SPIN SYSTEM FOR THE b-UNIVERSALITY AND THE ADDITIVE QUARK MODEL

By J. Szwed

Institute of Physics, Jagellonian University, Cracow*

(*Received July 27, 1976*)

The possibility that the spin systems of b-universality and of the additive quark model coincide, as suggested by previous analyses [1, 2], is investigated. A comparison of the predictions with the data for the processes $0^-\frac{1}{2}^+ \to 1^-\frac{3}{2}^+$ shows very good agreement with this hypothesis in both helicity and Gottfried-Jackson frames.

In recent papers [1, 2] we investigated the possibility of simultaneous application of b-universality [3, 4] and the additive quark model to the processes

$$0^-\tfrac{1}{2}^+ \to J^P\tfrac{3}{2}^+ \tag{1}$$

(J^P is the spin and parity of the peripherally produced meson). It turned out that the crucial point of the analysis was the relative position of the spin reference frames in which both hypotheses were formulated. To recall, it was suggested that the additivity frame [6] coincides with the Gottfried-Jackson frame although agreement with data was obtained also in other spin systems [7]. The b-universality was always assumed in the s-channel helicity frame [3–5], however it was shown [2] to work also in the Gottfried-Jackson frame. These facts allowed us to relax the assumption concerning the spin reference frames. Thus we did not limit ourselves to the originally mentioned possibilities for the b-universality and additivity frames.

The analysis consisted of two parts. In the case when both frames fulfill the constraint

$$\vartheta_B - \vartheta_{B_*} \neq k\pi, \quad k = 0, \pm 1, \pm 2, \ldots \tag{2}$$

(ϑ_B and ϑ_{B*} are the spin rotation angles from the additivity to the b-universality frame for the barion $B(\frac{1}{2}^+)$ and $B^*(\frac{3}{2}^+)$ respectively) we obtained strong predictions concerning amplitudes and density matrix elements of process (1). In particular the t-dependence of the amplitudes was determined up to a few arbitrary constants and the density matrix elements fulfill several constraints (e.g. in $1^-\frac{3}{2}^+$ production natural exchange should vanish). All these constraints are not valid when $\vartheta_B - \vartheta_{B*} = k\pi$.

* Address: Instytut Fizyki, Uniwersytet Jagielloński, Reymonta 4, 30-059 Kraków, Poland.

The main result of [1, 2] followed from the comparison of the obtained predictions with the data of the processes $\pi^+p \to \pi^0\Delta^{++}$ at 5 GeV/c [8] and 16 GeV/c [9] and $\pi^+p \to \varrho^0\Delta^{++}$ and $\pi^+p \to \omega\Delta^{++}$ at 7 GeV/c [10]. The conclusion is that the b-universality and additivity frames should fulfill the relation

$$\vartheta_B - \vartheta_{B^*} = 2k\pi, \qquad k = 0, \pm 1, \pm 2, \ldots, \tag{3}$$

with condition (2) being definitely ruled out by the data. It should be stressed that the quark model relations for the amplitudes are invariant under rotations satisfying Eq. (3). Thus we are not able to distinguish whether both frames simply coincide or fulfil a weaker condition (3).

In this letter we investigate further the suggested case where

$$\vartheta_B - \vartheta_{B^*} = 2k\pi$$

by comparing it with the existing data. To define our input we write down two sets of relations in the b-universality frame. The first follows from the additive quark model [11]

$$\begin{aligned} M^{\mu}_{\frac{3}{2}\frac{1}{2}}(s, t) &= \sqrt{3}\, M^{\mu}_{\frac{1}{2}-\frac{1}{2}}(s, t), \\ M^{\mu}_{\frac{1}{2}\frac{1}{2}}(s, t) &= M^{\mu}_{-\frac{1}{2}-\frac{1}{2}}(s, t), \\ M^{\mu}_{-\frac{3}{2}\frac{1}{2}} &= 0, \\ \mu &= -J, -J+1, \ldots, J-1,]. \end{aligned} \tag{4}$$

$M^{\lambda_c}_{\lambda_d\lambda_b}$ denotes the amplitudes with the spins projected in the reaction plane. λ_b, λ_c and λ_d are the spin projections of the baryon $B(\frac{1}{2}^+)$, meson J^P and baryon $B^*(\frac{3}{2}^+)$ respectively. The second set of relations is built up by the derivative relations

$$M_{n'}(s, t) = C_{n'n}(s) \sqrt{-t'}^{\,n'} \left(\frac{1}{\sqrt{-t'}} \frac{\partial}{\partial\sqrt{-t'}}\right)^{n'-n} \left(\frac{M_n(s, t)}{\sqrt{-t'}^{\,n}}\right) \tag{5}$$

where $n = |\lambda_b + \lambda_c - \lambda_d|$ and $C_{n'n}$ is an arbitrary function of c. m. energy. The above formulas are compared with the data for the processes

$$\left.\begin{array}{llll} \pi^+p \to \varrho^0\Delta^{++} & \text{at} & 7\ \text{GeV}/c & [10] \\ \pi^+p \to \omega\Delta^{++} & \text{at} & 7\ \text{GeV}/c & [10] \\ K^+p \to K^{+0}\Delta^{++} & \text{at} & 12\ \text{GeV}/c & [12] \end{array}\right\} \tag{6}$$

$$\left.\begin{array}{llll} \pi^+p \to \omega\Delta^{++} & \text{at} & 3.7\ \text{GeV}/c & [13] \\ \pi^+p \to \omega\Delta^{++} & \text{at} & 3.7\ \text{GeV}/c & [13] \\ \pi^+p \to \omega\Delta^{++} & \text{at} & 13\ \text{GeV}/c & [15] \\ \pi^+p \to \varrho^0\Delta^{++} & \text{at} & 16\ \text{GeV}/c & [14] \end{array}\right\} \tag{7}$$

For processes (6) the differential cross-section and density matrix elements of the vector meson were used, whereas experiments (7) provide 19 density matrix elements. All data are given in two spin frames (helicity and Gottfried-Jackson). This allows us to check whether one of these frames is preferred as the common frame for two hypotheses.

The method we use is similar to the model independent test of Ref. [2]. The amplitudes have the form

$$M_n(s, t) = C_{\{\lambda_i\}}\left(-\frac{\partial}{\partial\sqrt{-t'}}A(t)\right)^n e^{-A(t)}\sum_{k=0}^{N} f_k L_k^n(A(t)) \tag{8}$$

with $A(t) = \alpha+\beta t'$. L_k^n are the Laguerre polynomials and α, β and f_k are arbitrary independent constants. For each k the b-universality is automatically assured. In addition we impose on the amplitudes the constraints of Eq. (4). This reduces the number of unknown parameters $C_{\{\lambda_i\}}$.

The results are shown in Figs 1 and 2. In all cases very good agreement is obtained. Because there is no qualitative difference in the fits for different reactions listed above

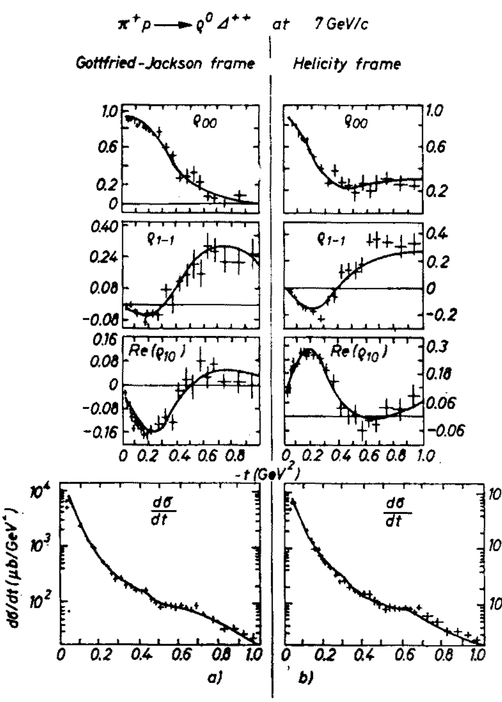

Fig. 1

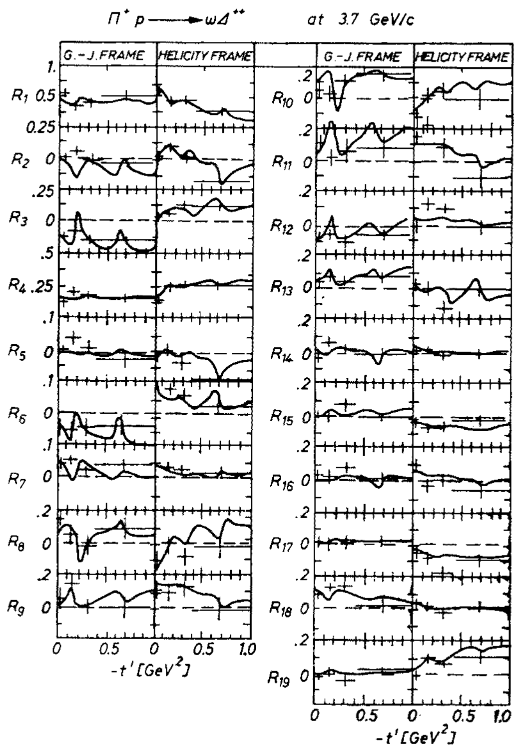

Fig. 2

Fig. 1. The fit to the reaction $\pi^+p \to \varrho^0\Delta^{++}$ [10] at 7 GeV/c: (a) in the Gottfried-Jackson frame; (b) in the helicity frame

Fig. 2. The fit to the 19 density matrix elements (listed in Table I) in the process $\pi^+p \to \omega\Delta^{++}$ at 3.7 GeV/c [13]: (a) in the Gottfried-Jackson frame; (b) in the helicity frame

we present the results for only one reaction of each group. It is seen that the combined models work equally well in the helicity and in Gottfried-Jackson frames. The series in Eq. (8) is rapidly convergent and in practice the fit does not depend on N for $N > 7$.

TABLE I

The 19 measurable combinations of density matrix elements and their moments in the proces $0^-\frac{1}{2}^+ \to 1^-\frac{3}{2}^+$

Term	D. m. elements	Moments
R_1	ϱ^{00}	$\frac{1}{2}(5\cos^2\theta_v - 1)$
R_2	ϱ^{1-1}	$-\frac{5}{4}\sin^2\theta_v \cos 2\phi_v$
R_3	Re ϱ^{10}	$-\frac{5}{4\sqrt{2}}\sin 2\theta_v \cos\phi_v$
R_4	ϱ_{33}	$\frac{1}{8}(7-15\cos^2\theta_v)$
R_5	ϱ_{3-1}	$-\frac{5}{8}\sqrt{3}\sin^2\theta_v \cos\phi_\Delta$
R_6	Re ϱ_{31}	$-\frac{5}{8}\sqrt{3}\sin 2\theta_\Delta \cos\phi_\Delta$
R_7	$\varrho^{-}_{33} - \varrho^{-}_{11}$	$\frac{25}{8}(1-3\cos^2\theta_\Delta)(1-3\cos^2\theta_\Delta)$
R_8	Re ϱ^{-}_{31}	$-\frac{25}{8}\frac{\sqrt{3}}{2}(1-3\cos^2\theta_\Delta)\sin 2\theta_\Delta \cos\phi_\Delta$
R_9	Re ϱ^{-}_{3-1}	$-\frac{25}{16}\sqrt{3}(1-3\cos^2\theta_\Delta)\sin^2\theta_\Delta \cos 2\phi_\Delta$
R_{10}	Re ϱ^{10}_{-}	$\frac{25}{8\sqrt{2}}\sin 2\theta_v \cos\phi_v (1-3\cos^2\theta_\Delta)$
R_{11}	ϱ^{1-1}_{-}	$-\frac{25}{8}\sin^2\theta_v \cos 2\phi_v (1-3\cos^2\theta_\Delta)$
R_{12}	Re $(\varrho^{10}_{31} - \varrho^{0-1}_{31})$	$\frac{25}{32}\sqrt{6}\sin 2\theta_v \sin 2\theta_\Delta \cos(\phi_v + \phi_\Delta)$
R_{13}	Re $(\varrho^{01}_{31} - \varrho^{-10}_{31})$	$\frac{25}{32}\sqrt{6}\sin 2\theta_v \sin 2\theta_\Delta \cos(\phi_v - \phi_\Delta)$
R_{14}	Re ϱ^{1-1}_{31}	$\frac{25}{32}\sqrt{3}\sin^2\theta_v \sin 2\theta_\Delta \cos(2\phi_v + \phi_\Delta)$
R_{15}	Re ϱ^{-11}_{31}	$\frac{25}{32}\sqrt{3}\sin^2\theta_v \sin 2\theta_\Delta \cos(2\phi_v - \phi_\Delta)$
R_{16}	Re $(\varrho^{10}_{3-1} - \varrho^{0-1}_{3-1})$	$\frac{25}{32}\sqrt{6}\sin 2\theta_v \sin^2\theta_\Delta \cos(\phi_v + 2\phi_\Delta)$
R_{17}	Re $(\varrho^{01}_{3-1} - \varrho^{-10}_{3-1})$	$\frac{25}{32}\sqrt{6}\sin 2\theta_v \sin^2\theta_\Delta \cos(\phi_v - 2\phi_\Delta)$
R_{18}	Re ϱ^{1-1}_{3-1}	$\frac{25}{32}\sqrt{3}\sin^2\theta_v \sin^2\theta_\Delta \cos 2(\phi_v + \phi_\Delta)$
R_{19}	Re ϱ^{-11}_{3-1}	$\frac{25}{32}\sqrt{3}\sin^2\theta_v \sin^2\theta_\Delta \cos 2(\phi_v - \phi_\Delta)$

The above results allow us to draw the following conclusions:

1) The b-universality and the additive quark model work very well when formulated in the spin frames connected by Eq. (3) (in particular when both frames coincide).

2) The agreement with the data is good in both the helicity and Gottfred-Jackson frames. Thus our analysis does not answer the question whether one of these spin systems should be preferred as the b-universality (and perhaps also additivity) frame.

The author would like to thank Professor A. Białas for helpful discussions.

REFERENCES

[1] J. Szwed, *Acta Phys. Pol.* **B7**, 577 (1976).
[2] J. Szwed, preprint Cracow TPJU-10/76.
[3] J. P. Ader et al., *Nuovo Cimento* **27A**, 385 (1975).
[4] B. Schrempp, F. Schrempp, *Nucl. Phys.* **B96**, 307 (1975).
[5] S. Humble, *Nucl. Phys.* **B98**, 267 (1975).

[6] A. Białas, A. Kotański, K. Zalewski, *Nucl. Phys.* **B28**, 1 (1971).
[7] see e. g. G. S. Abrams et al., *Phys. Rev.* **D7**, 1385 (1973).
[8] I. J. Bloodworth et al., *Nucl. Phys.* **B81**, 231 (1974).
[9] M. Deutschmann et al., *Nucl. Phys* **B85**, 31 (1975).
[10] S. U. Chung et al., *Phys. Rev.* **D12**, 693 (1975).
[11] K. Zalewski, *Acta Phys. Pol.* **B1**, 423 (1970).
[12] G. Ciapetti et al., *Nucl. Phys.* **B64**, 58 (1973).
[13] K. W. Barnham et al., *Phys. Rev.* **D7**, 1384 (1973).
[14] R. Honecker et al., Aachen–Berlin–Bonn–CERN–Cracow–Heidelberg Collaboration, *Study of the Reaction* $\pi^+p \rightarrow \varrho^0\Delta^{++}$ *at* **16** *GeV/c*.
[15] J. A. Gaidos et al., *Nucl. Phys.* **B72**, 253 (1974).