MAGNETIC MOMENTS OF THE CHARMED VECTOR BOSONS

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The magnetic moments of the charmed vector bosons and those of the usual ones have been compared assuming that the magnetic moment operator transforms as the (15,3) component of SU(8) symmetry. If the ψ-particle can be identified as $c\bar{c}$, the charmed quark-antiquark state, then we find $\mu(\psi) = 4\mu(\varrho^0)$.

It is the purpose of this note to report on the consequences of assuming SU(8) symmetry for the ratios of (*i*) the magnetic moments of the vector mesons including the charmed particles, (*ii*) the transition moments between vector and pseudoscalar mesons. As proposed by several authors [1], if we consider the ψ particle to be $c\bar{c}$ combination of the charmed quark c and antiquark $\bar{c}$, the magnetic moment of ψ can be compared with that of ϱ^0-meson.

Starting with SU(4) symmetry [2, 3], we assume that the mesons are obtained by the quark-antiquark combination $q\bar{q}$ and that they belong to the $\underline{1}+\underline{15}$-multiplet representation. We have to introduce here eight more charmed particles[1], for both pseudoscalar and vector mesons. In order to incorporate the spin of the particles, we enlarge the symmetry group to SU(8) [4]. The quark-antiquark combination in terms of the SU(4)$\times$SU(2) characterization, can be expressed as

$$8 \otimes \bar{8} = \underline{1}+\underline{63};\ \underline{63} = (\underline{1}+\underline{15}, 3)+(\underline{15}, \underline{1}). \tag{1}$$

Since the $\underline{63}$-representation is associated with Young's tableau [2111111], we can represent the particles in the $\underline{63}$-multiplet with a tensor $B_{\{ABCDEFG\}H}$, where $A = (\alpha, a)$, $B = (\beta, b)$ etc. The Greek alphabet is used for SU(4) and the Latin one for SU(2) labels. Hence α-s run from 1 through 4, whereas $a-s$ from 1 to 2. The tensor B is antisymmetric with respect to the simultaneous interchange of the indices within the curly bracket. We can, however, rewrite B in terms of a mixed tensor as follows:

$$B_{\{ABCDEFG\}H} = \frac{1}{\sqrt{7!}}\,\varepsilon_{ABCDEFGK}\mathscr{B}^{K}{}_{H}, \tag{2}$$

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[1] For exact nomenclature of these particles see reference [3]. We follow their identification.

where $\varepsilon_{ABCDEFGH}$ is the Levi–Civita tensor in eight dimensions. For $\mathscr{B}^A_B$ we can put

$$\mathscr{B}^A{}_B = V^\alpha{}_\beta \chi_{ab} + \frac{1}{\sqrt{2}} P^\alpha{}_\beta \varepsilon_{ab} \chi_0. \tag{3}$$

In expression (3) V stands for the multiplet $\underline{1}+\underline{15}$ vector mesons and P stands for the pseudoscalar mesons. They are given by the following 4×4 matrices [3]:

$$V^\alpha{}_\beta = \begin{bmatrix} \frac{1}{\sqrt{2}}(\omega+\varrho^0) & \varrho^+ & K^{*-} & D^{*0} \\ \varrho^- & \frac{1}{\sqrt{2}}(\omega-\varrho^0) & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & F^{*-} \\ D^{*0} & D^{*+} & F^{*+} & \psi \end{bmatrix} \tag{4}$$

and

$$P^\alpha{}_\beta = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} + \frac{\eta_c}{2\sqrt{3}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} + \frac{\eta_c}{2\sqrt{3}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{2\eta^0}{\sqrt{6}} + \frac{\eta_c}{2\sqrt{3}} & F^- \\ D^0 & D^+ & F^+ & -\frac{\sqrt{3}}{2}\eta_c \end{bmatrix} \tag{5}$$

In the expression (2), χ_{ab} are the wavefunctions with intrinsic spin one and are given by

$$\chi_{11} = u_1, \quad \chi_{12} = \chi_{21} = \frac{1}{\sqrt{2}} u_0, \quad \chi_{22} = u_{-1}, \tag{6}$$

where u_i's are assumed to be normalized to one. On the other hand χ_0 stands for a state of spin zero particle and also is normalized to one.

The most general current, that we can form out of B and $\bar{B}$, can now be written as [5]

$$J^{A'}{}_A = \mu_1 \bar{\mathscr{B}}_B{}^{A'} \mathscr{B}^B{}_A + \mu_2 \bar{\mathscr{B}}_A{}^B \mathscr{B}^{A'}{}_B + g_0 \delta^{A'}{}_A \bar{\mathscr{B}}_B{}^C \mathscr{B}^B{}_C. \tag{7}$$

The tracelessness of $J^{A'}{}_A$ gives us

$$\mu_1 + \mu_2 = 8g_0. \tag{8}$$

We would notice in the following that the term involving g_0 in equation (7) would not appear in the magnetic moment tensor at all.

Following Bég, Lee and Pais [6], we assume that the magnetic moment in the low frequency limit transforms as the (15, 3) component of a tensor (see also [5]). We can write

$$M = \mu_0 J^{A'}{}_A \vec{\sigma}^a{}_{a'} \cdot \vec{n} Q^\alpha{}_{\alpha'}, \tag{9}$$

where $\vec{n}$ is the vector $\vec{q}\times\vec{\varepsilon}$. The vector $\vec{q}$ is the momentum of the bosons and $\vec{\varepsilon}$ is the polarization vector of electromagnetic waves perpendicular to $\vec{q}$. The operator Q is the diagonal charge operator: $Q^\alpha{}_{\alpha'} = q_{\alpha'}\delta^\alpha{}_{\alpha'}$, with $q_1 = q_4 = \frac{2}{3}$ and $q_2 = q_3 = -\frac{1}{3}$.

Substituting (7) in (9) we get

$$M = M(\bar{V}V)+M(\bar{P}P)+M(\bar{V}P)+M(\bar{P}V), \tag{10}$$

where

$$M(\bar{V}V) = \mu_0 \vec{n}\cdot\langle\bar{\chi}\vec{\sigma}\chi\rangle_1[\mu_x(\bar{V}V_F)^{\alpha'}{}_\alpha+\mu_y(\bar{V}V_D)^{\alpha'}{}_\alpha]Q^\alpha{}_{\alpha'}, \tag{10a}$$

$$M(\bar{P}P) = 0, \tag{10b}$$

$$M(\bar{V}P) = \frac{1}{\sqrt{2}}\mu_0\vec{n}\cdot\langle\bar{\chi}_1\vec{\sigma}\chi_0\rangle\,[\mu_x(\bar{V}P_F)^{\alpha'}{}_\alpha+\mu_y(\bar{V}P_D)^{\alpha'}{}_\alpha]Q^\alpha{}_{\alpha'} \tag{10c}$$

and

$$M(\bar{P}V) = \frac{1}{\sqrt{2}}\mu_0\vec{n}\cdot\langle\bar{\chi}_0\vec{\sigma}\chi_1\rangle\,[\mu_x(\bar{P}V_F)^{\alpha'}{}_\alpha+\mu_y(\bar{P}V_D)^{\alpha'}{}_\alpha]Q^\alpha{}_{\alpha'}. \tag{10d}$$

In expressions (10a), (10c) and (10d), the abbreviation $(\bar{V}V_F)^{\alpha'}{}_\alpha$ stands for the F-current and $(\bar{V}V_D)^{\alpha'}{}_\alpha$ for the D-current without the trace term. We have also set

$$\mu_x = \tfrac{1}{2}(\mu_1-\mu_2), \qquad \mu_y = \tfrac{1}{2}(\mu_1+\mu_2), \tag{10e}$$

$$\langle\bar{\chi}\vec{\sigma}\chi\rangle_1 = \bar{\chi}^{ab}\vec{\sigma}_a{}^d\chi_{bd} \tag{10f}$$

and

$$\langle\bar{\chi}_1\vec{\sigma}\chi_0\rangle = \bar{\chi}^{ab}\vec{\sigma}_b{}^d\varepsilon_{ad}\chi_0. \tag{10g}$$

Defining $\mu(X) = \langle X; J = 1, J_z = 1|M|X; J = 1, J_z = 1\rangle$, we find from (10a) that for the vector mesons the following relations hold:

$$\mu(\omega) = \mu(\varrho^0) = -\tfrac{1}{2}\mu(\phi) = \tfrac{1}{4}\mu(D^{*0}) = -\tfrac{1}{2}\mu(K^{*0}) = -\tfrac{1}{2}\mu(\bar{K}^{*0})$$
$$= \tfrac{1}{4}\mu(\bar{D}^{*0}) = \tfrac{1}{4}\mu(\psi) = \tfrac{1}{3}\mu_0 n_3\mu_y, \tag{11a}$$

$$\mu(\varrho^+) = \mu(D^{*+}) = \mu(K^{*+}) = \mu(F^{*+}) = \tfrac{1}{3}\mu_0 n_3(-3\mu_x+\mu_y) \tag{11b}$$

and

$$\mu(\varrho^-) = \mu(K^{*-}) = \mu(D^{*-}) = \mu(F^{*-}) = \tfrac{1}{3}\mu_0 n_3(3\mu_x+\mu_y) \tag{11c}$$

The $V-V$ transition moment is defined as $\langle X|\mu|Y\rangle = \langle X; J=1, J_z=1|M|Y; J=1, J_z=1\rangle$

$$\langle\omega|\mu|\varrho^0\rangle = \langle\varrho^0|\mu|\omega\rangle = 3\mu(\varrho^0). \tag{11d}$$

For $P-V$ transitions, we define $\langle X|\mu|Y\rangle = \langle X; J=1, J_z=0|M|Y; J=0, J_z=0\rangle$ where X is a vector and Y is a pseudoscalar meson. Using (10c), we get for the nonvanishing transition moments

$$-\frac{1}{3\sqrt{2}}\langle\omega|\mu|\pi^0\rangle = -\frac{\sqrt{3}}{\sqrt{2}}\langle\omega|\mu|\eta\rangle = -\sqrt{3}\langle\omega|\mu|\eta_c\rangle = -\frac{1}{\sqrt{2}}\langle\varrho^0|\mu|\pi^0\rangle$$

$$= -\frac{1}{\sqrt{6}}\langle\varrho^0|\mu|\eta\rangle = -\frac{1}{\sqrt{3}}\langle\varrho^0|\mu|\eta_c\rangle = -\frac{1}{4\sqrt{2}}\langle D^{*0}|\mu|D^0\rangle$$

$$= \frac{1}{2\sqrt{2}}\langle\bar{K}^{*0}|\mu|\bar{K}^0\rangle = \frac{1}{2\sqrt{2}}\langle K^{*0}|\mu|K^0\rangle = \tfrac{3}{2}\langle\phi|\mu|\eta_c\rangle$$

$$= -\frac{1}{4\sqrt{2}}\langle\bar{D}^{*0}|\mu|\bar{D}^0\rangle = \frac{1}{2\sqrt{6}}\langle\psi|\mu|\eta_c\rangle = \tfrac{1}{3}\mu_0 n_3 \mu_y \tag{12a}$$

$$\langle\varrho^-|\mu|\pi^-\rangle = \langle F^{*-}|\mu|F^-\rangle = \langle K^{*-}|\mu|K^-\rangle = \langle D^{*-}|\mu|D^-\rangle = -\frac{\sqrt{2}}{3}\mu_0 n_3(3\mu_x+\mu_y) \tag{12b}$$

and

$$\langle\varrho^+|\mu|\pi^+\rangle = \langle D^{*+}|\mu|D^+\rangle = \langle K^{*+}|\mu|K^+\rangle = \langle F^{*+}|\mu|F^+\rangle = -\frac{\sqrt{2}}{3}\mu_0 n_3(-3\mu_x+\mu_y). \tag{12c}$$

We notice from (10b) that the pseudoscalar mesons do not have any magnetic moment and from (10d) that $\langle X|\mu|Y\rangle = \langle Y|\mu|X\rangle$.

Thirring [7] has already calculated the magnetic moments of the vector mesons in the SU(6) quark model. Our results coincide with his results for the known vector mesons if we set $\mu_y = 0$ and if we assume $\mu_0 n_3 \mu_x = 1$. Equation (11a) thus yields that all uncharged vector mesons have magnetic moment zero. Hence the magnetic moment of the ψ particle would then also become zero.

On the other hand, from the transition moments the imposition of the restriction $\mu_x = 0$, our results totally agree with the results of the quark model calculation of Thirring if we take $\sqrt{2}\mu_0 n_3 \mu_y$ as unity[2].

We can reproduce the results obtained by Thirring as follows: Let us assume that the vector mesons and the pseudoscalar mesons belong to different representations. Hence in equation (7) μ_1 and μ_2 are the coefficients of the $\bar{V}V$ current whereas in $\bar{V}P$ current terms for example we put constants μ'_1 and μ'_2 as the coefficients. Since the trace term

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[2] There is a sign anomaly of our results with that of Thirring. In equation (12b) and (12c) $\langle K^{*-}|\mu|K^-\rangle$ and $\langle K^{*+}|\mu|K^+\rangle$ are weighted negative in the results quoted by him.

does not appear in the magnetic moment matrix element, we need not worry about those terms at all. The above choice indicates that although we are calculating the moments in SU(8) symmetry, we label the constants in (7) in terms of the SU(4)×SU(2) irreducible representations. Thus in the equation (10c) we have to replace the constants by μ'_x and μ'_y, which are different from μ_x and μ_y of equation (10a). Our results then coincide with that of Thirring if we set $\mu_y = 0$ and $\mu_0 n_3 \mu_x = 1$ for the vector mesons and for the transition moments if we set $\mu'_x = 0$ and $\sqrt{2}\mu_0 n_3 \mu'_y = 1$.

We, however, see no justification for the above choice and claim our results to possess more general validity.

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