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BETA-DECAY OF PSEUDOSCALAR MESONS IN SPECTRUM-GENERATING SU(3)*

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Data on the beta-decays of pseudoscalar mesons are compared with the predictions of a spectrum-generating SU(3). When the experimental t-dependence of form-factor is taken into account, the predictions for overall factor and ξ are in agreement with the data on π_{e3}, $K^{\pm}_{l3}$, and K^{0}_{e3}. The difference of ξ in $K^{0}_{\mu3}$ from that in $K^{\pm}_{\mu3}$ is discussed with the possible existence of scalar interaction violating the $\Delta I = 1/2$ rule. The decay width for $K^{0}_{L} \to Ke\nu$ in the model is shown to be two orders of magnitude narrower than that in the Cabibbo model.

1. Introduction

Recently a new approach to the breaking of SU(3) has been proposed, in particular, for a weak transition of a pseudoscalar meson into the vacuum or another pseudoscalar meson [1]. Physical states are proposed to be specified by the four-velocity instead of the four-momentum. Under the assumption that the four-velocity operator commutes with the SU(3) generators, the physical state may be written as a direct product of the SU(3) state and the Poincaré state with the Poincaré group generated by the four-velocity. A similar assumption of the octet property for the anti-commutator of the physical current operator with the mass operator allows an exact use of the Wigner-Eckart theorem and, eventually, an estimate of the effect of SU(3) breaking on the physical matrix element.

The ambitious aim was to explain the suppression of the strangeness-changing current from the breaking of the SU(3) symmetry, which was expressed in terms of the masses of particles in the process. The purpose of this note is to study the predictions of the model for $\alpha \to \pi l\nu$ ($\alpha = \pi$ or K) in comparison with available data and seek possible means of discriminating the model from the usual Cabibbo model.

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In Section 2, we briefly state the main assumptions of the approach and the resulting predictions; it shall be pointed out among others that the ratio $\Gamma(K^0_{l3})/2\Gamma(K^{\pm}_{l3})$ shows an appreciable deviation from unity, even if the $\Delta I = 1/2$ rule is satisfied, because of the differences in the overall factor. In Section 3, we summarize the data on the parameters describing the distribution in the Dalitz-plot of K_{l3} decays and discuss the meaning of them. The results are somehow embarrasing; ξ seems to depend on the charge of initial kaon, which may mean the existence of scalar interaction violating the $\Delta I = 1/2$ rule. Therefore, in Section 4, we accept these results as they stand and compare our prediction for the overall factor with the data on widths. In the last section, we discuss the conclusions and the possible effects of radiative corrections, which are neglected throughout the text, and propose another method of discriminating the model from the Cabibbo model. The formulas for widths are summarized in Appendix together with those useful for the determination of the possible scalar interaction.

2. *Model and predictions*

First we assume

1. that the four-velocity operator $\hat{p}_\mu = p_\mu/M$ commutes with the SU(3) generators, and
2. that an operator of the form $\{M^p, J^V_\mu\}$ is the octet operator.

Throughout this note we set $p = -1$ as working hypothesis [1]. These two assumptions allow an exact use of the Wigner-Eckart theorem, which takes the following form for $\alpha \to \pi l \nu$ decay.

$$\langle \hat{p}_\pi, \pi| \{M^{-1}, J^V_\mu\} |\hat{p}_\alpha, \alpha\rangle = g(\text{C. G.}) \{F_+(\hat{t}) (\hat{p}_\alpha+\hat{p}_\pi)_\mu + F_-(\hat{t}) (\hat{p}_\alpha-\hat{p}_\pi)_\mu\}, \tag{1}$$

where g is the coupling constant, (C. G.) is the Clebsch–Gordan coefficient and

$$\hat{t} = [t-(m_\alpha-m_\pi)^2]/m_\alpha m_\pi. \tag{2}$$

In previous papers it has been shown that the relation between the matrix element (1) and the physical matrix element is [1]

$$\langle \hat{p}_\pi, \pi| \{M^{-1}, J^V_\mu\} |\hat{p}_\alpha, \alpha\rangle = m_\alpha m_\pi \left(\frac{1}{m_\alpha} + \frac{1}{m_\pi}\right) \langle p_\pi, \pi|J^V_\mu|p_\alpha, \alpha\rangle. \tag{3}$$

Besides 1 and 2, we assume

3. the CVC hypothesis for the octet current.

This condition leads to $F_- = 0$. The physical matrix element is finally given by

$$\mathfrak{M} = \frac{(\text{C. G.})g}{2m_\alpha m_\pi} F_+(\hat{t}) [(p_\alpha+p_\pi)+\xi(p_\alpha-p_\pi)]_\mu L_\mu, \tag{4}$$

where L_μ is the leptonic vector current and

$$\xi = -\frac{m_\alpha - m_\pi}{m_\alpha + m_\pi}. \tag{5}$$

This corresponds to the usual expression

$$\mathfrak{M} = (\text{C. G.})\frac{G}{\sqrt{2}}\begin{Bmatrix}\cos\theta_C\\ \sin\theta_C\end{Bmatrix}[f_+(t)(p_\alpha+p_\pi)+f_-(t)(p_\alpha-p_\pi)]_\mu L_\mu. \tag{6}$$

We thus have the following three predictions:

(*i*) the Cabibbo angle or the suppression factor S_{l3} at $\hat{t} = 0$ is given by the ratio of the overall mass-factor in (4),

(*ii*) $f_-(t)/f_-(0) = f_+(t)/f_+(0)$,

(*iii*) $f_-(0)/f_+(0) = \xi$ in (5).

TABLE I

Predictions for ξ and S_{l3}

	$K^+_{\mu3}$	$K^0_{\mu3}$
ξ	−0.5681	−0.5620

	$K^\pm_{l3}/\pi^\pm_{e3}$	$K^0_{l3}/\pi^\pm_{e3}$	$K^0_{l3}/K^\pm_{l3}$
S_{l3}	0.2827	0.2712	0.9593

The numerical values of the predictions (*i*) and (*iii*) are tabulated in Table I. It should be noted that, if the mass differences within isomultiplets are taken into account, the model predicts an apparent violation of the $\Delta I = 1/2$ rule, i.e., the 8% deviation from unity of the ratio $\Gamma(K^0_{l3})/2\Gamma(K^\pm_{l3})$; this deviation hopefully serves as a means of discriminating the model from the Cabibbo model. For the later purpose, we define the scalar interaction for $K \to \pi$ transition.

$$\mathfrak{M} = \frac{(\text{C. G.})g}{2m_K m_\pi} F_+(t) 2m_K f_S S, \tag{7}$$

where S is the leptonic scalar current and the mass factor is phenomenologically inserted as in the usual convention.

3. *Data on λ_+ and ξ*

3.1. λ_+

In order to make a comparison of the predictions with data, we must know the momentum-transfer dependence of the form-factors. Although the momentum-transfer dependence may be studied within the model, we here take it over from the experimental data. According to Ref. [2], most data on K_{l3} decays are consistent with the linear form-factor and there is no compelling evidence for the quadratic form-factor;[1] i.e., we develop $f_\pm(t)$ as

$$f_\pm(t) = 1+\lambda_\pm \frac{t}{m_\pi^2}\,. \tag{8}$$

The experimental data on λ_+ compiled by Particle Data Group are as follows [3].

$$\lambda_+ = \begin{cases} 0.0285 \pm 0.0043 & (K_{e3}^\pm) \\ 0.027 \pm 0.008 & (K_{\mu 3}^\pm) \\ 0.0288 \pm 0.0028 & (K_{e3}^0) \\ 0.034 \pm 0.006 & (K_{\mu 3}^0) \end{cases} \tag{9}$$

They show good consistency. In our model, $F(\hat{t})$ $(= F_+(\hat{t}))$ instead of $f_\pm(t)$ is SU(3) invariant and the universality for form-factor must be required for $F(\hat{t})$.

$$F(\hat{t}) = 1+b\hat{t} = 1-\frac{(m_\alpha - m_\pi)^2}{m_\alpha m_\pi} b + \frac{m_\pi}{m_\alpha} b \frac{t}{m_\pi^2}\,. \tag{10}$$

In the following we use this universality.

$$b = 0.087 \pm 0.011 \tag{11}$$

or

$$\lambda_+(K^\pm) = 0.0285 \pm 0.0043, \qquad \lambda_+(K^0) = 0.0290 \pm 0.0043, \tag{12}$$

where b follows from $\lambda_+^{\exp}(K_{e3}^\pm)$.

3.2. $\lambda_- = \lambda_+$

The equality of the momentum-transfer dependence of $f_\pm(t)$ is reduced to the relation $\lambda_- = \lambda_+$ in the linear approximation. It is difficult to deduce a meaningful result for this relation from the present data, because λ_- enters into formulas always with a rather small parameter $m_l^2/(m_K - m_\pi)^2$ (see Appendix A)[2]. Therefore, we simply cite recent results which are consistent with the relation or the independence of ξ from t

$$\frac{d\xi}{d(t/m_\pi^2)} = \begin{cases} -0.07 \pm 0.17 & \text{for } K_{\mu 3}^\pm \text{ [4]} \\ 0.03 \pm 0.07 & \text{for } K_{\mu 3}^0 \text{ [5]} \end{cases} \tag{13}$$

and assume that the prediction (*ii*) is not in disagreement with the present data.

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[1] We have tried to fit the data by various quadratic form-factors which are, in the region $0 \leqslant t \leqslant 5\, m_\pi^2$, contained in the area swept by the linear form-factor (11) with a few standard deviations. Since the differences between the quadratic and linear analyses are about 1% in widths, we cite only the results of linear analysis.

[2] The sensitivity of measurable quantities on $k = \lambda_-/\lambda_+$ is quite low. For example, $dR/dk \sim 0.01\,R$ and $d\varrho/dk \sim 0.01\varrho$, where $R = \Gamma(K_{\mu 3})/\Gamma(K_{e3})$ and ϱ is the Dalitz-plot density at $E_\pi = 180$ MeV and $E_\mu = 190$ MeV. These are evaluated at $k = 1$, $b = 0.09$, and $\xi = -0.57$.

3.3. ξ in $K^{\pm}_{\mu 3}$

Now we turn to the data on ξ. The magnitude of ξ can be determined by the following three methods: the Dalitz-plot analysis of $K_{\mu 3}$ decay (abbreviated as DP), polarization measurement of final muon from $K_{\mu 3}$ decay (PL), and comparison of branching ratios of $K_{\mu 3}$ and K_{e3} modes (BR). Since early experimental results are dispersed and include large error bar, we cite only a few recent data.

TABLE II

Recent data on ξ in $K^{\pm}_{\mu 3}$ obtained from the Dalitz-plot analysis and polarization measurement

Method	Ref.	Result	Average
Dalitz-plot analysis	[6]	$\xi = -0.8 \pm 0.8$ ($\lambda_+ = 0.025 \pm 0.03$ [b])	$\xi = -0.45 \pm 0.15$ ($\chi^2 = 0.744$)
	[7]	$\xi = -0.57 \pm 0.24$ ($\lambda_+ = 0.027 \pm 0.019$ [b])	
	[4]	$\xi = -0.34 \pm 0.20$ ($\lambda_+ = 0.027$ [a], $t = 6.6\ m_\pi^2$ [c])	
Polarization measurement	[8]	$\xi = -0.95 \pm 0.3$ ($t = 2.54\ m_\pi^2$ [c], $\xi =$ const. [a])	$\xi = -0.82 \pm 0.21$ ($\chi^2 = 0.525$)
	[7]	$\xi = -0.72 \pm 0.30$ ($\xi =$ const. [a])	
	[4]	$\xi = -0.25 \pm 1.20$ ($t = 4.2\ m_\pi^2$ [c])	

[a] assumption, [b] another quantity, [c] momentum-transfer used in the analysis.

We cite in Table II the recent results on ξ in $K^{\pm}_{\mu 3}$ obtained from the DP and PL methods. The results show consistency; i. e., the results of a single method are consistent with one another within one standard-deviation. Although the PL measurements give smaller ξ than that in the DP analysis, these two results are not in disagreement with each other.

TABLE III

Data on ξ in $K^{\pm}_{\mu 3}$ based on widths measurement. The values of $R^{\pm}$ are calculated from the average compiled in Ref. [3] and ξ's are evaluated at $\lambda_+ = \lambda_- = 0.0285$

Measured quantity	Average $R^{\pm} = \Gamma(K^{\pm}_{\mu 3})/\Gamma(K^{\pm}_{e3})$	ξ	Average ξ
$\Gamma(K_{l3})/\Gamma(K_{total})$	0.680 ± 0.036*	-0.03 ± 0.26	
$\Gamma(K_{l3})/\Gamma(K_{\pi 3})$	0.625 ± 0.064	-0.46 ± 0.54	-0.17 ± 0.19
$\Gamma(K_{l3})/\Gamma(K_{\mu 2})$	0.649 ± 0.040	-0.26 ± 0.31	($\chi^2 = 0.663$)
Fit by Particle Data Group	0.663 ± 0.018		-0.16 ± 0.14

* The ratio $\Gamma(K_{\mu 3})/\Gamma(K_{total})$ is based on sigle data by I. H. Chiang et al. [9] who obtained $\xi = +0.45 \pm 0.28$ and $\lambda_+ = \lambda_- = -0.006 \pm 0.015$.

Table III shows the ratio $R^{\pm} = \Gamma(K^{\pm}_{\mu 3})/\Gamma(K^{\pm}_{e3})$ obtained from various normalization methods and the resulting ξ. The results are, though consistent, rather sparse and the average ξ is in disagreement with that of PL. However, if the data normalized to $\Gamma(K^{\pm}_{\text{total}})$ are discarded, the other two data give an average $\xi = -0.31 \pm 0.27$, which is not in disagreement with the other two averages of ξ. The agreement of ξ's as well as the absence of scalar and tensor interactions is confirmed in a single experiment by Braun et al. [4]. We therefore conclude that our prediction $\xi = -0.57$ is in agreement with the present data. Our prediction for $R^{\pm}$ is

$$R^{\pm} = 0.612 \pm 0.005, \tag{14}$$

where $\xi = -0.57$ and $b = 0.087 \pm 0.011$ are used.

3.4. ξ in $K^0_{\mu 3}$

Table IV shows the recent data on ξ in $K^0_{\mu 3}$ obtained from the DP and BR methods. The data on the DP analysis show good consistency, while those on BR experiment are rather sparse. The average ξ's of these two methods are in agreement with each other. This agreement is, however, accidental; for $\Gamma(K^0_{e3})$ compiled in Ref. [3] includes the corresponding radiative events, which may appreciably change the value of ξ in BR method.

TABLE IV

Recent data on ξ in $K^0_{\mu 3}$

Method	Ref.	ξ	Average ξ
Dalitz-plot analysis	[10]	$\xi = -0.26 \pm 0.21$ ($\lambda_+ = 0.046 \pm 0.008$ [b])	$\xi = -0.131 \pm 0.064$ ($\chi^2 = 0.566$)
	[11]	$\xi = -0.11 \pm 0.07$ ($\lambda^+ = 0.030 \pm 0.0016$ [b])	
	[5]	$\xi = -0.20 \pm 0.22$ ($\lambda_+ = \lambda_- = 0.046 \pm 0.030$ [b])	
Branching-ratio experiment	[12]	$\xi = +0.5 \pm 0.4$ ($R = 0.741 \pm 0.044$ [b]) ($\lambda_+ = 0.019 \pm 0.013$ [b])	$\xi = -0.052 \pm 0.168$ ($\chi^2 = 2.33$)
	[13]	$\xi = -0.15 \pm 0.24$* ($R = 0.662 \pm 0.032$ [b])	
	[14]	$\xi = -0.20 \pm 0.29$ ($\lambda_+ = 0.03$ [a], $\lambda_- = 0$ [a])	

[a] assumption, [b] input data or another quantity, [c] momentum-transfer used in the analysis.
* evaluated by us at $\lambda_+ = \lambda_- = 0.029$.

The data on PL measurement still show inconsistency [15], [16]. Such inconsistency may stem from a scalar or tensor interaction with strong t-dependence. On the other hand, any kind of tensor interaction, which may lead to such inconsistency, is excluded by the result of Donaldson et al. [11]; the strongly t-dependent scalar interaction is also not conceivable, because the existence of a scalar interaction changes the apparent form of ξ into

$$\xi + (2m_K f_S / m_l) \tag{15}$$

and strong t-dependence of ξ is not observed as shown in (13). The origin of inconsistency thus may lie in systematic errors.

It may therefore be premature to use a definite value for ξ. Nevertheless, taking into account the above observations and the degree of consistency, we here rely on the data of the DP analysis.

$$\xi(K^{\pm}_{\mu 3}) = -0.45 \pm 0.15, \quad \xi(K^0_{\mu 3}) = -0.13 \pm 0.06. \tag{16}$$

Our prediction $\xi = -0.56$ leads to a value of $R(K^0_{l3})$ which is in disagreement with the present data by several standard-deviations[3]. The apparent disagreement between our prediction on ξ and the data does not necessarily mean the invalidity of our model, because ξ in $K^0_{\mu 3}$ obtained from the DP analysis is also definitely different from that in $K^{\pm}_{\mu 3}$ and the true value of ξ, which is in the model almost independent of the modes, can be significantly different from raw data.

3.5. Scalar interaction

The apparent dependence of ξ's on the charge of initial kaon can be induced if an interaction with abnormal property with respect to the isospin change coexists with the normal vector-interaction. The interaction may not be of tensor type, because of the severe limit on it in $K^0_{\mu 3}$ and t-independence of ξ's in (13). The abnormal vector-interaction leads to modification not only of ξ but also of the overall factor; whereas the prediction for the overall factor is in agreement with the data (see Section 4). We assume that the charge dependence of ξ's is due to the scalar interaction. The difference of ξ's in $K_{\mu 3}$, then, gives

$$f_S^0 - f_S^{\pm} = 0.034 \pm 0.017. \tag{17}$$

The interference term between S and V is proportional to the lepton mass and is negligible for K_{e3}. The square term of S might be appreciable (see Appendix A). For example, it gives rise to a correction factor to the t-tendence of form-factor analyzed in disregard of the scalar interaction:

$$\left(1 + f_S^2 \frac{6m_K^2 t}{(m_K^2 + m_\pi^2 - t)^2 - 4m_K^2 m_\pi^2}\right)^{\frac{1}{2}}. \tag{18}$$

For K^0_{e3}, we have

$$|f_S^0| = 0.038 \pm 0.008.^4 \tag{19}$$

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[3] Our prediction for R^0 is 0.612 ± 0.004, where $\xi = -0.56$ and $b = 0.087 \pm 0.011$ are used.

[4] This is obtained from the data of method 3 in Ref. [17]. The correction (18) is almost equal to 1 for $t \leqslant 0.08$ (GeV)^2 but appreciable for $t \geqslant 0.08$ (GeV)^2; in the latter region the probable value of $|f_S^0|$ in (19) reduces χ^2 by 6.3 for 9 experimental points compared with the case of $f_S^0 = 0$. The same analysis for the results of methods 1 and 2 gives $|f_S^0| = 0.015^{+0.008}_{-0.015}$ and $0.020^{+0.012}_{-0.020}$, respectively, with the decrease of χ^2 by less than unity.

As to the isospin property of the scalar interaction, there are two possibilities, i.e., $(2I, 2I_3) = (3, 3)$ or a combination of (1, 1) and (3, 1). The latter case may affect both $K^{\pm}$ and K^0 decays and might spoil the agreement of the prediction for ξ in $K^{\pm}_{\mu 3}$ with the data. The former case, which necessarily violates the $\Delta S = \Delta Q$ rule and changes only $\xi(K^0_{\mu 3})$, leads to a difference between δ_e and δ_μ defined as

$$\delta_l = \frac{\Gamma(K^0_{l+3}) - \Gamma(K^0_{l-3})}{\Gamma(K^0_{l+3}) + \Gamma(K^0_{l-3})}. \tag{20}$$

From the present data [18] and the formulas in Appendix B, we have

$$f^0_S = 0.0495 \pm 0.0665. \tag{21}$$

The above three results are consistent with the case

$$f^0_S = 0.034, \quad f^{\pm}_S = 0 \tag{22}$$

which, consequently, give $\xi = -0.45$ consistent with our model. However, they are not conclusive evidence for (22) and, furthermore, the data on $K^{\pm}_{e3}$ are too imprecise to allow the same kind of analysis as (19).

4. *Comparison of S_{l3} with data*

Since the factor S_{l3} is related with overall constants, the effect of it can be revealed only through widths; whereas widths are functions not only of S_{l3} but also of b and ξ. In order to separate the effect of S_{l3} from those of other parameters, we compare here the experimental widths with those calculated from the theoretical values of S_{l3} and the experimental values b and ξ obtained in the Dalitz-plot analysis. These are summarized in Table V.

The overall factor S_{l3} for K^0_{l3} in our model is different from that for $K^{\pm}_{l3}$ due to the mass difference within the isomultiplets and the difference gives an apparent violation of the $\Delta I = 1/2$ rule, which is stated in the Cabibbo model as the equality between $\Gamma(K^0_{l3})$ and $2\Gamma(K^{\pm}_{l3})$ except the small difference in the phase-volume. The theoretical ratio $\Gamma(K^0_{\mu 3})/2\Gamma(K^{\pm}_{\mu 3})$ obtained in the above method is larger than that of the electron mode because of the difference in ξ's.

The experimental value of $\Gamma(K^0_{e3})$ in Table V is obtained by subtracting $\Gamma(K^0 \to \pi e \nu \gamma)$ from $\Gamma(K^0_{e3})$ compiled in Ref. [3]. The width depends on the experimental conditions, i.e., how large fraction of the radiative events is included into the K^0_{e3} events; though we have calculated the ratio $\Gamma(K^0_{e3})/2\Gamma(K^{\pm}_{e3})$ by using the data by Evans et al. [13], which seem to include all the possible radiative events, we found no essential difference from that in Table V (a small decrease less than 1%). The experimental width $\Gamma(K^{\pm}_{\mu 3})$ strongly depends on the standard used in measurement (see Table III). If the width $\Gamma(K^{\pm}_{\mu 3})$ normalized to the total width of $K^{\pm}$ is discarded, the experimental ratio increases by 3%; but it may retain the same value if the most recent value of $\Gamma(K^0_{\mu 3})$ [14] is used instead of the world average.

TABLE V

Comparison of ratios of widths with data. The experimental ratio is the world average of Ref. [3]. The width $\Gamma(K^0_{e3})$ is obtained by subtracting the corresponding radiative width

	Experiment	B−W model	Cabibbo model
$\dfrac{\Gamma(K^\pm_{e3})}{\Gamma(\pi_{e3})}$	$(0.992\pm0.068)\times10^7$	$(1.100\pm0.041)\times10^7$	$(1.172\pm0.057)\times10^7$
$\dfrac{\Gamma(K^0_{e3})}{\Gamma(\pi_{e3})}$	$(1.852\pm0.134)\times10^7$	$(2.075\pm0.073)\times10^7$	$(2.374\pm0.115)\times10^7$
$\dfrac{\Gamma(K^0_{e3})}{2\Gamma(K^\pm_{e3})}$	0.933 ± 0.026	0.9436 ± 0.046	1.013 ± 0.022
$\dfrac{\Gamma(K^0_{\mu3})}{2\Gamma(K^\pm_{\mu3})}$	1.012 ± 0.034	1.002 ± 0.049	1.078 ± 0.049
Note	$\Gamma(K^0_{e3}) = \Gamma_{exp}(K^0_{e3}) - \Gamma(K^0_L \to \pi e\nu\gamma)$	$b = 0.087\pm0.011$ or $\lambda^\pm = 0.0285\pm0.0043$ $\lambda^0 = 0.0290\pm0.0043$ $S_{l3} = (m_i m_f)^{-1}$	$\lambda^\pm = 0.0285 \pm0.00453$ $\lambda^0 = 0.0305 \pm0.0047$ $S_{l3} = \begin{cases}\cos\theta_C\\ \sin\theta_C\end{cases}$ $\sin\theta_C = 0.236 \pm 0.005$ [19]

Because of these problems and large experimental uncertainties, any strong conclusion cannot be deduced from the comparison. If the comparison in Table V is taken at face value, the predictions of our model are in good agreement with the present data and the agreement is better than the Cabibbo model.

5. *Discussions*

Our model predicts definite values of S_{l3} and ξ and equality $f_+(t)/f_+(0) = f_-(t)/f_-(0)$. The prediction for S_{l3} is in agreement with the present data, the equality seems to be campatible, and ξ in $K^\pm_{\mu3}$ is agreement; but ξ in $K^0_{\mu3}$ is in disagreement. However, the experimental situation concerning ξ is not clear; some of results on $K^0_{\mu3}$ show inconsistency and the ξ's obtained in the Dalitz-plot analysis suggests the existence of abnormal scalar--interaction besides the normal vector-interaction. Although the scalar interaction of the magnitude compatible with the present limit may well explain the charge-dependence, the precise form of it with respect to the isospin change cannot be deduced from the present data; it could carry a spurion of the type $(2I, 2I_3) = (3, 3)$ or a combination of (1, 1) and (3, 1). The former case seems to be supported by the present data and gives ξ consistent with our model; however the latter case cannot be excluded.

Throughout the text we have neglected the radiative corrections. As to the difference in

ξ's, the corrections, which are different in $K_{l3}^{\pm}$ and K_{l3}^{0} due to the large Coulomb corrections in the latter, do not seem to explain it, because they are included in recent high-statistics experiments [11], [17]. The included corrections [20]–[23] were calculated in the framework of current-current interaction by using the Feynman cutoff and they contain the uncertainty due to the unknown properties of the cutoff effect of strong interaction [24]. Although this uncertainty may be well within the errors of the present data of Γ's and ξ's, it may be of the order of magnitude of the effect of possible scalar interaction to $\Gamma(K_{e3}^{0})/2\Gamma(K_{e3}^{\pm})$. In this sense, the isospin property of the scalar interaction cannot be deduced from this ratio even in the renormalizable theory of weak interaction, in which the cutoff is given by the mass of intermediate bosons and it has nothing to do with the strong interaction [25]. It is of the first importance to determine more precisely and systematically the values of ξ by the available three methods, the values of δ, and the t-dependence of form-factor in K_{e3}. Besides this principal difficulty, the published calculations of radiative corrections include minor problems; as pointed out by Donaldson et al. [11], the results of them cannot be directly applied to the data, because the conditions assumed in the calculations do not meet the experimental ones. However, the practical radiative corrections calculated under the experimental conditions do not seem to spoil the qualitative agreement of our prediction for S_{l3} or the ratio of widths with the data, because of the smallness of the corrections and the same sign of them[5].

The model predicts the ratio $\Gamma(K_{l3}^{0})/2\Gamma(K_{l3}^{\pm})$ smaller than that in the Cabibbo model by about 8%. Because of large uncertainties both in experiments and theoretical estimate of the radiative corrections, the comparison with the data cannot give a definite conclusion, but the present data seem to prefer our model than the Cabibbo model. There is another interesting comparison which may be used for the discrimination between these two models. The weak current $K_L^0 \to K^{\pm}$ in the Cabibbo model contains $\cos\theta_C$ just as the $\pi^{\pm} \to \pi^0$ current, while our $K_L^0 \to K^{\pm}$ current contains a factor $(m_{K^0} m_{K^{\pm}})^{-1}$ compared with the factor $(m_{\pi^{\pm}} m_{\pi^0})^{-1}$ in the $\pi^{\pm} \to \pi^0$ current. The corresponding branching ratios are different by orders of magnitude; the Cabibbo model predicts $\Gamma(K_L^0 \to Ke\nu)/\Gamma(K_{L'\mathrm{total}}^0) \sim 5 \times 10^{-9}$ and our model gives 3×10^{-11}. The universality of form-factor is also different between these two models; the universality of λ/m_π^2 or b. Although this difference can be, in principle, revealed by experiments, it is not possible for the moment.

In order to deduce more definite conclusion about the validity of the model, we must wait for more precise and unambiguous experiments in future. Since the model cannot be excluded by the present data, it is hoped to make unbiased analysis of experimental result.

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[5] The radiative corrections, which may be applicable to the data, are found in Ref. [20] for π_{e3} (-1.2%), in Ref. [20] (Fig. 4) for $K_{e3}^{\pm}$ (-0.5%), and in Ref. [11] for $K_{\mu3}^{0}$ (-2.4%). The numbers in parenthesis refer to the correction δ defined as $\Gamma_{\mathrm{bare}} = \Gamma_{\mathrm{exp}}\,(1+\delta)$.

APPENDIX A

Numerical formulas for widths

The widths can be calculated from the matrix element (4)

$$\Gamma(\alpha \to \beta l\nu) = \frac{1}{24\pi^3}\left[\frac{(\text{C. G.})g}{2m_\alpha m_\beta} F_+(0)\right]^2 \left(\frac{m_\alpha+m_\beta}{2m_\alpha}\right)^3 (m_\alpha-m_\beta)^5 I, \tag{A1}$$

where $F_+(\hat{t}) = F_+(0)\,(1+b\hat{t})$. The effective Clebsch-Gordan coefficients are $\sqrt{2}$, 1, 1, $1/\sqrt{2}$ for $\pi^\pm \to \pi^0$, $K_L^0 \to K^\pm$, $K_L^0 \to \pi^\pm$, $K^\pm \to \pi^0$, respectively. The intensity function I is a function of b and kinematical parameters $\eta = m_l^2/(m_\alpha-m_\beta)^2$ and $\Delta = (m_\alpha-m_\beta)^2/(m_\alpha+m_\beta)^2$.

$$I = \int_\eta^1 dx \left|\frac{F_+(\hat{t})}{F_+(0)}\right|^2 \left(\frac{x-\eta}{x}\right)^2 [(1-x)(1-\Delta x)]^{\frac{1}{2}}$$
$$\times \left\{\left(2+\frac{\eta}{x}\right)(1-x)(1-\Delta x) + \frac{3\eta}{x}(1+\sqrt{\Delta}\,\xi)^2\right\}, \tag{A2}$$

where $x = t/(m_\alpha-m_\beta)^2$. The parameters η in $K \to \pi e\nu$ and Δ in $\pi^\pm \to \pi^0$ and $K_L^0 \to K^\pm$ can be safely neglected. In the following we give the numerical expressions of I's calculated from the masses in Ref. [3] and under the assumption of universality of the form-factor $F_+(t)$.

$$I(\pi^\pm \to \pi^0) = 0.7531, \tag{A3}$$

$$I(K_L^0 \to K^\pm) = 0.7385, \tag{A4}$$

$$I(K^\pm \to \pi e\nu) = 0.6925 - 1.975 b_+ + 1.514 b_+^2, \tag{A5}$$

$$I(K^\pm \to \pi\mu\nu) = 0.4470 - 1.008 b_+ + 0.6308 b_+^2$$
$$+\xi[0.08757 - 0.07910(b_+ + b_-) + 0.08679 b_+ b_-]$$
$$+\xi^2[0.01330 - 0.01947 b_- + 0.009202 b_-^2], \tag{A6}$$

$$I(K_L^0 \to \pi e\nu) = 0.6956 - 1.895 b_+ + 1.388 b_+^2, \tag{A7}$$

$$I(K_L^0 \to \pi\mu\nu) = 0.4488 - 0.9654 b_+ + 0.5771 b_+^2$$
$$+\xi[0.08644 - 0.07472(b_+ + b_-) + 0.07838 b_+ b_-]$$
$$+\xi^2[0.01298 - 0.01813 b_- + 0.008187 b_-^2], \tag{A8}$$

where $b_\pm$ are meant to be the linear coefficients of the form-factors before $(p_K \pm p_\pi)_\mu$, respectively; the model predicts $b_- = b_+$. If there is a scalar interaction of the form (7) in addition to the vector interaction (4), ξ in (A6) and (A8) should be read as

$$\xi_{\text{bare}} + (2m_K f_S/m_\mu) \tag{A9}$$

and the following terms should be added to (A5) and (A7)

$$0.0029 f_S + 0.0013 \xi_{\text{bare}} f_S + 1.23 f_S^2 \tag{A10}$$

up to the linear terms in m_e.

APPENDIX B

Decay asymmetry of K^0_{l3} *and* $\Delta S = -\Delta Q$ *scalar interaction*

The states of neutral kaons, which are relevant for weak decays, are defined as

$$|K^0_L\rangle = \frac{p|K^0\rangle + q|\overline{K}^0\rangle}{\sqrt{|p|^2+|q|^2}}, \qquad |K^0_S\rangle = \frac{p|K^0\rangle - q|\overline{K}^0\rangle}{\sqrt{|p|^2+|q|^2}}. \tag{B1}$$

If $\Delta S = \Delta Q$ amplitude is the vector type of (4) and $\Delta S = -\Delta Q$ one is the scalar type of (7), the decay amplitudes take the following form.

$$\mathfrak{M}(K^0_{L,l+3}) \sim \frac{p}{\sqrt{|p|^2+|q|^2}} \left\{ (p_K+p_\pi) + \left(\xi^* + \frac{2m_K}{m_l} \frac{q}{p} f_S^* \right)(p_K - p_\pi) \right\}_\mu L_\mu, \tag{B2}$$

$$\mathfrak{M}(K^0_{L,l-3}) \sim \frac{q}{\sqrt{|p|^2+|q|^2}} \left\{ (p_K+p_\pi) + \left(\xi + \frac{2m_K}{m_l} \frac{q}{p} f_S \right)(p_K - p_\pi) \right\}_\mu L_\mu. \tag{B3}$$

Then the decay asymmetry parameter

$$\delta_l = \frac{\Gamma(K^0_{l+3}) - \Gamma(K^0_{l-3})}{\Gamma(K^0_{l+3}) + \Gamma(K^0_{l-3})} \tag{B4}$$

is given by the following formula up to the first order in ε, where $\varepsilon = (p-q)/(p+q)$.

$$1 - \frac{\delta_l}{2\,\mathrm{Re}\,\varepsilon} = \frac{2m_K}{m_l} \frac{\beta f_S + 2\gamma f_S \left(\xi + \frac{2m_K}{m_l} f_S \right)}{I(K^0_{l+3}) + I(K^0_{l-3})}, \tag{B5}$$

where the coupling constants are assumed to be real. The denominator of (B5) is given by

$$I(K^0_{l+3}) + I(K^0_{l-3}) = \alpha + \beta \left(\xi + \frac{2m_K}{m_l} f_S \right) + \gamma \left(\xi + \frac{2m_K}{m_l} f_S \right)^2, \tag{B6}$$

where the numerical values of α, β, γ can be obtained from Appendix A. From the present data in (11) and (16), we have

$$\delta_e = 2\,\mathrm{Re}\,\varepsilon[1 - 0.005 f_S - 4.57 f_S^2] \tag{B7}$$

up to the square terms in f_S, and

$$\delta_\mu = 2\,\mathrm{Re}\,\varepsilon[1 - 1.866 f_S]. \tag{B8}$$

If the $\Delta S = \Delta Q$ rule is satisfied, $\delta_l = 2\,\mathrm{Re}\,\varepsilon$.

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