THE SPIN-ISOSPIN SYMMETRY ENERGY OF NUCLEAR MATTER

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The spin-isospin symmetry energy of nuclear matter, $\varepsilon_{\sigma\tau}$, is calculated within the frame of the K matrix theory, in an approximation in which the K matrix depends on a single density. Results obtained for $\varepsilon_{\sigma\tau}$ with the Brueckner–Gammel Thaler and the Reid soft core potential, together with previous results for the isospin and spin symmetry energies, ε_τ and ε_σ, are presented and discussed. The most reliable result is: $\varepsilon_\tau = 53$ MeV, $\varepsilon_\sigma = 65$ MeV, $\varepsilon_{\sigma\tau} = 76$ MeV, in a reasonable agreement with that obtained with the "empirical" Landau parameters.

1. Introduction

Nuclear matter is a four component system composed of $N_\uparrow$ neutrons with spin up, $N_\downarrow$ neutrons with spin down, $Z_\uparrow$ protons with spin up, and $Z_\downarrow$ protons with spin down. We shall call this system polarized nuclear matter. The term unpolarized nuclear matter shall be used in the special case, when $N_\uparrow = N_\downarrow = Z_\uparrow = Z_\downarrow = A/4$. All the nucleons are contained in a periodicity box of volume Ω. The composition of the system may be characterized by $A = N_\uparrow + N_\downarrow + Z_\uparrow + Z_\downarrow$, by the neutron (or isospin) excess parameter $\alpha_\tau = (N_\uparrow + N_\downarrow - Z_\uparrow - Z_\downarrow)/A$, by the spin excess parameter $\alpha_\sigma = (N_\uparrow + Z_\uparrow - N_\downarrow - Z_\downarrow)/A$, and by the spin-isospin excess parameter $\alpha_{\sigma\tau} = (A_+ - A_-)/A$, where $A_+ = N_\uparrow + Z_\downarrow$, and $A_- = N_\downarrow + Z_\uparrow$.

The ground state energy of polarized nuclear matter, expanded in powers of the α's, has the form [1] (hereafter referred to as DH)

$$E/A = \varepsilon_{\text{vol}} + \tfrac{1}{2}(\varepsilon_\tau \alpha_\tau^2 + \varepsilon_\sigma \alpha_\sigma^2 + \varepsilon_{\sigma\tau} \alpha_{\sigma\tau}^2), \tag{1.1}$$

where powers higher than quadratic are neglected. Apart from the volume energy, ε_{vol}, and the usual (isospin) symmetry energy denoted by ε_τ, we have in (1.1) the spin symmetry energy ε_σ, and the spin-isospin symmetry energy $\varepsilon_{\sigma\tau}$.

The knowledge of ε_τ, ε_σ, and $\varepsilon_{\sigma\tau}$ is important in determining the effective nucleon-

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-nucleon interaction to be used in nuclear Hartree-Fock calculations. Such an effective interaction should be adjusted so as to give not only the right saturation density and binding energy of nuclear matter, but it is important that it also leads to the right values of the three symmetry energies. This point has been discussed recently in connection with the Skyrme type interaction in [2, 3, 4].

Among the three symmetry energies, the isospin symmetry energy ε_τ is most directly related to experiment. Because of Coulomb forces, the medium and heavy nuclei have in their ground state an appreciable neutron excess, and thus by analysing binding energies of stable nuclei, we may get a reliable empirical estimate of ε_τ.

The spin and spin-isospin symmetry energies are not related directly to the properties of nuclear ground states (all even-even nuclei have zero spin in their ground state). One possibility of a semi-empirical estimate of ε_σ and $\varepsilon_{\sigma\tau}$ (and also ε_τ) is connected with the fact that ε_σ and $\varepsilon_{\sigma\tau}$, similarly as ε_τ, may be related to the properties of certain collective excited states (giant resonances). Within a specific model of these excitations (generalized Goldhaber-Teller model), we may obtain a rough estimate of ε_σ and $\varepsilon_{\sigma\tau}$ from the measured excitation energies [1]. Another possibility of estimating ε_σ and $\varepsilon_{\sigma\tau}$ is rendered by a relation between ε_σ and $\varepsilon_{\sigma\tau}$, and the spin dependent part of the optical model potential U_{ss} [5, 6]. However, the magnitude of U_{ss} has not been determined experimentally with any reasonable accuracy so far.

In this situation, it is important to calculate ε_σ and $\varepsilon_{\sigma\tau}$ with realistic nuclear forces. It seems that such calculated values of ε_σ and $\varepsilon_{\sigma\tau}$ are at the moment the most reliable ones to which any effective nucleon-nucleon interaction should be adjusted. Such a calculation with the Brueckner-Gammel-Thaler (BGT) [7] and the Reid soft core (RSC) [8] potential has been presented in DH.

To calculate any of the symmetry energies, one should consider nuclear matter composed of two components, e. g., of nucleons with spin up and down in the case of ε_σ. With the help of an approximation (single density approximation) applied previously in calculating ε_τ in [9], the values of ε_σ and $\varepsilon_{\sigma\tau}$ have been deduced in DH from the properties of unpolarized nuclear matter. Whereas this single density approximation of DH for calculating ε_σ is a simple modification of the analogous approximation of [9], the case of $\varepsilon_{\sigma\tau}$ is slightly more involved. In fact, the way in which the single density approximation was applied in DH for calculating $\varepsilon_{\sigma\tau}$ has not been completely correct and should be improved. It is the purpose of the present paper to clarify and improve the single density approximation for calculating $\varepsilon_{\sigma\tau}$, and to see the effect of this improvement on the resulting value of $\varepsilon_{\sigma\tau}$.

In Section 2, we derive the correct expression for $\varepsilon_{\sigma\tau}$ in the single density approximation. The expression contains the effective interaction in unpolarized nuclear matter K, and its derivatives with respect to the Fermi momentum.

In Section 3, we present the results obtained for $\varepsilon_{\sigma\tau}$ with K calculated within the Brueckner theory with the BGT and RSC nucleon-nucleon forces. Results obtained previously for ε_τ and ε_σ are also presented. The agreement with results obtained with "empirical" Landau parameters is pointed out. The magnitude of the spin-spin part of the optical model potential U_{ss} is also discussed.

2. Single density approximation for $\varepsilon_{\sigma\tau}$

To calculate $\varepsilon_{\sigma\tau}$, we may put $\alpha_\tau = \alpha_\sigma = 0$, and we have a two component system with $N_\uparrow = Z_\uparrow = A_+/2$, and $N_\downarrow = Z_\downarrow = A_-/2$. The two corresponding densities, ϱ_+, ϱ_-, and the two corresponding Fermi momenta, κ, λ (in units of $\hbar$) are

$$\kappa^3 = 3\pi^2\varrho_+ = k_F^3(1+\alpha_{\sigma\tau}),$$

$$\lambda^3 = 3\pi^2\varrho_- = k_F^3(1-\alpha_{\sigma\tau}), \tag{2.1}$$

where k_F is the Fermi momentum for $\alpha_{\sigma\tau} = 0$,

$$k_F^3 = \tfrac{3}{2}\pi^2\varrho, \tag{2.2}$$

where $\varrho = \varrho_+ + \varrho_- = A/\Omega$.

The contribution of the kinetic energy to $\varepsilon_{\sigma\tau}$, $\varepsilon_{\sigma\tau}^{\mathrm{KIN}}$, is well known

$$\varepsilon_{\sigma\tau}^{\mathrm{KIN}} = \tfrac{2}{3}\varepsilon_F, \tag{2.3}$$

where ε_F is the Fermi energy, $\varepsilon_F = \hbar^2 k_F^2/2M$.

To calculate the contribution of the potential energy to $\varepsilon_{\sigma\tau}$, $\varepsilon_{\sigma\tau}^{\mathrm{POT}}$, we have to calculate the potential part of the energy of our system, E_{POT}, from which we then obtain

$$\varepsilon_{\sigma\tau}^{\mathrm{POT}} = (\partial^2(E_{\mathrm{POT}}/A)/\partial\alpha_{\sigma\tau}^2)_0 \tag{2.4}$$

where the subscript "0" indicates the value of the derivative at the point $\alpha_{\sigma\tau} = 0$.

To calculate E_{POT}, we apply the Brueckner theory. However, in our derivation of the expression for $\varepsilon_{\sigma\tau}$, we only assume that E_{POT} may be obtained from an effective two-body interaction in nuclear matter, the K matrix. The particular way, in which K is determined from nuclear forces, is irrelevant for our derivation.

We start with the expression

$$E_{\mathrm{POT}} = \tfrac{1}{2}\left[2\sum_{k_1}^{\kappa} V(\boldsymbol{k}_1\uparrow\mathrm{n}) + 2\sum_{k_1}^{\lambda} V(\boldsymbol{k}_1\downarrow\mathrm{n})\right], \tag{2.5}$$

where, e.g., $V(\boldsymbol{k}_1\uparrow\mathrm{n})$ is the single nucleon potential of a spin up neutron with momentum $\boldsymbol{k}_1$ (in units of $\hbar$), and where the superscript $\kappa(\lambda)$ means that the sum is restricted to $k_1 < \kappa(\lambda)$. Notice that

$$V(\boldsymbol{k}_1\uparrow\mathrm{n}) = V(\boldsymbol{k}_1\downarrow\mathrm{p}), \qquad V(\boldsymbol{k}_1\downarrow\mathrm{n}) = V(\boldsymbol{k}_1\uparrow\mathrm{p}), \tag{2.6}$$

where, e.g., $V(\boldsymbol{k}_1\downarrow\mathrm{p})$ is the single particle potential of a spin down proton. Relations (2.6) have been used in writing expression (2.5), and give rise to the factors 2.

For the single particle potentials, we have

$$V(\boldsymbol{k}_1\uparrow\mathrm{n}) = \sum_{k_2}^{\kappa}\{(\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{n}|K(\kappa)\,|\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{n})$$

$$+(\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{p}|K(\kappa)\,|\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{p})-\mathrm{exch}\}$$

$$+\sum_{k_2}^{\lambda}\{(\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{n}|K(\tilde{k}_F)\,|\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{n})+(\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{p}|K(\tilde{k}_F)\,|\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{p})-\mathrm{exch}\},$$

$$V(\boldsymbol{k}_1\downarrow\mathrm{n}) = \sum_{k_2}^{\kappa} \{(\boldsymbol{k}_1\downarrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{n}|K(\tilde{k}_\mathrm{F})\,|\boldsymbol{k}_1\downarrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{n})$$
$$+(\boldsymbol{k}_1\downarrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{p}|K(\tilde{k}_\mathrm{F})\,|\boldsymbol{k}_1\downarrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{p})-\mathrm{exch}\}$$
$$+\sum_{k_2}^{\lambda}\{(\boldsymbol{k}_1\downarrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{n}|K(\lambda)\,|\boldsymbol{k}_1\downarrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{n})+(\boldsymbol{k}_1\downarrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{p}|K(\lambda)\,|\boldsymbol{k}_1\downarrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{p})-\mathrm{exch}\}. \quad (2.7)$$

In nuclear matter with $\alpha_{\sigma\tau} \neq 0$, i.e., with $\kappa \neq \lambda$, the effective nucleon-nucleon interaction depends on two Fermi momenta, $K = K(\kappa, \lambda)$. Consequently, in Eqs (2.7), we should have everywhere $K(\kappa, \lambda)$. Instead, we have replaced the reaction matrices $K(\kappa, \lambda)$ by K matrices which depend on single Fermi momenta (i.e., on corresponding single densities): κ, λ, and

$$\tilde{k}_\mathrm{F} = \sqrt{(\kappa^2+\lambda^2)/2}\,. \quad (2.8)$$

This replacement constitutes the single density approximation, and Eqs (2.7), (2.5) express the potential energy of our system with $\alpha_{\sigma\tau} \neq 0$ in this approximation.

The justification of the single density approximation goes as follows. If we have two nucleons whose Fermi momenta are the same, the effective interaction between them is determined predominantly by their common Fermi momentum, e.g.,

$$(\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{p}|K(\kappa,\lambda)|\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{p}) \cong (\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{p}|K(\kappa)|\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\downarrow\mathrm{p}), \quad (2.9)$$

where $K(\kappa)$ is the K matrix in unpolarized nuclear matter with the Fermi momentum κ. In second order perturbation treatment of K, where the whole density dependence of K is due only to the Pauli principle, relation (2.9) is exact.

If we have two nucleons whose Fermi momenta are different, we expect that the effective interaction between them may be approximated by an interaction which depends on an average Fermi momentum $\tilde{k}_\mathrm{F}$, e.g.,

$$(\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{p}|K(\kappa,\lambda)|\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{p}) \cong (\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{p}|\,K(\tilde{k}_\mathrm{F})|\boldsymbol{k}_1\uparrow\mathrm{n}\boldsymbol{k}_2\uparrow\mathrm{p}), \quad (2.10)$$

where $K(\tilde{k}_\mathrm{F})$ is the K matrix in unpolarized nuclear matter with the Fermi momentum $\tilde{k}_\mathrm{F}$. Approximation (2.10), although plausible, is less justified than approximation (2.9).

By inserting expressions (2.7) into Eq. (2.5), we get

$$E_\mathrm{POT} = \sum_{k_1}^{\kappa}\sum_{k_2}^{\kappa}(\boldsymbol{k}_1\boldsymbol{k}_2|2K(11,1;\kappa)+\tfrac{1}{2}\sum_s\sum_T K(s0,T;\kappa)\,|\boldsymbol{k}_1\boldsymbol{k}_2)$$
$$+\sum_{k_1}^{\lambda}\sum_{k_2}^{\lambda}(\boldsymbol{k}_1\boldsymbol{k}_2|2K(11,1;\lambda)+\tfrac{1}{2}\sum_s\sum_T K(s0,T;\lambda)\,|\boldsymbol{k}_1\boldsymbol{k}_2)$$
$$+2\sum_{k_1}^{\kappa}\sum_{k_2}^{\lambda}(\boldsymbol{k}_1\boldsymbol{k}_2|\sum_s K(s0,1;\tilde{k}_\mathrm{F})+\sum_T K(11,T;\tilde{k}_\mathrm{F})\,|\boldsymbol{k}_1\boldsymbol{k}_2), \quad (2.11)$$

with the notation

$$(\boldsymbol{k}_1\boldsymbol{k}_2|K(sm_s,T;\kappa)|\boldsymbol{k}_1\boldsymbol{k}_2) = (\boldsymbol{k}_1\boldsymbol{k}_2sm_sT|K(\kappa)|\boldsymbol{k}_1\boldsymbol{k}_2sm_sT) \quad (2.12)$$

for the K matrix in the representation of the total spin and its third component s, m_s, and of the total isospin T of the two interacting nucleons. In obtaining Eq. (2.11), we have used the fact that in unpolarized nuclear mattter K does not depend on the third component of the total isospin, and on the sign of m_s.

When we calculate $\varepsilon_{\sigma\tau}^{\mathrm{POT}}$ according to Eq. (2.4), we have to take into account two ways in which E_{POT}, Eq. (2.11), depends on $\alpha_{\sigma\tau}$: first, through the upper limits of the sums over $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, and second, through the intrinsic dependence of K on κ, λ, and $\tilde{k}_{\mathrm{F}}$. For $\varepsilon_{\sigma\tau}^{(0)\mathrm{POT}}$, the part of $\varepsilon_{\sigma\tau}^{\mathrm{POT}}$ which arises entirely from the first type of the dependence, we get

$$\varepsilon_{\sigma\tau}^{(0)\mathrm{POT}} = 2\int \frac{d\hat{k}_{\mathrm{F}}}{4\pi} S^{\sigma\tau}(\boldsymbol{k}_{\mathrm{F}}) + \tfrac{1}{3} k_{\mathrm{F}} \left[\frac{\partial V_0(k)}{\partial k}\right]_{k=k_{\mathrm{F}}}, \tag{2.13}$$

where

$$S^{\sigma\tau}(\boldsymbol{k}) = \frac{A}{8}\int \frac{d\hat{k}_{\mathrm{F}}}{4\pi}\left(\boldsymbol{k}\boldsymbol{k}_{\mathrm{F}}\left|\sum_T (2T-1)\left[K(11,T) - \tfrac{1}{2}K(10,T) - \tfrac{1}{2}K(00,T)\right]\right|\boldsymbol{k}\boldsymbol{k}_{\mathrm{F}}\right), \tag{2.14}$$

and $V_0(k)$ is the single particle potential in unpolarized nuclear matter (see, e.g., [6]), and where we use the notation $K(sm_s, T) = K(sm_s, T; k_{\mathrm{F}})$.

For $\Delta\varepsilon_{\sigma\tau}$, the part of $\varepsilon_{\sigma\tau}^{\mathrm{POT}}$ which arises from the second type of the dependence, we get

$$\Delta\varepsilon_{\sigma\tau} = \Delta_0\varepsilon + \Delta_1\varepsilon_{\sigma\tau}, \tag{2.15}$$

where

$$\Delta_0\varepsilon = -\tfrac{1}{3}A^{-1}\sum_{k_1}^{k_{\mathrm{F}}}\sum_{k_2}^{k_{\mathrm{F}}}\left(\boldsymbol{k}_1\boldsymbol{k}_2\left|k_{\mathrm{F}}\frac{d}{dk_{\mathrm{F}}}\sum_T (2T+1)\sum_{sm_s} K(sm_s, T)\right|\boldsymbol{k}_1\boldsymbol{k}_2\right)$$

$$= -\tfrac{2}{3}V_{0\mathrm{R}}(k_{\mathrm{F}}), \tag{2.16}$$

where $V_{0\mathrm{R}}(k_{\mathrm{F}})$ is the rearrangement potential at the Fermi surface in unpolarized nuclear matter (see, e. g., [6]), and where

$$\Delta_1\varepsilon_{\sigma\tau} = \Delta_{1a}\varepsilon_{\sigma\tau} + \Delta_{1b}\varepsilon_{\sigma\tau}, \tag{2.17}$$

where

$$\Delta_{1a}\varepsilon_{\sigma\tau} = \tfrac{2}{3}\int\frac{d\hat{k}_{\mathrm{F}}}{4\pi}\sum_{k_2}^{k_{\mathrm{F}}}\left(\boldsymbol{k}_{\mathrm{F}}\boldsymbol{k}_2\left|k_{\mathrm{F}}\frac{d}{dk_{\mathrm{F}}}\left[2K(11,1) + \tfrac{1}{2}\sum_s\sum_T K(s0,T)\right]\right|\boldsymbol{k}_{\mathrm{F}}\boldsymbol{k}_2\right), \tag{2.18}$$

$$\Delta_{1b}\varepsilon_{\sigma\tau} = \tfrac{2}{9}A^{-1}\sum_{k_1}^{k_{\mathrm{F}}}\sum_{k_2}^{k_{\mathrm{F}}}\left(\boldsymbol{k}_1\boldsymbol{k}_2\left|k_{\mathrm{F}}\frac{d}{dk_{\mathrm{F}}}\left[\sum_T K(11,T) + \sum_s K(s0,1)\right]\right.\right.$$

$$\left.\left. + k_{\mathrm{F}}^2\frac{d^2}{dk_{\mathrm{F}}^2}\left[2K(11,1) + \tfrac{1}{2}\sum_s\sum_T K(s0,T)\right]\right|\boldsymbol{k}_1\boldsymbol{k}_2\right). \tag{2.19}$$

By adding all the contributions to $\varepsilon_{\sigma\tau}$, we get

$$\varepsilon_{\sigma\tau} = \varepsilon_{\sigma\tau}^{\mathrm{KIN}} + \varepsilon_{\sigma\tau}^{(0)\mathrm{POT}} + \Delta\varepsilon_{\sigma\tau}. \tag{2.20}$$

When we compare expressions (2.18), (2.19) with expression (28) of DH for $\Delta_1\varepsilon_{\sigma\tau}$, we notice that the second part of (2.18), the term with $\frac{1}{2}\sum_{s}\sum_{T} K$, is missing in expression (28) of DH. This is the only difference between the present results and those of DH. The starting point for the single density approximation in DH was the correct expression for $\Delta_1\varepsilon_{\sigma\tau}$ in terms of the K matrix depending on two Fermi momenta, Eq. (22) of DH. Into this expression, approximations (24a) — (24d) of DH have been introduced. Among them, approximation (24d) went beyond the single density approximation. This, in turn, led to the final expression for $\Delta_1\varepsilon_{\sigma\tau}$, Eq. (28) of DH, without the term with the first derivative of $\frac{1}{2}\sum_{s}\sum_{T} K$. This shortcoming of DH is corrected in the present paper in which the single density approximation is introduced in a consequent way at the beginning of our derivation, in Eq. (2.7).

Because of the single density approximation, Eq. (2.9) and in particular Eq. (2.10), our result for $\varepsilon_{\sigma\tau}$ is approximate. To avoid this approximation, one would have to determine an effective interaction, $K(\kappa, \lambda)$, in a two component system with two Fermi momenta, κ and λ. Analogous calculations have been performed in case of ε_τ [10, 11, 12]. Such involved calculations could also be performed in case of $\varepsilon_{\sigma\tau}$ (and ε_σ), where, however, an additional complication arises because of the deformation of the two Fermi spheres in spin-isospin polarized nuclear matter [13]. The effect of this deformation on $\varepsilon_{\sigma\tau}$ (and ε_σ) has been estimated in [13] to be very small, and is neglected in the present paper.

3. Numerical results and discussion

In our numerical calculations, we use the BGT and RSC nucleon-nucleon potentials. To determine the K matrix elements, needed for calculating $\varepsilon_{\sigma\tau}^{(0)\mathrm{POT}}$, we apply the Brueckner theory. The calculation has been described and performed in DH. The value of $\Delta_0\varepsilon$ has been determined by applying the Hugenholtz–Van Hove theorem [14]. In calculating $\Delta_1\varepsilon_{\sigma\tau}$ for the BGT potential, we have used the Fermi momentum dependence of the K matrix determined in [15], and for the RSC potential, we have used the effective interaction G–O of Sprung and Banerjee [16]. Except for the term with the first derivative of $\frac{1}{2}\sum_{s}\sum_{T} K$ in Eq. (2.8), the calculation is identic with that described in DH and in [6].

The calculated values of $\varepsilon_{\sigma\tau}$ at the equilibrium densities, determined for the BGT potential in [7] and for the RSC potential in [17], are given in Table I. Values of ε_τ and ε_σ, calculated in DH, are also shown in Table I. The present way of calculating $\varepsilon_{\sigma\tau}$ (according to Eq. (2.18), with the term $\frac{1}{2}\sum_{s}\sum_{T} K$) leads to bigger values of $\varepsilon_{\sigma\tau}$ compared to the result of DH, where we have obtained $\varepsilon_{\sigma\tau} = 76.9$ MeV for the BGT potential and $\varepsilon_{\sigma\tau} = 73.0$ MeV for the RSC potential.

The results obtained with the more up to date RSC potential, and with the rearrangement contributions $\Delta\varepsilon_x (x = \tau, \sigma, \sigma\tau)$ calculated with the carefully adjusted effective interaction [16], are more reliable than those obtained with the old BGT potential. The RSC values of ε_x have been calculated at the equilibrium density for the RSC potential ($k_\mathrm{F} = 1.43\ \mathrm{fm}^{-1}$ [17]). To obtain the RSC values of $\varepsilon_{\sigma\tau}$ at the empirical density ($k_\mathrm{F} =$

$= 1.36\,\mathrm{fm}^{-1}$), we have proceeded in the following way. The part containing $S^{\sigma\tau}$, Eq. (2.13), has been obtained from its value calculated at $k_F = 1.43\,\mathrm{fm}^{-1}$ by assuming a linear dependence in ϱ. The term $\frac{1}{3} k_F(\partial V_0/\partial k)_{k_F}$ has been obtained from the values of the effective mass as a function of k_F, given in [17]. The rearrangement part $\Delta\varepsilon_{\sigma\tau}$ has been calculated with the help of the effective interaction [16] with $k_F = 1.36\,\mathrm{fm}^{-1}$. An analogous pro-

TABLE I

Results for ε_x and U_x (in MeV)

Potential	BGT	RSC
$k_F(\mathrm{fm}^{-1})$	1.49[a]	1.43[b]
ε_τ	64.1[c]	60.5[d]
ε_σ	64.9[d]	74.1[d]
$\varepsilon_{\sigma\tau}$	92.5	86.6
$U_\tau(k_F)$	126.1[c]	129.8[c]
$U_\sigma(k_F)$	128.7[c]	177.8[c]
$U_{\sigma\tau}(k_F)$	239.5	225.3

[a] Determined in [7]. [b] Determined in [17]. [c] Determined in [9]. [d] Determined in [1]. [e] Determined in [5].

cedure has been applied to ε_τ and ε_σ. In this way, we obtain for $k_F = 1.36\,\mathrm{fm}^{-1}$ the following RSC results:

$$\varepsilon_\tau = 53\ \mathrm{MeV}, \quad \varepsilon_\sigma = 65\ \mathrm{MeV}, \quad \varepsilon_{\sigma\tau} = 76\ \mathrm{MeV}, \tag{3.1}$$

which we consider our best calculated values of the three symmetry energies. Notice that we have $\varepsilon_\tau < \varepsilon_\sigma < \varepsilon_{\sigma\tau}$.

Our value of ε_τ agrees reasonably with the liquid drop model estimate of Mayers and Swiatecki [18]: $\varepsilon_\tau = 56$ MeV (however, by applying the droplet model, they get a rather large value of $\varepsilon_\tau = 73$ MeV [19]).

As pointed out in § 1, an empirical estimate of ε_σ and $\varepsilon_{\sigma\tau}$ is very hard. An indirect estimate of ε_σ and $\varepsilon_{\sigma\tau}$ (and also of ε_τ) may be obtained with the help of the "empirical" values of the Landau parameters, $F_0' = 0.7$, $G_0 = 1.15$, $G_0' = 1.45$, $F_1 = -0.6$, adjusted in [20] (see also [3]) to low lying states of ^{208}Pb. These values inserted into the Landau theory expressions for ε_x,

$$\varepsilon_x = \tfrac{2}{3}\,\varepsilon_F(1+F_1/3)^{-1} \times \begin{cases} (1+F_0'), & x = \tau, \\ (1+G_0), & x = \sigma, \\ (1+G_0'), & x = \sigma\tau, \end{cases} \tag{3.2}$$

lead for $k_F = 1.36\,\mathrm{fm}^{-1}$ to the following values

$$\varepsilon_\tau = 54\ \mathrm{MeV}, \quad \varepsilon_\sigma = 69\ \mathrm{MeV}, \quad \varepsilon_{\sigma\tau} = 78\ \mathrm{MeV}, \tag{3.3}$$

which agree reasonably well with our best calculated values (3.1). The agreement could

not have been achieved with the single density approximation applied for $\varepsilon_{\sigma\tau}$ in DH (the RSC result for $k_F = 1.36\,\mathrm{fm}^{-1}$ would then be $\varepsilon_{\sigma\tau} = 64$ MeV). Also, no agreement could be achieved, in particular for $\varepsilon_{\sigma\tau}$, if we used in Eq. (3.2) Landau parameters calculated from nuclear forces in [21] and [22] (see [23]).

Now, let us discuss the consequences of our present new value of $\varepsilon_{\sigma\tau}$ on the estimate of the spin dependent part U_{ss} of the single nucleon potential U. The single nucleon potential U in nuclear matter is defined here, together with the kinetic energy, as the removal energy (it differs from the single nucleon model potential V by the rearrangement potential). In polarized nuclear matter, the single particle potential of a nucleon (n or p) with momentum $\boldsymbol{k}$, and with spin up or down ($\uparrow$ or $\downarrow$), has the form [5, 6]

$$U(k\uparrow_{\mathrm{p}}^{\mathrm{n}}) = U_0(k)+\tfrac{1}{4}\left[\pm\alpha_\tau U_\tau(k)+\alpha_\sigma U_\sigma(k)\pm\alpha_{\sigma\tau}U_{\sigma\tau}(k)\right],$$

$$U(k\downarrow_{\mathrm{p}}^{\mathrm{n}}) = U_0(k)+\tfrac{1}{4}\left[\pm\alpha_\tau U_\tau(k)-\alpha_\sigma U_\sigma(k)\mp\alpha_{\sigma\tau}U_{\sigma\tau}(k)\right], \tag{3.4}$$

where only linear terms in $\alpha_x (x = \sigma, \tau, \sigma\tau)$ are retained. Strictly speaking, U depends on the direction of $\boldsymbol{k}$, and $U(k\uparrow(\downarrow)_{\mathrm{p}}^{\mathrm{n}})$ in Eqs (3.4) denotes the value of U averaged over the directions of $\boldsymbol{k}$.

As has been shown in [5], and [6], one may calculate $U_x(k_F)$ from the relations

$$\varepsilon_x = \tfrac{2}{3}\varepsilon_F+\tfrac{1}{3}k_F(\partial U_0/\partial k)_{k_F}+\tfrac{1}{4}U_x(k_F), \quad x = \tau, \sigma, \sigma\tau, \tag{3.5}$$

if one knows ε_x.

The results obtained for $U_{\sigma\tau}(k_F)$, together with values of $U_\tau(k_F)$ and $U_\sigma(k_F)$ obtained before, are shown in Table I. The new values of $U_{\sigma\tau}(k_F)$ are much bigger than the values 175 MeV and 171 MeV, respectively for the BGT and RSC potential, obtained in [5] from the values of ε_σ and $\varepsilon_{\sigma\tau}$ of DH. In particular, we have now $U_{\sigma\tau} > U_\sigma$, which is consistent with the result obtained in [6] in the phase-shift approximation (see Fig. 2 of [6]). This, in turn, affects the estimate of U_{ss}.

In the valence nucleon model, in which the total spin $\boldsymbol{I}$ of the nucleus is equal to the spin $\boldsymbol{j}$ of a valence nucleon, the single nucleon potential (3.4) in the elastic channel may be written in the form [5]

$$U = U_0+A^{-1}(U_\tau \boldsymbol{t}_0\boldsymbol{T}+U_{ss}\boldsymbol{s}_0\boldsymbol{I}), \tag{3.6}$$

where

$$U_{ss} = (U_\sigma \pm U_{\sigma\tau})\times\begin{cases}(2I)^{-1} & \text{for } I = j = l+\tfrac{1}{2},\\ -[2(I+1)]^{-1} & \text{for } I = j = l-\tfrac{1}{2},\end{cases} \tag{3.7}$$

where the + sign applies to the case when the scattered and the valence nucleon are like nucleons (both neutrons or both protons), and the — sign to the opposite case, and where l is the orbital angular momentum of the valence nucleon, and $\boldsymbol{s}_0$ is the spin of the scattered nucleon ($\boldsymbol{t}_0$ is its isospin, and $\boldsymbol{T}$ is the total isospin of the target nucleus).

By inserting into Eq. (3.7) our RSC values of $U_\sigma(k_F)$ and $U_{\sigma\tau}(k_F)$ of Table I, we get $U_{ss} = -7$ MeV for the n — ^{59}Co scattering (valence proton configuration $(1f_{7/2})^{-1}$),

U_{ss} = 58 MeV for the p — ^{59}Co scattering, and U_{ss} = 81 MeV for the p — ^{27}Al scattering (valence proton configuration $(1d_{5/2})^{-1}$). Our present result for the n — ^{59}Co scattering, the case of unlike scattered and valence nucleons, which is determined by the difference $U_\sigma - U_{\sigma\tau}$, differs both in magnitude and sign from the result U_{ss} = 1 MeV of [5]. Our present result agrees better with Satchler's estimate, $U_{ss} \cong -12$ MeV [24]. As stated in [5] and [6], the experimental estimates of U_{ss} in the case of n — ^{59}Co are not conclusive. The difference between our present results and the results of [5] is less striking in cases of p — ^{59}Co, p — ^{27}Al, where U_{ss} is determined by $U_\sigma + U_{\sigma\tau}$ (like scattered and valence nucleons).

Both spin and spin-isospin dependent parts of the single particle potential contain tensor parts, $U_{\sigma,t}$ and $U_{\sigma\tau,t}$, besides the scalar parts, U_σ and $U_{\sigma\tau}$, discussed above. A partial estimate of $U_{\sigma\tau,t}$ given in [6] (Eq. (71) of [6]) would be modified within the present single-density approximation. We do not discuss here this modification, because it concerns only a part of $U_{\sigma\tau,t}$ whereas the complete $U_{\sigma\tau,t}$ has not been estimated so far. Furthermore, the magnitude of the tensor part of the spin-spin term in the optical model potential is even harder to determine experimentally than the scalar part [25].

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