LETTERS TO THE EDITOR

A POSSIBLE MODIFICATION OF THE DRELL-YAN-WEST RELATION DUE TO THE CORRELATION BETWEEN LONGITUDINAL AND TRANSVERSE MOMENTA OF PARTONS

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It is shown that correlations between longitudinal and transverse momentum of partons lead to a modified Drell-Yan-West (DYW) relation. In a particular model where the transverse momentum of the leading parton in the proton is $\sim 1/(1-x)^{1/4}$ for $x \to 1$, the modified DYW relation is compatible with both $\nu W_2 \sim (1-x)^4$ for $x \to 1$ and $F(Q) \to Q^{-4}$ for $Q \to \infty$.

Deep inelastic lepton nucleon scattering seems to be — at least phenomenologically — well understood within the framework of the parton model. It is however possible that new data, being rapidly accumulated at present, will modify or complete some of the present features of the parton model.

Quite recently a possible incompatibility of the data with the standard form of the Drell-Yan-West (DYW) relation [1] was found [2]. The threshold behaviour of the W_1 structure function indicated by the analysis of the new data seems to be $W_1 \sim (1-x_s)^4$ rather than the expected form $W_1 \sim (1-x_s)^3$. The variable $x_s = [\omega+1.42/Q^2]^{-1}$ with Q given in GeV/c.

In the present paper it will be shown that both this behaviour of W_1 and the $F(Q) \sim Q^{-4}$ asymptotic behaviour of the electromagnetic form-factors of the nucleon may be compatible with a modified DYW relation where the modification is due to a correlation between transverse and longitudinal momenta of the "leading" parton (the one with $x \to 1$).

We shall present the argument in a simple and transparent way [3] based on the correspondence between the infinite momentum physics and two-dimensional non-relativistic physics [3, 4]. In particular we shall use the analogy between the longitudinal momentum fraction x and the mass of a non-relativistic particle.

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Let us consider a non-relativistic system of two particles with masses m and M ($m \gg M$). Using the analogy mentioned above we denote

$$x_a \equiv \frac{m}{M+m}, \qquad x \equiv \frac{M}{M+m} = 1-x_a,$$

where x_a, x correspond to the longitudinal momentum fractions of the fast and the slow particles, respectively.

Let $r(x)$ be the distance between the two non-relativistic particles. In contradistinction to the derivation of the DYW relation given in [3] we shall suppose here that $r(x)$ is not a constant but a function of x. The wave function of the heavy particle is then spread over a distance

$$\Delta(x_a) = \frac{m}{M+m}\, r(x) = xr(x) = (1-x_a)\, r(x).$$

The form-factor of the heavy particle described by such a wave function will be larger for

$$Q < Q_0 \sim \frac{1}{\Delta(x_a)} = \frac{1}{(1-x_a)\, r(x)}.$$

Solving this equation for x_a one finds

$$x_{a,\,\min} \sim x_a(Q_0).$$

The asymptotic behaviour of the form-factor is then given by the probability of finding the two particle system in a configuration where $x_a > x_{a,\,\min}$. In this way [3]

$$F(Q) \sim \int\limits_{x_a(Q)}^{1} \nu W_2(x_a)\, dx_a \qquad \text{for } Q \to \infty.$$

Various forms of $r(x)$ lead to different modifications of DYW relation. The functional dependence $r = r(x)$ implies the existence of a correlation between the longitudinal momentum fraction x and the transverse position r of partons. This in turn implies a correlation between x and the transverse momentum.

Before proceeding further a few remarks are in order. We have considered here a two--body system while the parton model is a many-body problem. Asymptotic behaviour of both the structure function and the form-factor are given by the behaviour of the single "leading" parton. In such a situation we can represent the remaining partons by their centre of mass. The assumptions made above have therefore an interpretation (in terms of infinite momentum physics) only for the fast ($x_a \to 1$) partons and not for the slow ($x \to 0$) ones. If the dependence of the transverse distance r on the x_a is parametrized by

$$r(x) = x^k = (1-x_a)^k, \qquad x_a \to 1$$

and

$$\nu W_2(x_a) \sim (1-x_a)^g, \qquad x_a \to 1$$

we get

$$F(Q) \sim Q^{-a}, \qquad Q \to \infty \tag{1}$$

where

$$a = \frac{g+1}{k+1}.$$

For $k = 0$ ($r =$ const., no correlation) the usual form of the DYW relation results

$$\nu W_2 \sim (1-x_a)^g, \qquad F(Q) \sim Q^{-g-1}.$$

Other assumptions about the correlation between longitudinal and transverse momenta of partons lead to the modified[1] DYW relation (1). If it really turns out that, as indicated in [2], $\nu W_2 \sim (1-x_a)^4$ and if the present trend of the nucleon form-factor $F(Q) \sim Q^{-4}$ is maintained, the modified DYW relation (1) requires $k = 1/4$.

This would mean that the transverse momentum of the "leading" ($x_a \to 1$) parton is correlated with its longitudinal momentum fraction x_a by the relation

$$p_T \sim \left(\frac{1}{1-x_a}\right)^{1/4}. \tag{2}$$

Such a correlation should manifest itself in the transverse momentum distribution of hadrons produced in the deep inelastic lepton scattering in the current fragmentation region. This process is particularly suitable for studying problems of parton transverse momenta [5] because of its relatively simple kinematics. The longitudinal momentum fraction of the interacting parton in this process is given by the Bjorken scaling variable $x = x_{BJ} = Q^2/2M\nu$. The transverse momentum of the interacting parton should manifest itself [5] particularly in those events where, in the process of fragmentation, almost the whole parton momentum is received by a single hadron (the Feynman variable x_F should be large, $x_F \sim 1$).

For a correlation like Eq. (2) one has to expect that the transverse momenta of electroproduced hadrons with $x_F \sim 1$ increase with increasing x_{BJ} (for x_{BJ} close to 1). Such a prediction could be tested by data on deep inelastic ep and μp collisions. Unfortunately the dependence of $\langle p_T \rangle$ on x_{BJ} cannot be extracted from the published data. The correlation between p_T and p_L of partons could manifest itself also in multiparticle production where produced particles containing fast valence quarks should have larger transverse momenta than particles in the central region. Some evidence which may be interpreted as pointing in this direction is given in [6], where it is shown that in π^-p collisions at 147 GeV/c π^-'s produced with large c. m. rapidities have larger $\langle p_T \rangle$ than π^+'s, in the same rapidity region.

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[1] This result was obtained by using the non-relativistic analogy. One can arrive at the same result by using a more rigorous but less transparent formalism in which both the structure function and the form-factor are expressed in terms of parton wave functions.

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