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DIFFRACTIVE DISSOCIATION AND MULTIPLICITY DISTRIBUTION IN AN UNCORRELATED CLUSTER EMISSION MODEL

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Using an uncorrelated cluster emission model with a superposition of coherent states to describe non-diffractive processes, the diffractive production is estimated by means of shadow calculations, the results of which are shown to be linear as regards the probability distributions. The model describes fairly well the absolute magnitudes of cross-sections as well as the multiplicity distribution.

It is generally admitted (see e. g. Białas and Kotański's model [1]) that the diffractive production may be calculated from non-diffractive production amplitudes by means of the unitarity condition. Assuming that the non-diffractive production is described by the uncorrelated cluster emission model [2, 3], the high energy limit of diffractive cross-sections was obtained [4] through the formulae of Ref. [1] and by using de Groot's method [5]. Following the assumption that the clusters decay isotropically into 3 pions, numerical estimates for proton-proton collisions [6] showed that the model describes fairly well the absolute magnitude and the general behaviour of the diffractive cross-section. However, for the multiplicity distribution obtained within a two-component scheme (see e. g. Ref. [7]), the model fails badly. This was suggested to be due mainly to the fact that the model does not give a true description of the non-diffractive production.

In this paper we show that the discrepancy found earlier may indeed be avoided by taking as input a superposition of coherent states instead of just one of them. We follow in this respect an idea of Benecke, Białas and de Groot [8].

These authors have suggested that the observed cross-section should be represented as a sum over a number of component cross-sections, each of them satisfying the uncorrelated cluster emission model. We assume this to be true for the non-diffractive production.

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The observed cross-sections for the production of N clusters are then given by

$$\sigma_N^{\mathrm{ND}} = \int_A^{\infty} d\lambda \sigma_{\mathrm{N}}^{\mathrm{ND}}(\lambda), \tag{1}$$

where $\sigma_N^{\mathrm{ND}}(\lambda)$ are the cross-sections related to the different components. They are given by [6]

$$\sigma_N^{\mathrm{ND}}(\lambda) = \sigma^{\mathrm{ND}}(\lambda)\left(\frac{s}{\bar{\mu}^2}\right)^{-\lambda} \frac{(\lambda \ln s/\bar{\mu}^2)^{N+1}}{(N+2)N!}, \tag{2}$$

where

$$\sigma^{\mathrm{ND}}(\lambda) = \frac{\sigma^{\mathrm{ND}}}{\lambda[\Gamma(\lambda+1)]^2} \Bigg/ \int_A^{\infty} \frac{d\lambda}{\lambda[\Gamma(\lambda+1)]^2} \tag{3}$$

and where λ describes the energy dependence of the average multiplicity of the emitted clusters within the different components,

$$\bar{N}^{\mathrm{ND}}(\lambda) = \lambda \ln s/\bar{\mu}^2 - 1. \tag{4}$$

Parameter A, which is the lower bound of the integration over λ, has to be greater than zero as can be easily seen from the previous equation. Moreover in the above formulae, s is the squared total c. m. energy of the system and $\Gamma(z)$ the gamma function, while $\bar{\mu}$ is defined by

$$\ln \bar{\mu} = \int d^2q_\perp \ln (\sqrt{q_\perp^2 + \mathcal{M}^2}\, e^{\gamma}) f(q_\perp), \tag{5}$$

where $f(q_\perp)$ is the normalized transverse momentum distribution of a cluster, $\mathcal{M}$ its mass and γ Euler's constant equal to 0.5772.

Further, to get the observed probability for producing N clusters non-diffractively, we take a weighted average over the probabilities related to the different components. So we obtain

$$P_N^{\mathrm{ND}}(\lambda) = \int_A^{\infty} d\lambda \frac{\sigma^{\mathrm{ND}}(\lambda)}{\sigma^{\mathrm{ND}}} P_N^{\mathrm{ND}}(\lambda), \tag{6}$$

where

$$P_N^{\mathrm{ND}}(\lambda) = e^{-(\bar{N}^{\mathrm{ND}}(\lambda)+1)} \frac{(\bar{N}^{\mathrm{ND}}(\lambda)+1)^{N+1}}{(N+2)N!}, \tag{7}$$

as may be seen from equations (2) and (4).

In the same way, the observed average non-diffractive multiplicity is given by

$$\bar{N}^{\mathrm{ND}} = \int_A^{\infty} d\lambda \frac{\sigma^{\mathrm{ND}}(\lambda)}{\sigma^{\mathrm{ND}}} \bar{N}^{\mathrm{ND}}(\lambda). \tag{8}$$

We can also write this as

$$\overline{N}^{\text{ND}} = \lambda\!\!\!^{-} \ln s/\bar{\mu}^2 - 1, \tag{9}$$

where we define $\lambda\!\!\!^{-}$ by

$$\lambda\!\!\!^{-} = \int\limits_A^\infty \frac{d\lambda}{[\Gamma(\lambda+1)]^2} \Bigg/ \int\limits_A^\infty \frac{d\lambda}{\lambda[\Gamma(\lambda+1)]^2}\,. \tag{10}$$

In order to obtain the peak-plateau structure of the leading particle spectrum, we take $\lambda\!\!\!^{-}$ equal to 1 [8]. This gives us the value of 0.5160 for parameter A.

This method that we used for non-diffractive production, is also applied to diffractive production. But this procedure needs some justification.

We know from Ref. [1] that within the different components, shadow calculations allow to deduce the diffractive production from the non-diffractive one. We know also from Ref. [6] that within these different components, the diffractive probability shows almost a Furry distribution [9] i. e.

$$P_N^{\text{D}}(\lambda) \simeq \frac{(\overline{N}^{\text{D}}(\lambda))^N}{(\overline{N}^{\text{D}}(\lambda)+1)^{N+1}}, \tag{11}$$

where the average diffractive multiplicity at high energies is fairly well given by

$$\overline{N}^{\text{D}}(\lambda) = 0.02\lambda \ln s/\bar{\bar{\mu}}^2 + 4.5, \tag{12}$$

parameter $\bar{\bar{\mu}}$ being defined by

$$\langle q_\perp^2 \rangle \ln \bar{\bar{\mu}} = \int d^2 q_\perp \ln\left(\sqrt{q_\perp^2 + \mathcal{M}^2}\, e^{\gamma}\right) q_\perp^2 f(q_\perp), \tag{13}$$

where $\langle q_\perp^2 \rangle$ is the average squared transverse momentum of a cluster. Looking moreover at equation (7), we see that the non-diffractive probability distribution is approximately a Poisson. For simplicity we take it to be exactly so and the diffractive distribution to be exactly a Furry. Now, the average multiplicities involved being the diffractive ones, it may easily be shown that between both distributions following relation [10] within the different components holds at high energies

$$P_N^{\text{D}}(\lambda) = \int\limits_0^\infty dm\, e^{-m} P_N^{\text{ND}}(\lambda m). \tag{14}$$

In fact, this relation gives nothing but the effect of shadow calculations on the probability distributions. We apply it to the observed non-diffractive probability distribution to obtain the observed diffractive probability distribution. This gives us

$$P_N^{\text{D}}(\lambda\!\!\!^{-}) = \int\limits_0^\infty dm\, e^{-m} P_N^{\text{ND}}(\lambda\!\!\!^{-} m) \tag{15}$$

and finally, with equations (6) and (14),

$$P_N^{\text{D}}(\lambda\!\!\!^{-}) = \int\limits_A^\infty d\lambda \frac{\sigma^{\text{D}}(\lambda)}{\sigma^{\text{D}}} P_N^{\text{D}}(\lambda). \tag{16}$$

Here we used the fact that the weight factor $\sigma^{D}(\lambda)/\sigma^{D}$ (obtained with formula (17) and parameter values given below) is approximately equal to $\sigma^{ND}(\lambda)/\sigma^{ND}$, as shown in Fig. 1. Thus with equation (16) we proved that the procedure which was described for non-diffractive production may also be applied to diffractive production.

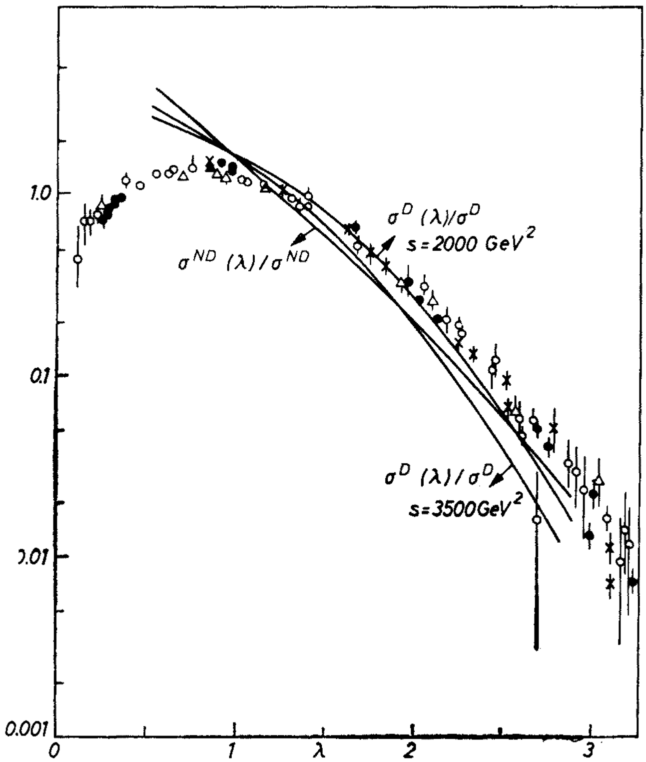

Fig. 1. Energy independent weight factor $\sigma^{ND}(\lambda)/\sigma^{ND}$ and weight factor $\sigma^{D}(\lambda)/\sigma^{D}$ compared with Wróblewski's compilation of experimental data [15] for the KNO scaling function $\psi'(\lambda)$

Here the cross-sections for production of N clusters related to the different components are given by [4]

$$\sigma_N^D(\lambda) = \sigma_{el}(\lambda)\frac{1}{N+1}\left(\frac{\lambda\langle q_\perp^2\rangle \ln s/\bar{\bar{\mu}}^2}{\Omega_1^2(\lambda)}\right)^N, \tag{17}$$

with

$$\Omega_1^2(\lambda) = \lambda\langle q_\perp^2\rangle\,(\ln s/\bar{\bar{\mu}}^2 - 2\psi(\lambda+1) + 1/(\lambda\alpha\langle q_\perp^2\rangle)),$$

for N even, and by

$$\sigma_N^D(\lambda) = 0 \tag{18}$$

for N odd. In formula (17), $\sigma_{el}(\lambda)$ is given by

$$\sigma_{el}(\lambda) = \frac{\sigma_{el}}{[\Gamma(\lambda+1)]^4\Omega_1^4(\lambda)} \Bigg/ \int\limits_A^\infty \frac{d\lambda}{[\Gamma(\lambda+1)]^4\Omega_1^4(\lambda)}, \tag{19}$$

$\psi(z)$ being the digamma function. Moreover, α equals to half of the slope of the elastic cross-section, which we take equal to 12. As the overall magnitudes of cross-sections (17) decrease rather rapidly with growing N, we sum them over the even N values up to 10 to obtain the total cross-sections $\sigma^D(\lambda)$.

In order to compare the multiplicity distribution predicted by the model with experimental data we assume first an isotropic cluster decay into 3 pions, then some definite transverse momentum distribution for the clusters. As earlier [6], we follow the suggestion of Barshay and Chao [11] and take $f(q_\perp)$ in the form

$$f(q_\perp) = \frac{\beta^2}{2\pi\Gamma(2, \beta\mathcal{M})} e^{-\beta\sqrt{q_\perp^2+\mathcal{M}^2}}, \tag{20}$$

where $\Gamma(a, z)$ is the incomplete gamma function and β a parameter which can be easily determined when $\langle q_\perp^2 \rangle$ and $\mathcal{M}$ are known. The last two parameters are not independent from each other, as has been shown in Ref. [12] by using the same assumption about the cluster decay. For the cluster mass $\mathcal{M}$ we choose the value 1.3 GeV in agreement with Ref. [2] where it was estimated that $\langle\mathcal{M}\rangle$ is equal to 1.3 GeV when the clusters decay on the average into 3 pions. As value of $\langle q_\perp^2 \rangle$ we obtain then 0.3279 (GeV/c)^2. This gives us the value of 9.2922 for the slope β. We find also $\bar{\mu}$ equal to 2.5141 and $\bar{\bar{\mu}}$ equal to 2.7274.

Now, having determined all parameters, we can see how the model agrees with experimental data. Absolute magnitudes of average multiplicities and cross-sections obtained at high energies are fairly well given by the model. As these results are very similar to those obtained earlier, we may refer to Ref. [6]. For the ratio σ^D/σ_{el}, however, which in the limit $s \to \infty$ is predicted by the model to be one, the experimentally observed value [13], we note that it is very sensitive to the cluster mass. It decreases rather rapidly with increasing $\mathcal{M}$.

Since we have the explicit formulae for the observed diffractive and non-diffractive production, it is useful to look at the multiplicity distribution, more precisely at the KNO scaling function [14], for which discrepancies were found in the earlier version of the model [6]. Therefore we use a two-component scheme as was suggested e. g. by Fiałkowski and Miettinen [7]. This gives for the average total cluster multiplicity

$$\overline{N} = \varrho\overline{N^D}+(1-\varrho)\overline{N^{ND}}, \tag{21}$$

where

$$\varrho = \sigma^D/\sigma_{in}. \tag{22}$$

Further, we use the fact that with our assumptions about the cluster decay the same scaling function describes cluster production and particle production. So we obtain, using Wróblewski's parametrization [15], the following equation for the KNO scaling function related to cluster production

$$\psi'(z') = (\overline{N}+0.5)\,\frac{\sigma_N^{\mathrm{D}}+\sigma_N^{\mathrm{ND}}}{\sigma_{\mathrm{in}}}\,, \tag{23}$$

where

$$z' = (N+0.5)/(\overline{N}+0.5). \tag{24}$$

In Fig. 2 we have plotted the function ψ' versus z' and compared to the data as compiled by Wróblewski. Because of the peculiar fact that for N odd σ_N^{D} is equal to zero, we took the arithmetic average of the separate curves for N odd and for N even at a given energy.

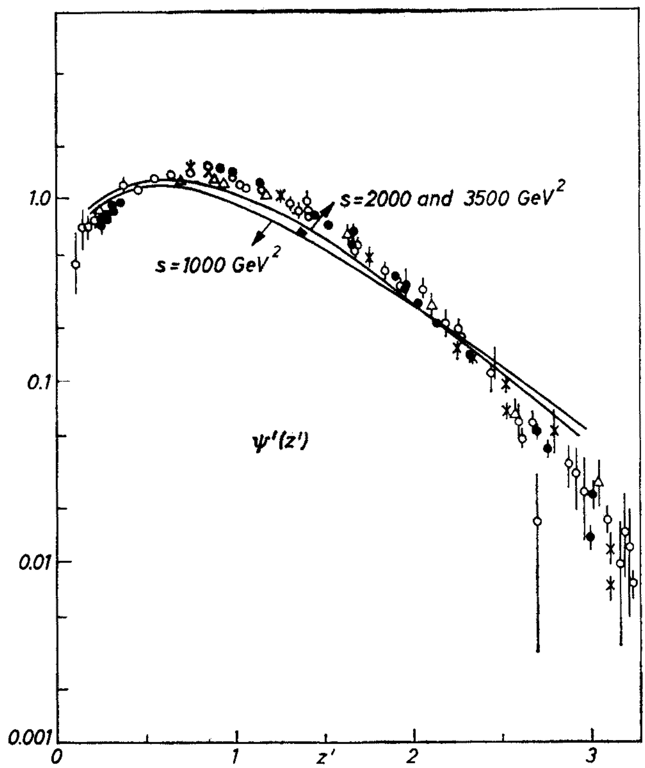

Fig. 2. KNO scaling function $\psi'(z')$ compared with experimental data [15]

We see that the agreement is fairly good, both for the shape of the curve and for its scaling property.

If we limit ourselves to non-diffractive production, we obtain, as shown in Fig. 3, a scaling function $\psi'_{\mathrm{ND}}(z')$ which seems to be a rather good approximation to the experimental data [15]. Moreover, it is in fair agreement with the formula which was derived by de Groot [16] from a unitary uncorrelated cluster model [17]. That formula, however, includes some more parameters.

Let us finally look again at Fig. 1 where we plotted the weight factor $\sigma^{ND}(\lambda)/\sigma^{ND}$. By comparison of it with the experimentally obtained data for the scaling function $\psi'(\lambda)$ [15], we see that it is quite different. This fact is in disagreement with the predictions of Ref. [8],

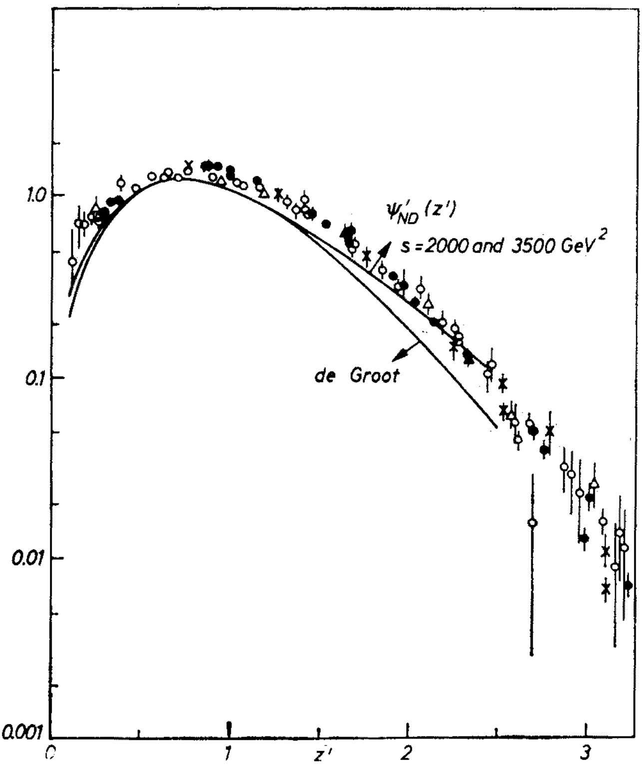

Fig. 3. KNO scaling function $\psi'_{ND}(z')$ compared with experimental data [15] for $\psi'(z')$ and with de Groot's fit [16]

which, however, were derived in the KNO limit, in which case the non-diffractive Poisson distributions become delta-functions.

Thus, comparing the results obtained earlier [6] with those we got here from confrontation of the multiplicity distribution with experimental data, we can say that clearly a superposition of coherent states should be used as the input when building up a model for multiparticle production.

To conclude, we would like to make some comments. The model of Białas and Kotański that we used, in which diffractive dissociation is generated as a shadow of non-diffractive interactions, gives a correct description of the diffractive production. This is true when for the non-diffractive production an uncorrelated cluster emission model is used, where a superposition of coherent states is taken instead of just one coherent state. The relation which exists then between diffractive and non-diffractive production as regards the probability distributions appears to be linear. The use of non-diffractive production only seems to be already a good approximation to describe the observed phenomena. The addition of diffractive production, within a two-component scheme, agrees fairly well with experiment.

Summarizing, we may say that an uncorrelated cluster emission model where superposed coherent states are used, describes correctly both the non-diffractive production and, as its shadow, the diffractive production.

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