A POSSIBLE INSTABILITY MECHANISM IN NEUTRON STARS

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The emission of photons by magnetic *Bremsstrahlung* of strongly interacting neutron pairs is considered. The emission rate is so high that it causes instabilities in the star's equilibrium.

Among all possible radiative processes which can take place in a neutron star (e.g. neutrino, graviton and black-body radiation) we shall consider in this paper electromagnetic radiation arising from magnetic *Bremsstrahlung* during nuclear interactions involving neutrons.

While magnetic interaction by itself (for instance magnetic dipole interaction between two neutrons) gives a negligible contribution to electromagnetic radiation, magnetic *Bremsstrahlung* by the intrinsic magnetic dipole of a neutron accelerated in a nuclear potential well gives, under suitable conditions, a very strong effect.

The aim of this work is to show how this emission of electromagnetic radiation can contribute strongly to the total radiation of a neutron star with a possible setting in of instabilities.

Neutron stars are thought to be formed at a very high temperature as a result of a supernova implosion. After the neutron star is formed, the temperature decreases quickly as a consequence of a cooling process, due mainly to neutrino emission from the interior and to electromagnetic radiation from the surface. In the first stage, when the temperature is very high, neutrino emission dominates, while at lower temperatures photon emission becomes important [1]. According to the view referred to above, a neutron star is thought to be in a state of nearly constant inner temperature ($\sim 10^9$ K) for about 10^3 years: we shall see now that this period can become troublesome for the stability of the star. Let

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us assume a typical neutron star with a volume $V \sim 10^{19}$ cm^3, a mass density $\delta \sim 10^{14}$ gr/cm^3 ($M \sim 1 M_\odot$) and neutron number density $n \sim 10^{38}$ cm^{-3}. The Fermi momentum is given by

$$n = \frac{P_F^3}{3\pi^2 \hbar^3}.$$

Then we have

$$P_F \simeq 1.3 \cdot 10^{-14} \text{ CGS units}$$

and the Fermi energy is

$$E_F = \frac{P_F}{2m} \sim 5 \cdot 10^{-5} \text{ erg} \sim 30 \text{ MeV}$$

so the neutron velocity is

$$v_0 = 0.75 \cdot 10^{10} \text{ cm/sec.}$$

Because of this great density, the mean distance between two neutrons is $3 \cdot 10^{-13}$ cm, therefore, there is a very high probability that scattering occurs.

Obviously, we need the potential responsible for the scattering between two particles which, in the present case, are 30 MeV neutrons. The potential must be in agreement with experimental data; from $p-p$ and $n-p$ scattering data, we can assume pure S-wave singlet scattering. In so doing, the error in the total cross section at 30 MeV is likely to be negligible. We shall, therefore, assume a Yukawa central potential

$$V(r) = -V_0 \beta \frac{\exp(-r/\beta)}{r} \tag{1}$$

with $\beta = 1.5 \cdot 10^{-13}$ cm and $V_0\beta = g^2 = \hbar c \cdot 0.008 = 2.5 \cdot 10^{-18}$ CGS units and where the spin-spin and the spin-orbit terms have been neglected because of the low energy involved.

As the neutron gas is degenerated one expects that the emitted photon energy is $E_\gamma \sim KT$ which is the energy available for neutrons above the Fermi energy. Now, for an inner temperature $\sim 10^9$ K $E_\gamma \ll E_F$ as $\dfrac{KT}{E_F(n)} \sim 3 \cdot 10^{-3}$ and this means we are dealing with "soft" photons and Weinberg's approximations [2] can be applied to our case.

On the other hand Carmeli [3] has shown that a classical treatment gives the same results as Weinberg's quantum theory. We are, therefore, justified in using Carmeli's classical approach.[1]

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[1] In fact, even if not like, the situation is very similar: Carmeli has an emission of gravitational waves through e. m. interaction, that is a weak radiation compared to the interaction mechanism; we have emission of e. m. waves through nuclear interaction which is still a weak radiation in respect to a strong interaction. For this reason we believe that we are justified in disregarding the quantum approach; on the other hand, quantum restrictions appear in our treatment as the vanishing of the magnetic dipole term is due to selection rules of the S- and P-waves. Once quantum restrictions are taken into account, the classical treatment, as usual, gives the correct order of magnitude.

According to this view, a neutron, when accelerated in the nuclear potential well (1), radiates electromagnetic waves because of its intrinsic magnetic moment $\mu \sim 10^{-23}$ erg/G. As the contribution of the magnetic dipole term in the usual multipole expansion vanishes, only the quadrupole term contributes to radiation, so:

$$Q_{ij} = \sum 3\mu_i x_j + 3\mu_j x_i - 2\delta_{ij}\vec{\mu} \cdot \vec{x}, \tag{2}$$

where Q_{ij} is the quadrupole momentum tensor, and $i, j = 1, 2, 3$. The sum is over neutron pairs.

First of all we shall evaluate the cross section σ_γ for the production of γ-rays by the collision of two neutrons which is given in [3] and [4]:

$$\sigma_\gamma = \frac{\chi}{E_F} = \frac{1}{E_F} \int_0^\infty \int_{-\infty}^{+\infty} I dt 2\pi \varrho d\varrho, \tag{3}$$

where $dt = \frac{dr}{v_r}$ is the collision time, ϱ the impact parameter and I the intensity, given by:

$$I = \frac{1}{180c^5} |\dddot{Q}_{ij}|^2. \tag{4}$$

For these calculations we shall use standard formulas, assuming $\vec{\mu}$ constant both in magnitude and direction. In fact, the radiation due to the variation of the direction of the magnetic moment is given only by the electromagnetic interaction of the two magnetic dipoles. But this interaction produces a radiation which has an intensity many orders of magnitude lower than the radiation produced because of the acceleration of a magnetic dipole by nuclear interaction.

Using formula (1), one gets:

$$\dddot{Q}_{ij} = \frac{V_0 \beta \exp(-r/\beta)}{m} \left\{ \frac{3\dot{r}a}{r^4} (\mu_i x_j + \mu_j x_i) - \frac{3b}{r^3} (\mu_i v_j + \mu_j v_i) - \frac{2\delta_{ij}}{r^3} \left(\frac{a\dot{r}}{r} \vec{\mu} \cdot \vec{r} - b\vec{\mu} \cdot \vec{v} \right) \right\}, \tag{5}$$

where

$$a = 3 + \frac{3r}{\beta} + \frac{r^2}{\beta^2}, \quad b = 1 + \frac{r}{\beta}$$

and all terms containing first and higher order derivatives of $\vec{\mu}$ have been neglected as such terms, being electromagnetic in origin, give negligible contributions.

As we limit ourselves to calculating the order of magnitude of the total emitted power, we shall consider the simplest case in which the magnetic dipole is perpendicular to the plane of motion.

Substituting (5) in (3) we get:

$$\sigma_\gamma = \frac{\chi}{E_F} = \frac{2\pi\mu^2 V_0^2 \beta^2}{10c^5 m_0^2 E_F} \int_{-\infty}^{+\infty} \frac{\exp(-2r/\beta)}{r^6}\, dr$$

$$\times \int_0^\infty \varrho d\varrho \cdot \frac{(4+8y+8y^2+4y^3+y^4)v^2-(3+6y+7y^2+4y^3+y^4)v_\varphi^2}{v_r}, \tag{6}$$

where

$$v_\varphi^2 = v^2 - v_r^2 = \frac{v_0^2}{r}\varrho^2, \quad v^2 = v_0 - \frac{2V_0 \exp(-r/\beta)}{m_0 r/\beta} - \frac{v_0^2}{r^2}\varrho^2,$$

$$m_0 = \frac{m}{2} \text{ (reduced mass)}$$

and $y = r/\beta$.
By performing the first integration, we get:

$$\sigma_\gamma = \frac{\chi}{E_F} = \frac{16\mu^2 V_0^2}{30c^5 m^2 v_0^2 \beta E_F} \cdot \frac{\exp(-2y)}{y^4}$$

$$\times \left(v_0^2 - \frac{2V_0}{m}\frac{\exp(-y)}{y}\right)^{3/2} (6+12y+10y^2+4y^3+y^4)dy. \tag{7}$$

An approximate evaluation of the last integral gives

$$\sigma_\gamma = \frac{\chi}{E_F} \sim \frac{8 \cdot 10^{-37}}{E_F} \sim 10^{-32}\ \text{cm}^2. \tag{8}$$

So far, the degeneracy of the neutron gas has not been taken into account: this implies, first of all, that only neutrons on the edge of their Fermi sea can undergo inelastic scattering. Thus a fraction of the order $\frac{KT}{E_F}$ of the neutrons can participate in the reaction. This argument does not apply to photons, but the photon phase space, which is proportional to E_γ^2, is reduced [1] by a factor $\left(\frac{KT}{E_F}\right)^2$.

Besides this, one must include one factor $\frac{KT}{E_F}$ for each final neutron that participates in the process.

The energy loss can so be written as:

$$L_\gamma \sim V n^2 \sigma_\gamma v_0 E_\gamma \left(\frac{KT}{E_F}\right)^6 \text{ erg/sec}. \tag{9}$$

As $E_\gamma \sim KT$, the total luminosity depends on the temperature as T^7. With our numerical values, $L_\gamma \sim 5 \cdot 10^{50}$ erg/sec[2], while the estimated rate for the neutrino emission is [1] $\sim 6 \cdot 10^{39}$ erg/sec. The above luminosity is computed assuming a constant temperature: see for instance [1].

The thermal energy of a neutron star is given approximately by

$$U = \frac{VKT}{E_F} nKT = \frac{n}{E_F}(KT)^2 \sim 5 \cdot 10^{47} \text{ erg}. \tag{10}$$

The photon emission process is therefore extremely quick.

The electron density in the inner part of neutron star in equilibrium with neutrons is [5] $\sim 10^{-3}$ n $\sim 10^{35}$ cm^{-3}; of these electrons only a fraction $\dfrac{KT}{E_F(e)} \sim \dfrac{KT}{E_F(n)} \sim 3 \cdot 10^{-3}$, i.e. $3 \cdot 10^{32}$ cm^{-3}, can interact with radiation [1]. Now the requirement of electrical neutrality [1] leads to the fact that the same number of protons is involved in the process: therefore, the total mass which interacts with the radiation is $3 \cdot 10^{27}$ grams and its gravitational energy is $2 \cdot 10^{47}$ erg which is of the same order of the total photon energy.

On the other hand this matter is opaque to radiation (as can be easily seen considering Thomson scattering) and because of the enormous radiation pressure, an instability phenomena can be produced e.g. by ejection of matter.

This problem is of the utmost interest and deserves further work.

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[2] Clearly, this rate is much greater than the Eddington limit that for a star of $1M_\odot$ is $\sim 10^{38}$ erg/sec and then we think that the charged matter can be pushed away before a thermalization is reached.