

## SHELL AND EXCITATION ENERGY EFFECTS IN THE POTENTIAL RADII DEDUCED FROM (d, p) REACTIONS ON s-d SHELL NUCLEI AT LOW DEUTERON ENERGY

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A systematic phase displacement has been observed between (d, p) reaction angular distributions measured and those calculated by the conventional DWBA method. It was found that this disagreement could be eliminated if shell and excitation energy dependent optical parameters were introduced.

Several years ago a work by Kaschl et al. [1] was published in which the authors reported a shell effect in the radius of the real part of the optical potential observed in the proton pick-up (d,  $^3\text{He}$ ) reaction on s-d shell nuclei. A similar effect was also observed in the neutron stripping (d, p) reaction on s-d shell nuclei at incident deuteron energies below the Coulomb barrier.

In the present work angular distributions of the products of (d, p) reaction on the nuclei:  $^{23}\text{Na}$ ,  $^{24}\text{Mg}$ ,  $^{26}\text{Mg}$ ,  $^{27}\text{Al}$ ,  $^{28}\text{Si}$ ,  $^{31}\text{P}$ ,  $^{32}\text{S}$ ,  $^{34}\text{S}$  and  $^{35}\text{Cl}$  have been measured at incident deuterons energies of 2.4, 2.5 and 2.6 MeV. To eliminate fluctuations of the cross sections of the competitive compound reaction mechanism, the measured distributions were averaged over energy. The angular distributions averaged in this way were compared with the theoretical ones, obtained as sums of the distributions calculated by the conventional DWBA method and of the distributions obtained in the Hauser-Feshbach formalism [ $\sigma_{\text{exp}}(\theta) = S \times \sigma_{\text{DWBA}}(\theta) + H \times \sigma_{\text{HF}}(\theta)$ , where  $S$  and  $H$  are free parameters in least square method]. The DWBA calculations were performed using the modified JULIE programme (providing for finite range and nonlocality in the local energy approximation [2]) and the values of the optical potential parameters taken from the work of Becchetti and Greenless [3] for protons and neutrons, and from the work of Schwandt and Haeberli [4] for deuterons.

Ample experimental material, consistently elaborated, make possible to conclude that the theoretical angular distributions are systematically shifted in phase with regard to the measured ones. The difference between the positions of the first minimum for  $s_{1/2}$

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transitions or first maximum for  $d_{3/2}$  and  $d_{5/3}$  transitions found experimentally and theoretically for selected (with a high spectroscopic factors) final nuclei states depending on their excitation energy is shown in Fig. 1a.

As the  $s-d$  shell nuclei reveal a strong deformation (cf., for instance, [5] and [6]), it may be expected that a strong coupling will appear in the entrance and exit channels modifying the angular distribution of the products of the  $(d, p)$  reaction. When in the

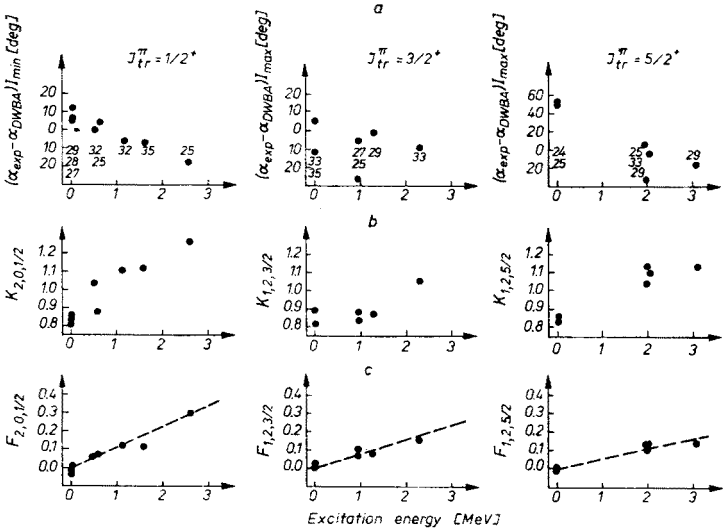


Fig. 1. Differences between positions of first minima (for  $l = 0$ ) or first maxima (for  $l = 2$ ) in angular distributions measured and calculated by the conventional DWBA method (a). Best-fit of  $K_{NlJ}$  values (b) and the proposed quantity  $F_{NlJ}$  (c), as a function of the excitation energy of the final nucleus. The points in Fig. 1a are provided with the mass numbers of the final nucleus

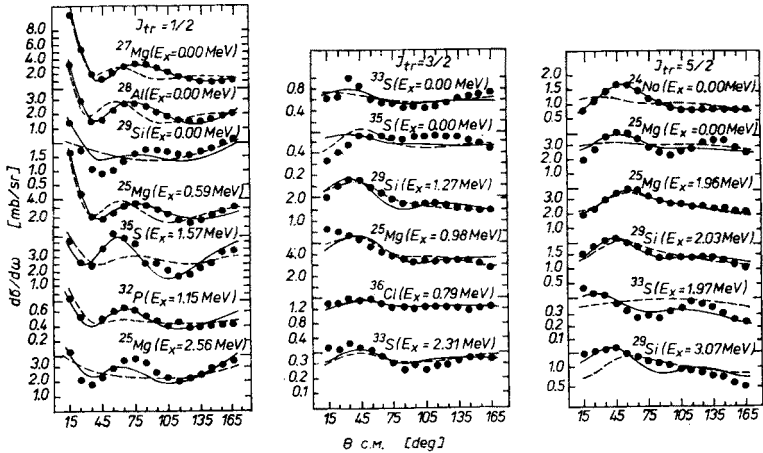


Fig. 2. Selected angular distributions measured for reactions with  $j_{tr}^{\pi} = 1/2^+, 3/2^+, 5/2^+$ . Dashed curves represent the calculations performed by the conventional, finite-range, non-local DWBA method. Solid curves represent finite-range, non-local DWBA calculations with modified radii

DWBA calculations the strong coupling was provided for in an approximate way (Kunz et al. method [7]), the existing disagreement between theoretical and experimental distributions was not eliminated. It was therefore decided to modify the parameter  $r_0$  of the real part of the optical potential radius by introducing a common factor for both channels, viz.  $k_{Nlj}(r_0 \rightarrow r_0 \times k_{Nlj})$ , where  $Nlj$  are: main quantum number, orbital momentum and total momentum, for which the "eye fit" of the theoretical angular distributions to the experimental ones is the best. The quality of the fits is shown in Fig. 2. An additional simultaneous change of the bound state radius parameter, practically not affecting the shape of the angular distribution, changes the absolute value of the cross section. Figs 1 b, c show the variation of the factor  $k_{Nlj}$  and of the quantity  $F_{Nlj}$  (a function of the found  $k_{Nlj}$  values) we have introduced with the excitation energy of the final nucleus. The  $F_{Nlj}$  function has the following form:

$$F_{Nlj} = (k_{Nlj} - k_{Nlj}^0) \times (k_{Nlj}^0)^2 = \beta_{Nlj} \times E_x, \quad (1)$$

where

$$k_{Nlj}^0 = \begin{cases} A/A_0 & \text{for } A \leq A_0 \\ 1 & \text{for } A > A_0 \end{cases} \quad (2)$$

$A$  is the mass number of the target nucleus and  $A_0$  is the mass number of the nucleus in which the subshells (proton and neutron ones) with quantum numbers  $N, l, j$  are closed.

For subshells  $1d5/2$ ,  $2s1/2$  and  $1d3/2$  the values of  $A_0$  are 28, 32 and 40, respectively.

Hence the factors  $k_{Nlj}$  obtained from the best fit can be described by the function:

$$k_{Nlj} = k_{Nlj}^0 + \beta_{Nlj} \times (k_{Nlj}^0)^{-2} \times E_x. \quad (3)$$

The  $\beta_{Nlj}$  parameters are:

$$\beta_{2,0,1/2} = 0.115, \quad \beta_{1,2,3/2} = 0.081, \quad \beta_{1,2,5/2} = 0.055.$$

Hence for the (d, p) reaction on s-d shell nuclei at the incident deuteron energies somewhat lower than the Coulomb barrier we observe a distinct dependence of the modified parameter  $r_0$  on the excitation energy, as well as a shell effect for this parameter. From these relations it follows that:

1. The angular distributions of the (d, p) reaction products corresponding to transitions to various final nucleus states cannot be fitted satisfactorily by one value of the  $r_0$  parameter, common for all these distributions. The value of this parameter depends on the quantum characteristics  $N, l$ , and  $j$  of the subshell to which the neutron was transferred and also on the final state excitation energy.

2. Parameter  $r_0$  increases with the increasing mass number  $A$  of the target nucleus and from the  $A_0$  value, corresponding to a nucleus with closed proton and neutron  $N, l, j$  shells,  $r_0$  is constant.

3. The higher the final state excitation energy, the larger the value of parameter  $r_0$ , the rate of its growth increasing as the  $N, l, j$  shells are further from being closed, and as

the  $j$  value of the shell (i.e., the number of particles which can be placed in the shell) decreases.

The effect of the  $r_0$  change observed in the (d,  $^3\text{He}$ ) pick-up analysis [1], where the subshell radius  $R_{ij}^N$  is independent of the nucleus mass number and of its excitation energy. Hence for the (d,  $^3\text{He}$ ) pick-up the changes of the  $r_0$  parameter are smaller and proceed in the direction opposite to that of the values of corrections obtained in the present work. In the case of (p, d) reaction on s-d shell nuclei [8] the changes of the neutron bound state radius, introduced to eliminate phase disagreements between theoretical and experimental angular distributions, are significant and have the same direction as the changes of the radii in both channels observed in the present work.

The modifications we have introduced make the parameter  $r_0$ , and hence the shape of the angular distribution, dependent not only on  $l$  but also on the  $j$  value (effect of Lee and Schiffer [9]). The angular distributions measured in our experiment as well as the fitted theoretical ones show a characteristic dependence on the  $j$  value the same as that of the Lee and Schiffer effect (cf., for instance, Figs 7 and 8 in Ref. [9]) in (d, p) reactions at higher energies of the incident deuterons: for the  $l = 2$  transition the first maximum for  $j = l - \frac{1}{2}$  lies at lower angles than in the case of  $j = l + \frac{1}{2}$ .

It is as yet difficult to propose a reliable explanation of the above related dependence of the modified parameter  $r_0$  on the  $N$ ,  $l$  and  $j$  values of the shell to which the nucleon

TABLE

Comparison between the  $k_{Nlj}$  values calculated from the given formula and those computed by the Kunz et al. method [ $k^{\text{KUNZ}} = 1 + |\beta_2| \times k(\beta_\lambda)$ ]

$E_x$ [MeV]	$J^\pi$	$\Omega^\pi[Nn_zA]$	$k_{Nlj}$	$k^{\text{KUNZ}}$
$^{25}\text{Mg}$ [ $\beta_2 = 0.39$ ; $\beta_4 = -0.015$ ]				
0.000	5/2 <sup>+</sup>	5/2 <sup>+</sup> [202]	0.857	0.901
0.585	1/2 <sup>+</sup>	1/2 <sup>+</sup> [211]	0.870	0.975
0.975	3/2 <sup>+</sup>	1/2 <sup>+</sup> [211]	0.819	1.015
1.960	5/2 <sup>+</sup>	1/2 <sup>+</sup> [211]	1.004	1.107
2.562	1/2 <sup>+</sup>	1/2 <sup>+</sup> [200]	1.275	1.113
2.801	3/2 <sup>+</sup>	1/2 <sup>+</sup> [200]	1.230	1.233
3.905	5/2 <sup>+</sup>	1/2 <sup>+</sup> [200]	1.150	1.426
$^{27}\text{Mg}$ [ $\beta_2 = 0.28$ ; $\beta_4 = -0.02$ ]				
0.000	1/2 <sup>+</sup>	1/2 <sup>+</sup> [211]	0.813	0.995
0.984	3/2 <sup>+</sup>	1/2 <sup>+</sup> [211]	0.940	0.996
$^{29}\text{Si}$ [ $\beta_2 = -0.38$ ; $\beta_4 = 0.08$ ]				
0.000	1/2 <sup>+</sup>	1/2 <sup>+</sup> [211]	0.875	0.951
1.273	3/2 <sup>+</sup>	3/2 <sup>+</sup> [202]	0.910	0.977
2.027	5/2 <sup>+</sup>	1/2 <sup>+</sup> [211]	1.111	1.002
2.426	3/2 <sup>+</sup>	1/2 <sup>+</sup> [211]	1.100	1.087
3.067	5/2 <sup>+</sup>	3/2 <sup>+</sup> [202]	1.169	1.101

is being transferred and on the excitation energy of the final state. The dependence may be due to the occurrence of a deformation in the nuclei studied (also of a deformation dependent on the nucleus excitation energy) and to the related effect of strong coupling of states in the initial and final reaction nuclei. The agreement in the direction of changes of parameter  $r_0$  (see Table), calculated in accordance with the formula in the paper, as well as the changes of  $r_0$  calculated according to the approximate method (Kunz et al. [7]), may constitute a premise for such an explanation.

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