— if there were no energy and momentum conservation, inelastic diffraction would vanish,
— the diffractively produced particles are fast.

(*iii*) The advantage of our approach as compared to the overlap matrix technique is that it quarantees the exact unitarity of the whole S-matrix. We see that the BC-type potential is able to describe qualitatively the s-dependence of amplitudes as opposed to the AASB-type one.

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