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DISCUSSION OF VARIOUS METHODS OF CALCULATING OF INERTIAL MASS PARAMETERS IN ADIABATIC APPROXIMATION

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Nuclear inertial mass parameters corresponding to quadrupole and hexadecapole deformation are investigated. Two different approach are applied, both making use of the adiabatic approximation for the collective motion. The results are presented for ^{240}Pu.

1. Introduction

Inertial mass parameters play an important role in describing the dynamics of the nuclear collective motions, namely fission and vibrations. In this paper the inertial mass parameters corresponding to multipole-multipole vibrations and those connected with fission mode are calculated. Both types of inertial mass parameters can be calculated in adiabatic approximation [1]. The inertial mass parameters for multipole-multipole interaction can also be obtained by harmonic approximation [2]. The formulae for the harmonic approximation in the limit of low frequency are the same as for the adiabatic one [3]. It is still possible to obtain the inertial mass parameters of a fissioning nucleus from the vibrational approximation, because each change of deformation can be described by the changes of multipole moments [3]. The aim of this paper is to check if there are any differences between the inertial mass parameters corresponding to fission mode calculated by adiabatic approximation when deformation parameters are treated as collective variables and those "extracted" from the vibrational ones.

An attempt to show that a mean square radius is significant dynamical variable is made. This variable should be taken into account especially for large quadrupole deformations corresponding to the second minimum and to the second saddle. It seems that such a variable as well as the quadrupole moment may play a significant role as the second collective variable in the description of the excited vibrational states in the second minimum of the potential energy.

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2. Description of the calculation

2.1. Formulae

The inertial mass parameter is calculated in the adiabatic approximation. The general formula for this parameter corresponding to the collective coordinates q_i and q_j is given by [1]

$$B_{q_i q_j} = 2\hbar^2 \sum_{m \neq 1} \frac{\langle 0| \frac{\partial}{\partial q_i} |m\rangle \langle m| \frac{\partial}{\partial q_i} |0\rangle}{\mathscr{E}_m - \mathscr{E}_0}. \tag{1}$$

$\mathscr{E}_m$ and $\mathscr{E}_0$ are the energies of the excited $|m\rangle$ and the ground $|0\rangle$ states, respectively.

After inclusion of the pairing interaction by the quasiparticle formalism and taking into account the coupling with pairing vibrations, formula (1) takes the form [1, 4]

$$B_{q_i q_j} = 2\hbar^2 \left\{ \sum_{\nu\omega} \frac{\langle \nu| \frac{\partial \hat{H}}{\partial q_i} |\omega\rangle \langle \omega| \frac{\partial \hat{H}}{\partial q_j} |\nu\rangle}{(E_\nu + E_\omega)^3} (u_\nu v_\omega + u_\omega v_\nu)^2 \right.$$

$$\left. + \tfrac{1}{8} \sum_\nu \frac{1}{E_\nu^5} \left[\Lambda_i^\nu \Lambda_j^\nu - \Delta \left(\Lambda_i^\nu \langle \nu| \frac{\partial \hat{H}}{\partial q_i} |\nu\rangle + \Lambda_j^\nu \langle \nu| \frac{\partial \hat{H}}{\partial q_i} |\nu\rangle \right) \right] \right\}, \tag{2}$$

where

$$\Lambda_i^\nu = \frac{\partial \lambda}{\partial q_i} \Delta + \frac{e_\nu - \lambda}{2\Delta} \frac{\partial \Delta^2}{\partial q_i}, \tag{3}$$

$\hat{H}$ is the single-particle Hamiltonian, u_ν and v_ν ($u_\nu^2 = 1 - v_\nu^2$) are the variational parameters of the BCS wave function corresponding to the single-particle state $|\nu\rangle$ with the energy e_ν, and E_ν is the quasiparticle energy of this state $E_\nu = \sqrt{(e_\nu - \lambda)^2 + \Delta^2}$. The quantities λ and Δ are the usual pairing parameters, i. e. the chemical potential and the energy gap, respectively. The derivatives $\frac{\partial \lambda}{\partial q_i}$ and $\frac{\partial \Delta^2}{\partial q_i}$ can be obtained from the following formulae

$$\frac{\partial \lambda}{\partial q_i} = D(ab + \Delta^2 cd), \qquad \frac{\partial \Delta^2}{\partial q_i} = -2\Delta^2 D(cb - ad), \tag{4}$$

where

$$a = \sum_\nu \frac{e_\nu - \lambda}{E_\nu^3}, \qquad b = \sum_\nu \frac{(e_\nu - \lambda) \langle \nu| \frac{\partial \hat{H}}{\partial q_i} |\nu\rangle}{E_\nu^3},$$

$$c = \sum_\nu \frac{1}{E_\nu^3}, \qquad d = \sum_\nu \frac{\langle \nu| \frac{\partial \hat{H}}{\partial q_i} |\nu\rangle}{E_\nu^3}, \qquad D = (a^2 + \Delta^2 c^2)^{-1}.$$

The terms in the second line of Eq. (2) describe the coupling with pairing vibrations.

One can get formula (2) only in the case when the single-particle Hamiltonian depends explicitly on q_i coordinates.

Now, let us assume that this is not the case. The Hamiltonian depends on another set of parameters σ_i and then the inertial parameter $B_{q_iq_j}$ is given by

$$B_{q_iq_j} = \sum_k \sum_l \frac{\partial \sigma_k}{\partial q_i} \frac{\partial \sigma_l}{\partial q_j} 2\hbar^2 \sum_{m \geq 0} \frac{\langle 0| \frac{\partial}{\partial \sigma_k} |m\rangle \langle m| \frac{\partial}{\partial \sigma_l} |0\rangle}{\mathscr{E}_m - \mathscr{E}_0} \tag{5}$$

after a simple change of variables in the derivation. The last formula can be written in the following form

$$B_{q_iq_j} = \sum_{k,l} B_{\sigma_k\sigma_l} \frac{\partial \sigma_k}{\partial q_i} \frac{\partial \sigma_l}{\partial q_j}. \tag{6}$$

Expression (5) is equivalent to formula (1) only when the parameters σ_i can describe the same type of vibration as that described by the set of the collective coordinates q_i. This happens when the set of the parametrs σ_i is sufficiently large.

The derivative $\frac{\partial \sigma_i}{\partial q_j}$ can be calculated in a microscopical way. Let us assume that the coordinate q_i is equal to the mean value of the operator $\hat{Q}_i$

$$q_i = \langle 0|\hat{Q}_i|0\rangle. \tag{7}$$

In this case the derivative $\frac{\partial q_i}{\partial \sigma_j}$ is given by [1]

$$\frac{\partial q_i}{\partial \sigma_j} = -2 \sum_{\nu,\omega} \frac{\langle \nu|\hat{Q}_i|\omega\rangle \langle \omega| \frac{\partial \hat{H}_{\mathrm{N}}}{\partial \sigma_j} |\nu\rangle}{E_\nu + E_\omega} (u_\nu v_\omega + u_\omega v_\nu)^2 + \sum_\nu \frac{\Delta}{E_\nu^3} \Lambda_j^\nu \langle \nu|\hat{Q}_i|\nu\rangle. \tag{8}$$

The sum in the second line of the last equation is due to the coupling with pairing vibrations. If the number of parameters q_i is equal to the number of the parameters σ_i then it is possible to get the derivatives $\frac{\partial \sigma_i}{\partial q_j}$ appearing in Eq. (5) in the standard way from the set of the derivatives $\frac{\partial q_i}{\partial \sigma_j}$ calculated according to formula (8).

All the calculations in the present paper are made with the use of the Nilsson potential [5, 6]

$$V(\varepsilon, \varepsilon_4) = \tfrac{1}{2} \hbar\omega_0(\varepsilon, \varepsilon_4)\varrho^2[1 - \tfrac{2}{3} \varepsilon P_2(\cos \vartheta_t) + 2\varepsilon_4 P_4(\cos \vartheta_t)]$$
$$- \hbar\mathring{\omega}_0 \kappa[2\boldsymbol{l}_t \cdot \boldsymbol{s} + \mu(\boldsymbol{l}_t^2 - \langle \boldsymbol{l}^2 \rangle_{\mathrm{N}_t})]. \tag{9}$$

One takes here as the dynamical variables q_i the quadrupole ε and hexadecapole ε_4 deformations and gets $B_{\varepsilon\varepsilon}$, $B_{\varepsilon\varepsilon_4}$ and $B_{\varepsilon_4\varepsilon_4}$ according to Eq. (2). The derivatives of the Nilsson Hamiltonian $\hat{H}_N$ with respect to the deformation parameters are equal to [7]

$$\frac{\partial \hat{H}_N}{\partial \varepsilon} = \hbar\omega_0\varrho^2 \left\{ [1-\tfrac{2}{3}\varepsilon P_2+2\varepsilon_4 P_4] \frac{1}{\omega_0}\frac{\partial\omega_0}{\partial\varepsilon} \right.$$

$$\left. -\tfrac{2}{3}\left[\left(1+\frac{\varepsilon_4}{\alpha}\right)P_2-\frac{\varepsilon_4}{\alpha}\left(\tfrac{5}{11}P_6+\left(\tfrac{5}{11}+\frac{\varepsilon}{3}\right)P_4\right)\right]\right\}; \tag{10}$$

$$\frac{\partial \hat{H}_N}{\partial \varepsilon_4} = \hbar\omega_0\varrho^2 \left\{ [1-\tfrac{2}{3}\varepsilon P_2+2\varepsilon_4 P_4] \frac{1}{\omega_0}\frac{\partial\omega_0}{\partial\varepsilon_4} + P_4 \right\},$$

where

$$\alpha = (1-\tfrac{2}{3}\varepsilon)(1+\tfrac{1}{3}\varepsilon).$$

The calculations can also be performed with another set of collective parameters Q_{00}, Q_{20} and Q_{40} which correspond to the mean values of the following operators

$$\hat{Q}_{00} = r^2, \quad \hat{Q}_{20} = 2r^2 P_2(\cos\vartheta), \quad \hat{Q}_{40} = \varrho^2 P_4(\cos\vartheta_t), \tag{11}$$

where r and ϑ are the spherical coordinates, ϱ and ϑ_t are the spherical coordinates defined in the stretched coordinates [5] ξ, η, ζ. To calculate the inertial mass parameter $B_{Q_{\lambda 0}Q_{\mu 0}}$ the term with the operators (11) is added to the Nilsson Hamiltonian $\hat{H}_N$, i. e.

$$\hat{H} = \hat{H}_N - \sum_{\lambda=0,2}^{4} (\sigma_\lambda - \sigma_\lambda^0)\hat{Q}_{\lambda 0}. \tag{12}$$

Obviously the new Hamiltonian $\hat{H}$ is equal to $\hat{H}_N$ when all the parameters σ_λ are equal to σ_λ^0. The parameters σ_λ^0 are the mean values of multipole fields obtained from a linearization of two-body multipole-multipole interactions. These mean values (σ_λ^0) correspond to the given deformations ε, ε_4.

One can calculate now the derivative of the Hamiltonian $\hat{H}$ with respect to σ_λ

$$\frac{\partial \hat{H}}{\partial \sigma_\lambda} = -\hat{Q}_{\lambda 0}. \tag{13}$$

According to Eq. (2) the inertial mass parameter $B_{\sigma_\mu\sigma_\lambda}$ is given by

$$B_{\sigma_\mu\sigma_\lambda} = 2h^2 \sum_{\nu,\omega} \frac{\langle\nu|\hat{Q}_{\mu 0}|\omega\rangle\langle\omega|\hat{Q}_{\lambda 0}|\nu\rangle}{(E_\nu+E_\omega)^3}(u_\nu v_\omega + u_\omega v_\nu)^2$$

$$+\text{terms which describe the coupling with pairing vibrations.} \tag{14}$$

According to (8) the derivative $\frac{\partial Q_{\lambda 0}}{\partial \sigma_\mu}$ is

$$\frac{\partial Q_{\lambda 0}}{\partial \sigma_\mu} = 2 \sum_{\nu,\omega} \frac{\langle \nu|\hat{Q}_{\lambda 0}|\omega\rangle \langle \omega|\hat{Q}_{\mu 0}|\nu\rangle}{E_\nu + E_\omega} (u_\nu v_\omega + u_\omega v_\nu)^2$$

+terms which describe the coupling with pairing vibrations. (15)

Thus one gets

$$B_{Q_{\lambda 0} Q_{\mu 0}} = \sum_{\tau,\varrho} B_{\sigma_\tau \sigma_\varrho} \frac{\partial \sigma_\tau}{\partial Q_{\lambda 0}} \frac{\partial \sigma_\varrho}{\partial Q_{\mu 0}}. \tag{16}$$

It is interesting to note that formulae (14)–(16) are identical with those obtained by the harmonic approximation in the limit of small frequency ω (cf. Ref. [3]). The notation used here is slightly different from that in Ref. [3], namely,

$$\frac{1}{\hbar^2} B_{\sigma_\lambda \sigma_\mu} \equiv 2\Sigma_3^{\lambda\mu} \quad \text{and} \quad \frac{\partial Q_{\lambda 0}}{\partial \sigma_\mu} \equiv 2\Sigma_1^{\lambda\mu}. \tag{17}$$

The calculations are made for protons and neutrons separately. Inertial mass parameter $B_{\varepsilon_i \varepsilon_j}$ is given by Eq. (2) and becomes the sum

$$B_{\varepsilon_i \varepsilon_j} = B^{\mathrm{p}}_{\varepsilon_i \varepsilon_j} + B^{\mathrm{n}}_{\varepsilon_i \varepsilon_j}, \tag{18}$$

where $B^{p(n)}_{\varepsilon_i \varepsilon_j}$ denotes inertial mass parameter for protons (neutrons). The situation becomes more complicated when one takes the parameter $B_{Q_{\lambda 0} Q_{\mu 0}}$ connected with the collective coordinates $Q_{\lambda 0}$ and $Q_{\mu 0}$ which are the sums of proton and neutron parts

$$Q_{\lambda 0} = Q^{\mathrm{p}}_{\lambda 0} + Q^{\mathrm{n}}_{\lambda 0}. \tag{19}$$

The derivative $\frac{\partial}{\partial Q_{\lambda 0}}$, which appears in formula (1), is equal to

$$\frac{\partial}{\partial Q_{\lambda 0}} = \tfrac{1}{2} \left(\frac{\partial}{\partial Q^{\mathrm{p}}_{\lambda 0}} + \frac{\partial}{\partial Q^{\mathrm{n}}_{\lambda 0}} \right). \tag{20}$$

One obtains this expression by introducing the difference $(Q^n_{\lambda 0} - Q^p_{\lambda 0})$ as a second collective variable. It can be seen from Eqs (6) and (20) that

$$B_{Q_{\lambda 0} Q_{\mu 0}} = \tfrac{1}{4} (B^{\mathrm{p}}_{Q_{\lambda 0} Q_{\mu 0}} + B^{\mathrm{n}}_{Q_{\lambda 0} Q_{\mu 0}}). \tag{21}$$

The inertial mass parameter $B_{\varepsilon_i \varepsilon_j}$ can be obtained from the set of the parameters $B_{Q_{\lambda 0} Q_{\mu 0}}$ with the help of the following relation [3]

$$\tilde{B}_{\varepsilon_i \varepsilon_j} = \sum_{\mu,\lambda} \left(B^{\mathrm{p}}_{Q_{\lambda 0} Q_{\mu 0}} \frac{\partial Q^{\mathrm{p}}_{\lambda 0}}{\partial \varepsilon_i} \frac{\partial Q^{\mathrm{p}}_{\mu 0}}{\partial \varepsilon_j} + B^{\mathrm{n}}_{Q_{\lambda 0} Q_{\mu 0}} \frac{\partial Q^{\mathrm{n}}_{\lambda 0}}{\partial \varepsilon_i} \frac{\partial Q^{\mathrm{p}}_{\mu 0}}{\partial \varepsilon_j} \right). \tag{22}$$

The tilde over $\tilde{B}_{\varepsilon_i\varepsilon_j}$ distinguishes it from the $B_{\varepsilon_i\varepsilon_j}$ obtained with the use of formulae (2) and (18). The derivative $\frac{\partial Q_{\lambda 0}^{p(n)}}{\partial \varepsilon_i}$ can be calculated in a microscopic way from Eq. (8) only if σ_j is identical with ε_i; or in a macroscopic way, for a given distribution of the density inside a nucleus, if it is assumed that the shape of the nuclear surface is identical with the equipotential surface for the potential (9). One of the aims of this paper is to show that $B_{\varepsilon_i\varepsilon_j}$ of Eqs (2) and (18) are identical with $\tilde{B}_{\varepsilon_i\varepsilon_j}$ of Eq. (22) if a sufficient number of multipole vibrations generated by multipole moments $Q_{\lambda 0}$ are taken into account and if the calculation is completely microscopic $\left(\text{i. e. with } \frac{\partial Q_{\lambda 0}}{\partial \varepsilon} \text{ calculated microscopically}\right)$.

2.2. Choice of parameters

The calculation is performed using a single-particle Nilsson potential. The set of the "A = 242" parameters [8] of the potential is used, i. e. $\kappa_p = 0.0577$, $\mu_p = 0.650$ for protons and $\kappa_n = 0.0635$, $\mu_n = 0.325$ for neutrons. The frequency of the spherical harmonic oscillator is

$$(\hbar\mathring{\omega}_0)_{p(n)} = 41/A^{1/3}\left(1 \mp \frac{N-Z}{3A}\right) \text{MeV}. \tag{23}$$

We do not take into account the matrix elements of the hexadecapole term ($\varrho^2 P_4$) of the Hamiltonian between different shells when diagonalizing the Hamiltonian. The strength of the pairing interaction is taken in the form [8]

$$G_{p(n)}A = \left(19.2 \pm 7.4\,\frac{N-Z}{A}\right) \text{MeV}. \tag{24}$$

For solving the pairing equations we take into account $2\sqrt{15Z}$ or $2\sqrt{15N}$ levels nearest to the Fermi level. The pairing strength G is assumed constant with the deformation.

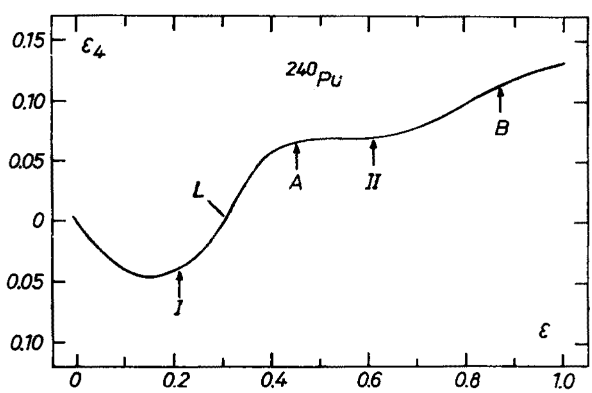

Fig. 1. The path L corresponding to the bottom of the potential energy valley of ^{240}Pu

The calculations are performed for ^{240}Pu. All the quantities presented in the next section are calculated along the path (L) in the (ε, ε_4)-plane (see Fig. 1). The path is taken from Refs. [9, 10]. It corresponds to the bottom of the potential energy valley of the nucleus. It goes through four points: first minimum (I), first saddle (A), second minimum (II), and second saddle (B) of the potential energy surface of ^{240}Pu taken from Ref. [9].

The matrix elements of the operators $\hat{Q}_{\lambda 0}$ and $\frac{\partial \hat{H}_N}{\partial \varepsilon_i}$ between the single-particle states from the same (N, N) shell and the shells differing by ± 2 (N, $N \pm 2$) are taken into account throughout the calculation.

The macroscopic calculations of $Q_{\lambda 0}$ and its derivatives $\frac{\partial Q_{\lambda 0}}{\partial \varepsilon_i}$ are performed assuming the uniform density distribution inside a nucleus.

3. *Results and discussion*

To illustrate the changes of the mean value of the square radius of a nucleus (Q_{00}) and the quadrupole moment (Q_{20}) with deformation, they are calculated as a function of ε (note that for each value of ε an appropriate value of ε_4 from the L path (Fig. 1) is

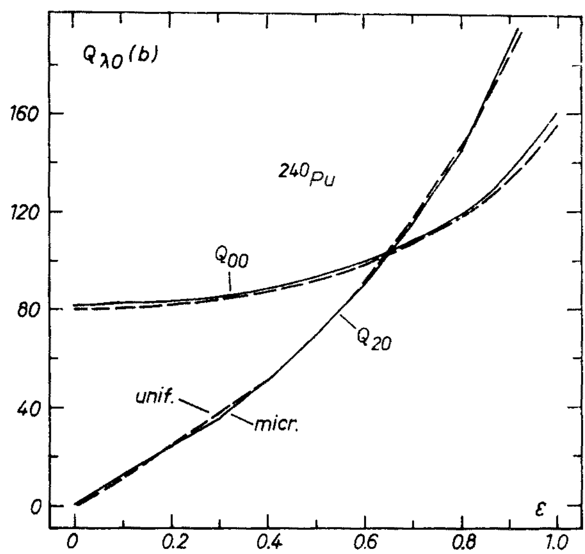

Fig. 2. Dependence of the mean square radius (Q_{00}) and quadrupole moment (Q_{20}) of ^{240}Pu on deformation, calculated in the microscopic way (solid line) and, also, macroscopically for uniform mass distribution (dashed line)

taken). The results are plotted in Fig. 2. The solid lines correspond to the microscopic and the dashed ones to the macroscopic values. The change of Q_{00} is negligible for small deformation (around the first minimum) but becomes more significant for larger deformations. With the help of definitions (11) it can be shown that in the limit of the largest quadrupole deformation the ratio of $\frac{\partial Q_{00}}{\partial \varepsilon}$ to $\frac{\partial Q_{20}}{\partial \varepsilon}$, both calculated for the uniform mass distribution, satisfies the relation

$$\lim_{\varepsilon \to 1.5} \left(\frac{\partial Q_{00}}{\partial \varepsilon} \Big/ \frac{\partial Q_{20}}{\partial \varepsilon} \right) = 0.5. \tag{25}$$

Fig. 3 shows that in the point corresponding to the second saddle ($\varepsilon = 0.87$, $\varepsilon_4 = 0.12$) this ratio is equal to 0.39. These results indicate that the collective coordinate Q_{00}, in

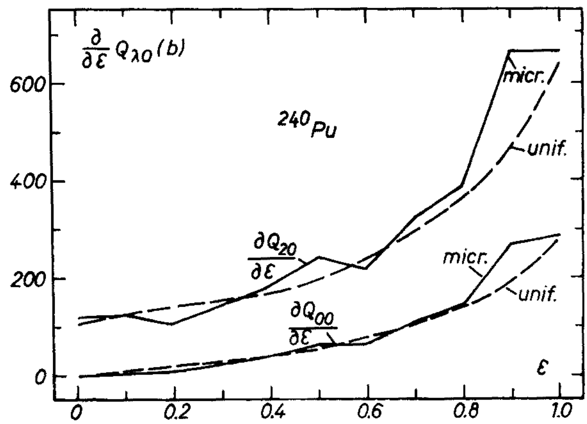

Fig. 3. The same as Fig. 2 for the derivatives $\frac{\partial Q_{00}}{\partial \varepsilon}$ and $\frac{\partial Q_{20}}{\partial \varepsilon}$

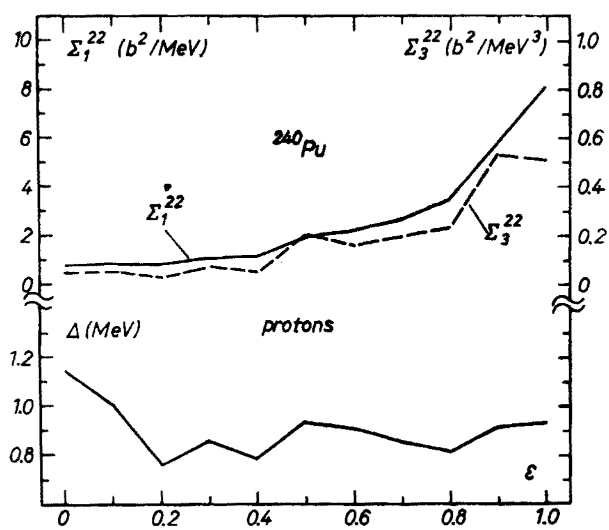

Fig. 4. Dependence of the proton energy gap Δ and Σ_1^{22}, Σ_3^{22} (cf. Eqs (14), (15) and (17)) on deformation for ^{240}Pu

addition to the coordinate Q_{20}, may play an important role for large deformations which appear in the fission process. The microscopic values of $\frac{\partial Q_{\lambda 0}}{\partial \varepsilon}$ (solid lines in Fig. 3) are close to the macroscopic ones (dashed lines in Fig. 3); the structure of the microscopic curves $\frac{\partial Q_{\lambda 0}}{\partial \varepsilon}$ is the shell effect.

Let us now investigate the influence of the Q_{00} and Q_{40} degrees of freedom on the value of the inertial mass parameters $B_{Q_{20}Q_{20}}$ (16) and $\tilde{B}_{\varepsilon\varepsilon}$ (22). The values of Σ_1^{22}, Σ_3^{22} (cf. Eqs (14), (15) and (17)) and Δ are shown in Figs 4 and 5 for protons and neutrons, respectively.

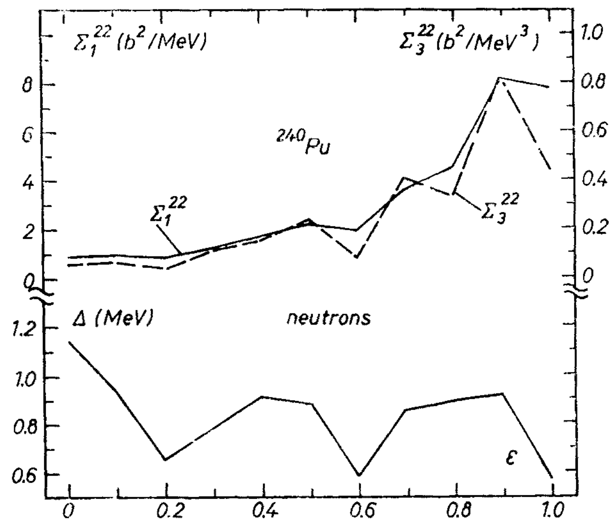

Fig. 5 The same as in Fig. 4 for neutrons

tively. It is seen that the structures of these three curves are correlated. The quantities Σ_1^{22} and Σ_3^{22} grow with the deformation, not monotonically, however. Each decrease (or increase) in Σ_i^{22} with respect to its mean behavior corresponds to the minimum (or maximum) in the Δ function. The single-particle structure is particularly visible for Σ_3^{22} and Δ. The inertial mass parameter $B_{Q_{20}Q_{20}}$ is presented in Fig. 6. It is calculated according to formulae (16) and (21) in the three cases. Assuming:

— the mixing of quadrupole Q_{20}, monopole Q_{00} and hexadecapole Q_{40} vibration (solid line),

— the mixing of vibrations Q_{20} and Q_{40} (dotted-dashed line),

— quadrupole vibration only (dashed line)

it can be seen that the effect of the vibration Q_{00} on $B_{Q_{20}Q_{20}}$ is much stronger than that of the Q_{40}-type vibration for the deformation $\varepsilon \geqslant 0.4$. For a smaller deformation both effects are comparable. Thus, it can be concluded that the coupling with the Q_{00}-type vibrations may be important in the treatment of the quadrupole vibrations in the second minimum. Of course, if one wishes to describe the multipole-multipole vibrations, one

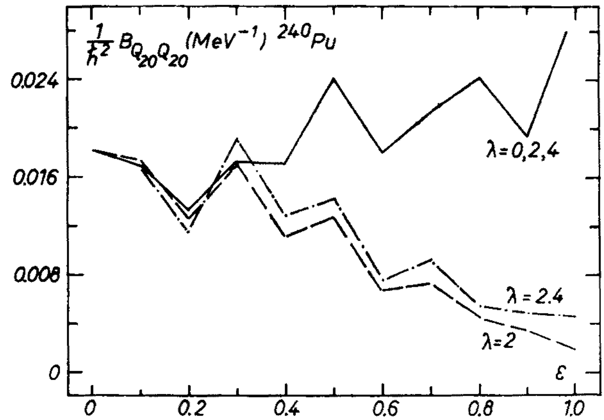

Fig. 6. Dependence of the quadrupole-quadrupole inertial mass parameter $B_{Q_{20}Q_{20}}$ of ^{240}Pu on deformation in the three cases: with coupling between monopole, quadrupole and hexadecapole vibrations taken into account (solid line), with coupling between two last ones (dotted-dashed line), and with quadrupole vibrations only (dashed line)

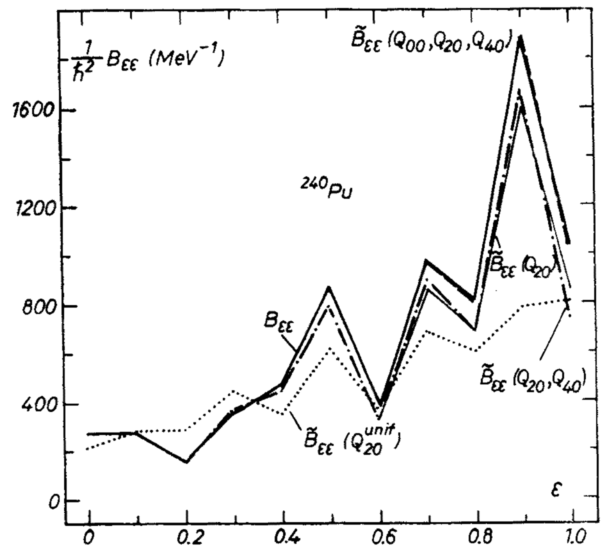

Fig. 7. Dependence of the inertial mass parameter $B_{\varepsilon\varepsilon}$ of ^{240}Pu on deformation. For details see text

has to put some kind of volume conservation condition on the vibrations. Then the monopole variable Q_{00} will depend on the other multipolarity variables.

The effect of taking into account the collective coordinates Q_{00}, Q_{20} and Q_{40} on $\tilde{B}_{\varepsilon\varepsilon}$ is shown in Fig. 7. The thick solid line represents the inertial mass parameter $B_{\varepsilon\varepsilon}$ calculated according to formula (2) (i.e. with the derivative of the Hamiltonian over deformation). The dashed line presents $\tilde{B}_{\varepsilon\varepsilon}$ of Eq. (22), calculated with the Q_{00}, Q_{20} and Q_{40} degrees of freedom taken into account. The quantities $B_{\varepsilon\varepsilon}$ and $\tilde{B}_{\varepsilon\varepsilon}$ are not distinguishable in the plot. The dashed-dotted line represents $\tilde{B}_{\varepsilon\varepsilon}$ calculated with only Q_{20} and Q_{40}. The thin solid line shows $\tilde{B}_{\varepsilon\varepsilon}$ when only Q_{20} is taken into account. The parameters $\tilde{B}_{\varepsilon\varepsilon}$ are also calculated using the macroscopic derivatives $\frac{\partial Q_{20}}{\partial \varepsilon}$ and assuming the quadrupole vibration only (dotted line in Fig. 7). $B_{\varepsilon\varepsilon}$ differs from $\tilde{B}_{\varepsilon\varepsilon}$ for Q_{20} only by about 4% in the first minimum and by about 16% in the second minimum. The coupling with the hexadecapole

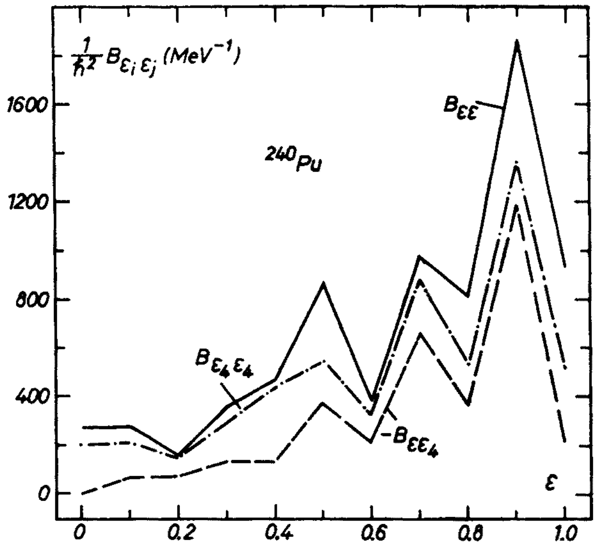

Fig. 8. Dependence of the inertial mass parameters $B_{\varepsilon\varepsilon}$, $-B_{\varepsilon\varepsilon_4}$ and $B_{\varepsilon_4\varepsilon_4}$ of ^{240}Pu on deformation

vibration contributes only to about 1.7% of the last difference while the rest (14.3%) of this difference is the effect of the coupling with the monopole vibration.

Finally, we present the results for all the three components of the mass tensor in the ε, ε_4 dynamical space, i.e., the parameters $B_{\varepsilon\varepsilon}$, $B_{\varepsilon\varepsilon_4}$ and $B_{\varepsilon_4\varepsilon_4}$ (see Fig. 8). One can see that the dependences of the three inertial mass parameters on deformation are similar. Here again, the values $\tilde{B}_{\varepsilon\varepsilon}$, $\tilde{B}_{\varepsilon\varepsilon_4}$ and $\tilde{B}_{\varepsilon_4\varepsilon_4}$ are not practically distinguishable, in the plot, from the values of $B_{\varepsilon\varepsilon}$, $B_{\varepsilon\varepsilon_4}$ and $B_{\varepsilon_4\varepsilon_4}$, respectively.

5. *Conclusions*

The following conclusions may be drawn:

(*i*) The monopole moment ($Q_{00} = \langle r^2 \rangle$) treated as a collective variable influences the values of the inertial mass parameter $B_{Q_{20}Q_{20}}$ corresponding to the quadrupole-quadrupole vibrations. This effect increases with the growth of the deformation and may be important for a dynamical calculation in the second minimum of the potential energy of fissioning nucleus.

(*ii*) The inertial mass parameter $\tilde{B}_{\varepsilon\varepsilon}$ "extracted" from the mass parameter corresponding to the multipole-multipole vibration is equal to that obtained from the adiabatic approximation for the dynamical variable ε.

(*iii*) The minima of the inertial mass parameters correspond to the minima of the potential energy of the nucleus (cf. Ref. [3]).

(*iv*) Deformation affects the inertial mass parameters $B_{\varepsilon\varepsilon}$, $B_{\varepsilon_4\varepsilon_4}$ and $-B_{\varepsilon\varepsilon_4}$ in a similar way; their minima and maxima are correlated.

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