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QUANTUM STATISTICAL CORRELATIONS IN THE FRAMEWORK OF AN UNCORRELATED JET MODEL

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The influence of Bose–Einstein statistics on pion production is studied in the framework of an uncorrelated jet model. Significant positive short range correlations of like pions are found. They are in good agreement with experimentally observed correlations in the azimuthal angle. An increase of the strength of such Bose-type correlations with increasing transverse momentum is predicted. Isospin invariance is shown to produce some Bose-type correlations between unlike pions too. They should be particularly pronounced in several decay channels of ψ mesons.

1. Introduction

In the present paper we will discuss some aspects of quantum statistics in multiparticle production. This analysis has been stimulated by recent experimental results on azimuthal two-pion correlations. In the case of like pairs ($\pi^+\pi^+$ and $\pi^-\pi^-$) the two-particle distribution shows a rather pronounced peak at small rapidity separation Δy and *small* azimuthal angles ϕ. No such effect is found for unlike pairs ($\pi^+\pi^-$). As we shall see, this is a natural consequence of the statistics of the pions, i.e. Bose–Einstein statistics. This interpretation is widely accepted now. One should also say, however, that the operation of Bose–Einstein statistics in production processes is not really well understood. The reason is the lack of a unique understanding of strong interaction dynamics.

Some attempts [1–3] of incorporation of Bose–Einstein statistics start from an analogy with radio-astronomy, the so-called Hanbury Brown–Twiss effect [4]. In this case an extended source *randomly* emits identical particles (photons or, in the case of interest here, pions of equal charge). If they are observed by two separate detectors intensity correlations will be found. This effect is called second-order interference. It is caused by a path ambiguity due to the identity of the observed particles. There are two indistinguishable ways for

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the two particles (emitted simultaneously with nearby momenta) to reach the two detectors [5]. This yields an interference term which is constructive in the case of bosons. The observed interference pattern depends on the spacetime distribution of the luminosity of the source. Due to the discussed path ambiguity second-order interference builds up, roughly speaking, *after* the emission from the source. In this way even a random emission process leads to correlations. But in general, there will be additional effects due to "internal" correlations of the source. In radio-astronomy there is no such effect (distant points on the surface of a star do not know from each other). In hadronic collisions, however, the situation changes drastically. The wave length of the emitted pions and the radius of the interaction region are now of the same size, and correlated emission may become a significant (or even the dominant) effect. Such internal correlations will in general depend on the details of the production dynamics. In this case the observed phenomena cannot be reduced to a path ambiguity between source and detectors, i.e. to a simple quantum mechanical effect.

A rigorous treatment of Bose–Einstein statistics would have to take into account the requirement of permutation invariance of strong interactions and the space-time development of the production process in a unique way. Such a demanding analysis is not the aim of the present paper, however. It seems to be a step forward already to explore the consequences of Bose statistics in the framework of reasonable dynamical schemes leaving away their space-time properties. This point of view will be justified by some results of the present paper. Studying an uncorrelated jet model with Bose statistics significant effects of kinematical and dynamical origin will be found which are not present in a simple second-order interference scheme.

The influence of Bose–Einstein statistics has been particularly studied in the framework of statistical models, probably starting with GGLP [6]. The analysis done in the present paper is based on a recent detailed calculation of the level density of an ideal relativistic quantum gas [7, 8]. In order to apply these calculations to high energy hadron collisions we additionally incorporate a transverse momentum cut-off. This model is formulated in Chapter 2. In Chapter 3 a phenomenological analysis of short range azimuthal correlations of like pions is performed. There is strong support for the interpretation that the observed effects are a manifestation of Bose–Einstein statistics (see also Refs [9–11]). In Chapter 4 consequences of the interplay between isospin invariance and Bose–Einstein statistics are investigated. It is proposed to search for Bose type correlations in ψ decay. In Chapter 5 our main results are summarized. Some experimental possibilities are discussed allowing a better understanding of the operation of Bose–Einstein statistics in multiparticle production and a distinction between different schemes.

2. *An uncorrelated jet model including Bose statistics*

The influence of Bose–Einstein statistics can be rather easily studied in the framework of statistical models. In any statistical model the distribution functions of the produced particles are determined by the available level density. The uncorrelated jet model is a model of this type. In the usual formulation the level density is given by the phase space integrals

$$\tilde{\Omega}_N(Q) = \int \delta^{(4)}\left(Q - \sum_{j=1}^{N} p_j\right) \frac{1}{N!} \prod_{j=1}^{N} d^4 p_j B f(p_{\perp j}^2) \delta_{\perp}(p_j^2 - m^2), \qquad (1)$$

where N is the total number of particles, Q is the total four-momentum and a transverse cut-off $f(p_{\perp}^2)$ is taken into account.

Strictly speaking, this gives a reasonable description only in the Boltzmann limit, i.e. in the high temperature-low density limit. For most applications this is sufficient, but identical particle correlations due to Bose–Einstein statistics are missed in this case. A treatment of this problem is given in the following.

The phase space available for an ideal gas of N identical bosons of total four-momentum Q is given by

$$\Omega_N(Q) = \mathrm{Tr}_N\, P^Q. \qquad (2)$$

P^Q is the projection operator ensuring energy-momentum conservation

$$P^Q = \frac{1}{(2\pi)^4} \int d^4 a e^{-iaQ} \mathscr{U}(a), \qquad \mathscr{U}(a) = e^{ia\hat{Q}}. \qquad (3)$$

The trace Tr_N refers to the subspace of given particle number

$$\mathrm{Tr}_N\, A = \sum_{\{n_p\}} \delta(N - \sum_p n_p) \langle \{n_p\} | A | \{n_p\} \rangle \qquad (4)$$

with

$$|\{n_p\}\rangle = \prod_p \frac{a_p^{+n_p}}{\sqrt{n_p!}} |0\rangle. \qquad (5)$$

The subscript p of the occupation number n_p indicates the single particle momentum state. Bose statistics is taken into account by using the appropriate commutation relations of the creation and annihilation operators

$$[a_p, a_{p'}^+] = \delta_{pp'}. \qquad (6)$$

In order to calculate the expression (2) one essentially has to evaluate a vacuum expectation value. Using the following transformation property of the creation operators

$$\mathscr{U}(a) a_p^+ \mathscr{U}^{-1}(a) = e^{iap} a_p^+ \qquad (7)$$

one obtains, after some calculations [8], the representation

$$\Omega_N(Q) = \frac{1}{(2\pi)^4} \int d^4 a e^{-iaQ} \sum_{\{L_n\}} \delta\left(N - \sum_n L_n \cdot n\right) \prod_n \frac{1}{L_n!} \left(\frac{1}{n} \sum_p e^{iapn}\right)^{L_n}. \qquad (8)$$

The set of parameters $\{L_n\} = \{L_1, L_2, \ldots\}$ takes on all non-negative integers

$$L_n = 0, 1, 2, \ldots .$$

In fact, only a finite subset contributes because of the δ-function.

So far, we have not explicitly indicated which momentum states p (with which densities) are available for a single particle. Now we specify this single particle momentum density introducing a cut-off in the transverse momentum

$$\sum_p \to \int B d^4p f(p_\perp^2)\delta_\perp(p^2-m^2). \tag{9}$$

This is the dynamical input of this model. Performing in Eq. (8) the configuration space integration d^4a we obtain the final result for the level density $\Omega_N(Q)$

$$\Omega_N(Q) = \sum_{\{L_n\}} \delta(N-\sum_n nL_n) \int \delta^{(4)}(Q-\sum_n \sum_{j_n}^{L_n} np_{j_n})$$

$$\times \prod_n \frac{1}{L_n!} \prod_{j_n}^{L_n} d^4p_{j_n} \frac{B}{n} f(p_{\perp j_n}^2)\delta_\perp(p_{j_n}^2-m^2). \tag{10}$$

The term with

$$L_1 = N, \quad L_n = 0$$

gives the Boltzmann contribution (1) usually called phase space. It dominates in the limit Q^2 large, B large, N fixed (compare the discussion in Chapter 3). In general, more terms of the cluster decomposition (10) are relevant. If $L_n = 1$ for some n the corresponding particle momentum contributes n times to the total four-momentum, and there are effectively n particles in one and the same momentum state. Such virtual states will be called Bose clusters. Introducing the cluster momenta $q_n = np$ the procedure of obtaining the total phase space can be phrased in the following way: Besides of the particles themselves one has to count all virtual states (Bose clusters) of mass nm, transverse cut-off $f(q_{n\perp}^2/n^2)$ and coupling parameter ("effective volume") B/n^3. The parameter L_n denotes the number of such n-particle Bose clusters.

The calculation of the particle momentum distributions according to the level density (10) is most efficiently done introducing a generating functional $F[Q, N|\phi]$

$$F[Q,N|\phi] = \frac{1}{\Omega_N(Q)} \sum_{\{L_n\}} \delta\left(N-\sum_n nL_n\right) \int \delta^{(4)}\left(Q-\sum_n \sum_{j_n}^{L_n} np_{j_n}\right)$$

$$\times \prod_n \frac{1}{L_n!} \prod_{j_n}^{L_n} d^4p_{j_n} \frac{B}{n} f(p_{\perp j_n}^2)\delta_\perp(p_{j_n}^2-m^2)\phi^n(\vec{p}_{j_n}). \tag{11}$$

With a n-particle Bose cluster we have to associate a product of n test functions at the same momentum p

$$\phi^n(\vec{p})$$

one for each particle in the cluster.

The l-particle momentum distribution functions are found as functional derivatives of the generating functional (11) taken at $\phi \equiv 1$

$$\prod_{j=1}^{l} (2E_j) \frac{dN}{d^3 p_1 \dots d^3 p_l} = \frac{\delta}{\delta\phi(\vec{p}_1) \dots \delta\phi(\vec{p}_l)} F[Q, N|\phi]|_{\phi\equiv 1}. \tag{12}$$

In particular, we obtain the single particle distribution

$$2E \frac{dN}{d^3 p} = \frac{Bf(p_\perp^2) \sum_{K=1}^{\infty} \Omega_{N-K}(Q-Kp)}{\Omega_N(Q)} \tag{13}$$

and the two-particle distribution

$$2E_1 2E_2 \frac{dN}{d^3 p_1 d^3 p_2} = \frac{B^2 f(p_{\perp 1}^2) f(p_{\perp 2}^2) \sum_{K,K'=1}^{\infty} \Omega_{N-K-K'}(Q-Kp_2-K'p_2)}{\Omega_N(Q)}$$
$$+ \frac{Bf(p_{\perp 1}^2) 2E_1 \delta^{(3)}(\vec{p}_1-\vec{p}_2) \sum_{K=1}^{\infty} (K-1)\Omega_{N-K}(Q-Kp_1)}{\Omega_N(Q)}. \tag{14}$$

The sums over $K(K')$ are actually limited by the total particle number and the available energy. We emphasize that the phase space terms $\Omega_M(Q')$ in Eqs (13), (14) are given by the cluster decomposition (10) and not only by the Boltzmann contribution (1).

Eq. (14) contains terms of two different types. The first one corresponds to the emission of the two observed particles from different Bose clusters of order K and K', respectively. δ-like contributions appear if the two particles come from one and the same Bose cluster. It is a significant result of the present analysis that these terms contain only one transverse cut-off factor. In this case one has to expect Bose-type correlations increasing with the transverse momentum.

3. Analysis of like-pion azimuthal correlations

The model developed in Chapter 2 predicts the existence of pronounced positive correlations between like pions nearby in momentum space. Some observable effects should result, e.g. some angular correlations or an enhancement in the two-pion effective mass plot near the threshold. In fact, corresponding experimental observations have been reported (compare e.g. Ref. [21] and further references given there). The topic of this chapter will be a discussion of correlations in the transverse angle ϕ. This should provide some understanding of the order of magnitude of a possible Bose effect, and there are interesting data to be understood.

Before turning to the model calculations we should briefly discuss these data. In order to enable some high energy approximations we will content ourselves with studying some data in the energy range 40 ... 200 GeV/c [9, 12–15]. The quantity experimentally most easily accessible is the azimuthal asymmetry A defined as

$$A = \frac{\int_{\pi/2}^{\pi} \frac{dN}{d\phi}\, d\phi - \int_{0}^{\pi/2} \frac{dN}{d\phi}\, d\phi}{\int_{0}^{\pi} \frac{dN}{d\phi}\, d\phi} . \tag{15}$$

$dN/d\phi$ is the two-particle azimuthal distribution. For large rapidity separation $\Delta y = |y_1 - y_2|$, the asymmetry appears to be flat with a value of about

$$A^{LR} \sim \begin{cases} 0.08 & 40\ \text{GeV/c}, \\ & \text{at} \\ 0.05 & 200\ \text{GeV/c}. \end{cases}$$

It is found to be independent of the charge combinations of the pion pair (compare Figs 1 a, b). Such a behaviour is expected in the framework of an uncorrelated jet model.

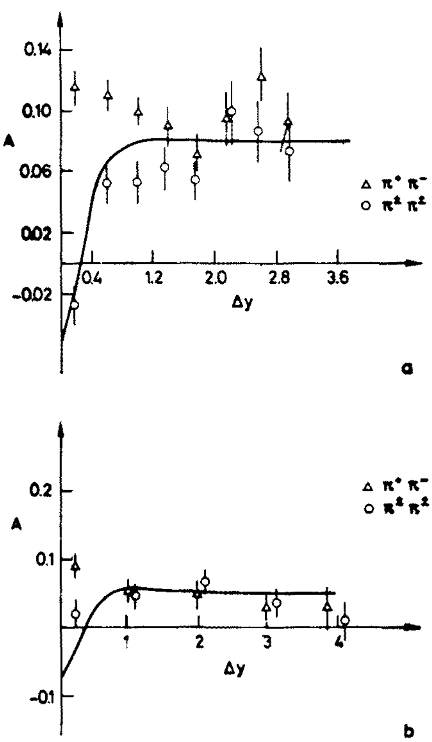

Fig. 1. Azimuthal asymmetry as function of $\Delta y = y_1 - y_2$: a) $\pi^- p \to \pi\pi + x$ at 40 GeV/c [9], b) pp $\to \pi\pi + x$ at 200 GeV/c [13]

If the rapidity difference is small ($\Delta y \lesssim 1$) the asymmetry shows a peak for unlike pairs and a small dip for like pions (Figs 1 a, b). In the case of *unlike* pions this effect is probably caused by resonance (or cluster) production. In order to understand the correlations of *like* pions better, less integrated quantities are required. Fig. 2 displays the dependence of the divided correlation function R^{--} on Δy for different ranges of the transverse angle ϕ.

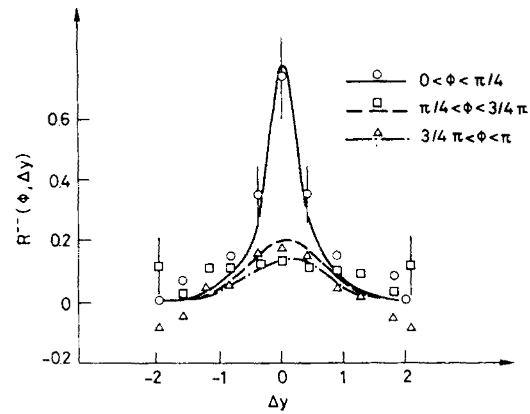

Fig. 2. Divided correlation function R^{--} as function of Δy for different ranges of the azimuthal angle ϕ [14]

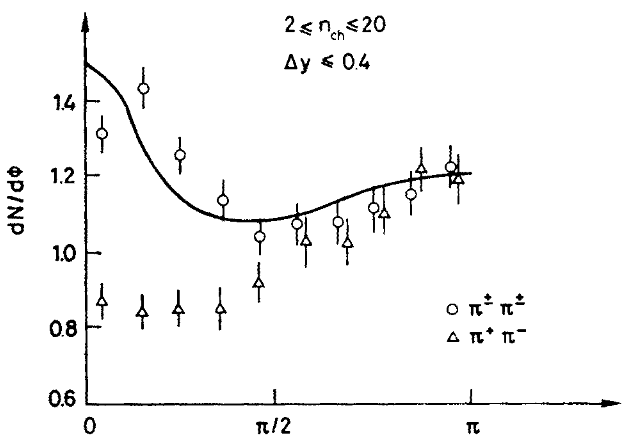

Fig. 3. $dN/d\phi$ for like and unlike pairs [9]

The striking feature is a narrow peak (of a correlation length of about 0.4 units in rapidity [14]) seen for small ϕ angles only. Such a peak in the ϕ distribution is also observed at 40 GeV/c (Fig. 3). Moreover, the dependence of the asymmetry on the transverse momentum separation $\Delta p_\perp$ indicates that this peak is enhanced at small $\Delta p_\perp$ (compare Figs 4 a, b).

Such a behavior, a peak at small $\Delta y, \phi, \Delta p_\perp$ for like pions is qualitatively expected from quantum statistics. In the framework of the uncorrelated jet model of Chapter 2 it is caused by the δ-like contribution to the two-pion distribution (14). In fact, this δ-peak will be smeared out because of the uncertainty relation and the smallness of the hadronic interaction volume

$$2E_1\delta^{(3)}(\vec{p}_1-\vec{p}_2) = 2\delta(y_1-y_2)\delta^{(2)}(\vec{p}_{\perp 1}-\vec{p}_{\perp 2})$$

$$\rightarrow D(\Delta y, p_{\perp 1}, p_{\perp 2}, \phi) = \frac{2}{\pi^{3/2}\delta y_1(\delta p_\perp)^2}\, e^{-\left(\frac{\Delta y}{\delta y_1}\right)^2} e^{-\frac{(\vec{p}_{\perp 1}-\vec{p}_{\perp 2})^2}{(\delta p_\perp)^2}}. \tag{16}$$

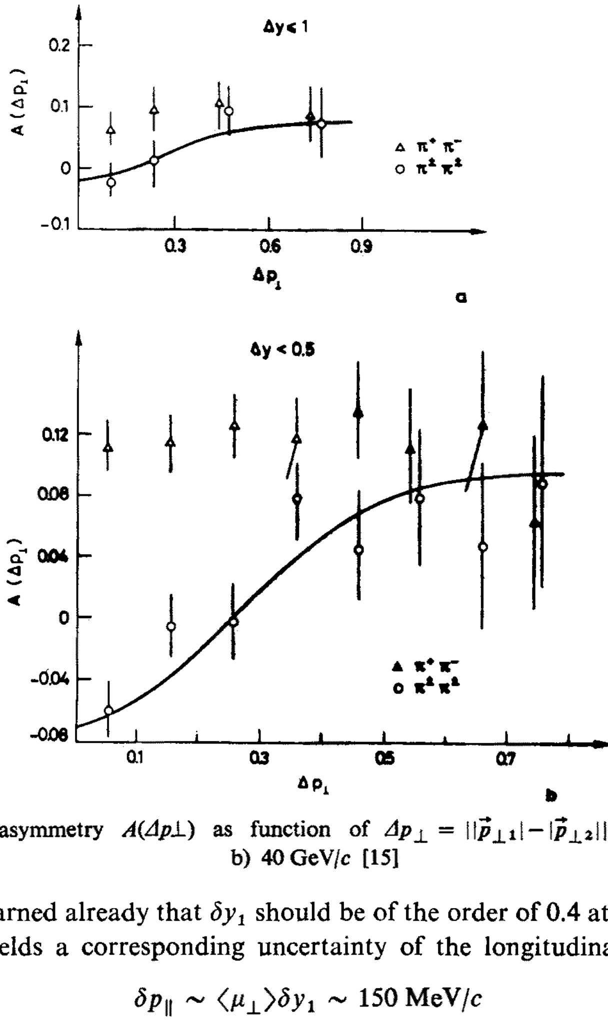

Fig. 4. Azimuthal asymmetry $A(\Delta p\perp)$ as function of $\Delta p_\perp = ||\vec{p}_{\perp 1}|-|\vec{p}_{\perp 2}||$: a) 102 GeV/$c$ [12], b) 40 GeV/c [15]

From Fig. 2 we learned already that δy_1 should be of the order of 0.4 at 200 GeV/c. A very rough estimate yields a corresponding uncertainty of the longitudinal momentum

$$\delta p_\parallel \sim \langle \mu_\perp \rangle \delta y_1 \sim 150\ \text{MeV}/c$$

which looks reasonable. A similar value for $\delta p_\perp$ is expected. In order to perform a more comprehensive analysis, some high energy approximations will be used bringing the expression (14) into a simple and manageable form. In the extremely relativistic limit the phase space integrals (1) behave like [16]

$$\tilde{\Omega}_N(Q) \sim \text{const}\, I_N(\vec{Q}_\perp)\frac{1}{Q^2}(\log Q^2)^{N-2} \tag{17}$$

independent of the particle masses. The transverse part is given by

$$I_N(\vec{Q}_\perp) = \int (\prod_{i=1}^{N} d^2p_{\perp i} Bf(p_{\perp i}^2))\delta^{(2)}(\vec{Q}_\perp - \sum \vec{p}_{\perp i}). \tag{18}$$

Applying this approximation to the representation (14) we obtain the simple expression

$$E_1E_2 \frac{dN}{d^3p_1d^3p_2} \sim \left[B^2f(p_{\perp 1}^2)f(p_{\perp 2}^2) + Bf\left(\left(\frac{\vec{p}_{\perp 1}+\vec{p}_{\perp 2}}{2}\right)^2\right) D(\Delta y, p_{\perp 1}, p_{\perp 2}, \phi)\right]$$
$$\times I_{N-2}(\vec{p}_{\perp 1}+\vec{p}_{\perp 2})\left(1+O\left(\frac{1}{B\log Q^2}\right)\right), \tag{19}$$

where $D(\Delta y, p_{\perp 1}, p_{\perp 2}, \phi)$ is given by Eq. (16). The contribution of a K-particle Bose cluster is of the order of

$$(B\log Q^2)^{-K}.$$

At very high energies only two terms will survive: the Boltzmann term and a term corresponding to a configuration with only one two-particle Bose cluster which just yields the two observed pions.

At present energies the approximation (19) is not a very precise one. Therefore, some non-asymptotic effects are taken into account introducing an additional normalization factor W_1 (which should not be very far from one, however)

$$E_1E_2 \frac{dN}{d^3p_1d^3p_2} \sim \left[B^2f(p_{\perp 1}^2)f(p_{\perp 2}^2) + W_1Bf\left(\left(\frac{\vec{p}_{\perp 1}+\vec{p}_{\perp 2}}{2}\right)^2\right) D(\Delta y, p_{\perp 1}, p_{\perp 2}, \phi)\right]$$
$$\times \left(1 - \frac{p_{\perp 1}p_{\perp 2}}{\langle p_\perp\rangle^2}\,\frac{\pi}{2}\,A^{\mathrm{LR}}\cos\phi\right). \tag{20}$$

In Eq. (20), an additional approximation [19]

$$I_{N-2}(\vec{p}_{\perp 1}+\vec{p}_{\perp 2}) \sim \left(1 - \frac{p_{\perp 1}p_{\perp 2}}{\langle p_\perp\rangle^2}\,\frac{\pi}{2}\,A^{\mathrm{LR}}\cos\phi\right) \tag{21}$$

with

$$A^{\mathrm{LR}} \sim \frac{1}{N} \tag{22}$$

has been used. Strictly speaking, this holds for large N and not too large $p_{\perp 1}p_{\perp 2}/\langle p_\perp\rangle^2$ only.

A further inspection at Fig. 2 shows the presence of a small background term enhanced at small Δy and not strongly dependent on ϕ. It might be a manifestation of some kinematical rapidity correlations [14], of the interplay of diffractive and non-diffractive channels

[17] and of additional dynamical correlations expected, e.g., in the framework of independent cluster models [18]. This effect is small, and we do not try to investigate it in more detail. But it will be taken into account in a phenomenological way adding a contribution

$$W_2 \frac{1}{\sqrt{\pi}\,\delta y_2} e^{-\frac{(y_1-y_2)^2}{(\delta y_2)^2}} B^2 f(p_{\perp 1}^2) f(p_{\perp 2}^2) \left(1 - \frac{p_{\perp 1} p_{\perp 2}}{\langle p_\perp \rangle^2} \frac{\pi}{2} A_2 \cos\phi\right) \tag{23}$$

to Eq. (20). We choose $W_2 = 0.25$, $y_2 = 1.3$, $A_2 = 0.15$. Our results are not sensitive to the details of the parametrization (23) since its overal effect is small.

We still have to specify the cut-off function $f(p_\perp^2)$ and the coupling parameter B. In order to be able to perform some integrations analytically we use

$$f(p_\perp^2) = \frac{c}{\pi} e^{-cp_\perp{}^2} \tag{24}$$

with

$$c = \frac{1}{\langle p_\perp^2 \rangle} \sim 6.5(\mathrm{GeV}/c)^{-2}. \tag{25}$$

B may be determined considering the total average multiplicity

$$\langle n \rangle \sim B/2 \log Q^2. \tag{26}$$

$B \approx 6$ is a reasonable choice.

Now we are prepared to look for the data again. The parameters we still have to determine are the transverse momentum uncertainty $\delta p_\perp$ and the weight factor W_1. A rather good agreement with the data shown in Figs 1—4 is obtained using

$$W_1 = 0.5, \quad \delta p_\perp = 250\ \mathrm{MeV}/c.$$

The model calculations are shown in the corresponding figures.

Some comments are in order. First of all, with one and the same set of parameters (δy_1, $\delta p_\perp$, W_1) data from different experiments at different energies (40 ... 200 GeV/c can be described. It would be interesting, however, if a more refined analysis using data of higher statistics would exhibit some energy dependence.

Since the rapidity correlation length $\delta y_1 \approx 0.4$ [14] is found to be rather small, the considered quantities, e.g. the azimuthal asymmetry, should show a significant Δy dependence at small Δy. This suggests a more detailed experimental study of this Δy dependence using bins as small as possible. The same can be said about the ϕ and $\delta p_\perp$ dependence.

The data shown in Figs 4 a, b allow only a rough estimate of $\delta p_\perp$. Its value $\delta p_\perp \simeq 250$ MeV/c is in agreement with earlier estimates [10]. It might be a bit too large, however, to be comfortable for a Bose statistics interpretation. Thus one cannot exclude that other dynamical effects contribute to the observed short range correlations of like pions. Resonance production, not properly taken into account in the present analysis, is a possible candidate. At low and medium energies ϱ production influences (via sum rules) like pion correlations too (compare Ref. [20]). Moreover, high mass resonances may contribute directly. In our opinion, it seems unlikely that such a mechanism could offer an explanation of the discussed ϕ distributions. However, it might somewhat screen Bose statistics correlations. This problem deserves further investigation. An extreme point of view would be to consider Bose statistics correlations only between directly produced pions (and between identical resonances). This would be correct if hadronic resonances were very narrow.

So far, the striking feature of the present model, i.e. the increasing strength of the Bose effect with increasing transverse momentum has not been tested explicitly. The considered quantities are not really sensitive to this property. A simple possibility to explore

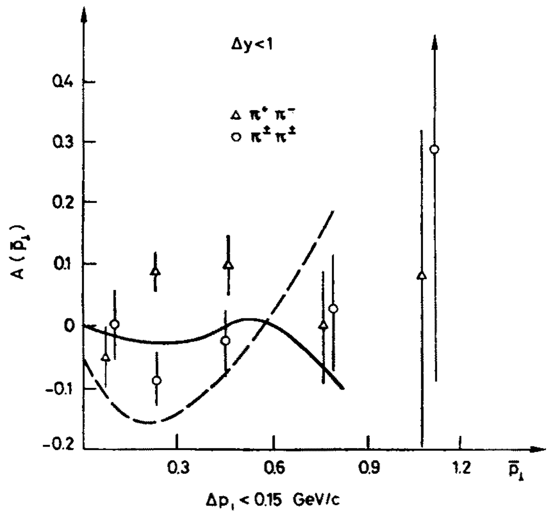

Fig. 5. Azimuthal asymmetry $A(\bar{p}_\perp)$ as function of $\bar{p}_\perp = \frac{p_{\perp 1}+p_{\perp 2}}{2}$, $\Delta p_\perp < 0.15$ GeV/c [12]; ——— this model, — — — Ref. [15]

this dependence is the study of the azimuthal asymmetry as a function of the typical transverse momentum $\bar{p}_\perp$ [12]

$$\bar{p}_\perp = \frac{p_{\perp 1}+p_{\perp 2}}{2}.$$

This quantity $A(\bar{p}_\perp)$ is plotted in Fig. 5 for small $\Delta p_\perp$ where Bose-type correlations should be significant. Unfortunately, the available data do not allow to discriminate between the present model and a behaviour of the type [10, 15]

$$E_1 E_2 \frac{dN}{d^3p_1 d^3p_2} \sim f(p_{\perp 1}^2) f(p_{\perp 2}^2)(1+D(\Delta y, p_{\perp 1}, p_{\perp 2}, \phi)) \tag{27}$$

not yielding an increase of the Bose effect with increasing $\bar{p}_\perp$. In the available range of $\bar{p}_\perp$

both models are in rough agreement with data (Fig. 5). Data at higher $\bar{p}_\perp$ values (1...2 GeV/c) including ϕ distributions in dependence of $\bar{p}_\perp$ would be of great interest. In fact, such an analysis of the $\bar{p}_\perp$ dependence would also require more reasonable model calculations. The approximation (21) is not justified at large $\bar{p}_\perp$. Also the ansatz (24) is not a very realistic one and tends to overestimate Bose type correlations at large $\bar{p}_\perp$.

At the moment the experimental situation is not conclusive. The discussed data neither support nor disprove an increase of Bose statistics correlations with increasing transverse momentum.

5. *Consequences of isospin conservation*

In the previous chapter internal symmetries have not been taken into account explicitly. Such a treatment will be given now. As we will see the inclusion of isospin conservation does not change the behaviour of the like-pion correlation function (at least in leading order). However, a new and interesting effect arises. Isospin conservation leads to quantum statistical correlations also between unlike charge states of pions. This has been noted already in Ref. [6]. If only charge conservation is taken into account this effect disappears. There are two questions to be answered in this chapter:

(*i*) Why have such unlike-pion Bose correlations not been observed so far?

(*ii*) Are there possibilities to detect them?

We will proceed in a similar way as in Chapter 2. The level density of an ideal gas of $N_+\pi^+$, $N_0\pi^0$ and $N_-\pi^-$ is given by

$$\Omega_{N_+N_0N_-}(Q^2, I) = \mathrm{Tr}_{N_+N_0N_-} P^Q P^I_i, \tag{28}$$

where (I, i) is the isospin of the N-pion gas. P^I_i is the isospin projection operator

$$P^I_i = \frac{(2I+1)}{8\pi^2} \int dR D^I_{ii}(R^{-1}) \mathcal{U}(R). \tag{29}$$

R and $\mathcal{U}(R)$ denote isospin rotations and the corresponding unitary transformations, respectively. The D^I_{ii} are diagonal elements of the rotation matrix belonging to isospin I. The basic new feature introduced by the isospin transformations is the mixing of creation operators a^+_{pt} of different charge ($t = +, 0, -$)

$$\mathcal{U}(R) a^+_{pt} \mathcal{U}^{-1}(R) = D^T_{tt'}(R) a^+_{pt'} \tag{30}$$

with the pion isospin $T = 1$. In distinction to the transformation law (30) abelian symmetries (corresponding to energy-momentum or charge conservation) lead to phase factors only (compare Eq. (7)). This mixing of different charges has the important consequence that the cluster decomposition of the level density (28) contains Bose clusters of like as well as unlike pions. For instance, there are $\pi^+\pi^+$, $\pi^+\pi^-$, $\pi^+\pi^0$, $\pi^+\pi^0\pi^-$ Bose clusters etc. We refer to Ref. [8] where this cluster decomposition has been discussed in detail.

Using the explicit expression for the level density given in Ref. [8] we can calculate the corresponding two-particle distribution functions for several charge combinations. In order to study their qualitative behaviour the leading contribution (compare the discussion in Chapter 3) should be sufficient. We obtain expressions of the type

$$E_1E_2\frac{dN^{t_1t_2}}{d^3p_1d^3p_2} \sim \{B^2f(p_{\perp 1}^2)f(p_{\perp 2}^2)+W_{t_1t_2}^{Ii}(N_+, N_0, N_-)Bf(p_{\perp 1}^2)2E_1\delta^{(3)}(\vec{p}_1-\vec{p}_2)\} \times \tilde{\Omega}_{N-2}(Q-p_1-p_2) \tag{31}$$

with

$$N = N_+ + N_0 + N_-$$

and

$$t_1, t_2 = +, 0, -.$$

The weight coefficients $W_{t_1t_2}^{Ii}(N_+, N_0, N_-)$ are the result of isospin conservation. We find

$$W_{t_1t_2}^{Ii}(N_+, N_0, N_-) = \begin{cases} 1 & (t_1, t_2) \quad ++, 00, -- \\ 1-\dfrac{\tilde{G}^{Ii}(N_+-1, N_0+1, N_--1)}{\tilde{G}^{Ii}(N_+, N_0, N_-)} & +- \\ 1-\dfrac{\tilde{G}^{Ii}(N_+, N_0-1, N_-)}{\tilde{G}^{Ii}(N_+, N_0, N_-)} & +0, -0 \end{cases} \tag{32}$$

where the $\tilde{G}^{Ii}(N_+, N_0, N_-)$ are generalized Clebsch-Gordan coefficients (see, e.g., Ref. [22])

$$\tilde{G}^{Ii}(N_+, N_0, N_-) = \frac{(2I+1)}{8\pi^2}\int dRD_{ii}^I(R^{-1})\,\{D_{11}^1(R)\}^{N_+}\{D_{00}^1(R)\}^{N_0}\{D_{-1-1}^1(R)\}^{N_-}. \tag{33}$$

First of all, we note that in the case of identical pions ($\pi^+\pi^+$, $\pi^0\pi^0$ and $\pi^-\pi^-$) the results of Chapter 2 remain valid. This also justifies the analysis of Chapter 3.

We now estimate the order of magnitude of quantum statistical effects for unlike charge combinations taking the $\pi^+\pi^-$ distribution function as an example. In Table I the corresponding weight coefficients $W_{+-}^{Ii}(N_+, N_0, N_-)$ are given for $(I, i) = (0, 0), (1, 0), (1, 1)$. In general, they are small and positive (apart from particular channels to be discussed later). If the total pion number N as well as the number of charged pions N_c becomes large they are of the order

$$W_{+-}^{Ii}(N_+, N_0, N_-) \sim O(1/N^2) \tag{34}$$

independent of the isospin. For charged-neutral combinations a behaviour

$$W_{+0}^{Ii}(N_+, N_0, N_-) \sim -\frac{1}{N-N_c/2}+O(1/N^2) \tag{35}$$

is found.

In this case an inclusive study of unlike pion distributions will not show significant quantum statistical correlations

$$\langle W^{Ii}_{\text{unlike}}(N_+, N_0, N_-)\rangle_{\text{inclusive}} \lesssim O\left(\frac{1}{\log Q^2}\right). \tag{36}$$

At best one can expect an effect of the order of some per cent compared with like-pion Bose-type correlations. Such an effect should not have been seen. The expected effect might be somewhat larger for some low multiplicity channels but in this case a statistical treatment is doubtful.

TABLE I

Some weight coefficients $W^{Ii}_{+-}(N_1, N_0, N_-)$ (compare Eq. (32))

N_+	N_0	N_-	$I = 0$	$I = 1,\ i = 0$
1	2	1	1	−1
2	0	2	0.17	0
1	3	1	−1	1
2	1	2	0	0.05
1	4	1	1	−1
2	2	2	0.05	0
3	0	3	0.07	0.02
1	5	1	−1	1
2	3	2	0.00	0.02
3	1	3	0.02	0.02

N_+	N_0	N_-	$I = 1,\ i = 1$
2	1	1	0.
2	2	1	0.05
3	0	2	0.07
2	3	1	0.00
3	1	2	0.02
2	4	1	0.02
3	2	2	0.02
4	0	3	0.03

Strictly speaking, we have studied only N-pion systems, produced e.g. in $\overline{\text{N}}$N annihilation. However, our conclusions remain valid for other types of high energy hadron-hadron scattering. In this case the isospin weights (33) have to be replaced by more general expressions [22] taking into account the presence of one or two isospin one-half particles (nucleous or kaons). The quantum statistical correlations between unlike pions found in this way are not more pronounced than those of pure pion final states. Thus the model is not in contradiction to the fact that Bose type correlations between unlike pions have not been identified so far. Nevertheless, it would be of basic interest to find such correlations. An outstanding possibility is suggested by the present model. For channels containing only one π^+ and one π^- (and an arbitrary number of π^0) the weight coefficients $W^{Ii}_{+-}(N_+, N_0, N_-)$ are not small but

$$W^{Ii}_{+-}(1, N_0, 1) = (-1)^{N_0+I}, \quad I = 0, 1, \quad i = 0. \tag{37}$$

The most obvious idea would be the study of such channels in $\bar{p}p$ annihilation. In this case, however, $I = 0$ and $I = 1$ initial states both contribute. Moreover, the number of π^0 is hard to estimate, and one would have to add channels of different number of π^0. This mixing of different isospin states and of exclusive channels will probably destroy Bose statistics effects completely.

However, a promising possibility of detecting such an effect has been opened up recently with the discovery of the $I = 0$, $G = -1\,\psi$ (3.1) particles. As discussed, the decay channels

$$\psi \rightarrow \pi^+\pi^- + \pi^0\text{'s}$$

are of particular interest. In this case the number of π^0 is automatically odd (apart from a small background) because of G-parity. Thus, since the isospin is zero, significant *negative* $\pi^+\pi^-$ quantum statistical correlations are expected (compare Eq. (37)). They should disappear for the non-resonant background at neighbouring energies.

This experiment probably offers the best chance to find such correlations. In any case, the observation or non-observation of an *unlike*-pion Bose effect would essentially help to clarify which mechanism is indeed responsible for the observed *like*-pion correlations.

5. *Summary and discussion*

Throughout this paper we have emphasized that significant quantum statistical correlations of like pions are expected in high energy production processes. In fact, several effects have been reported indicating an enhancement of pions of equal charge in momentum space. We have especially discussed short range correlations observed in like-pion azimuthal distributions. We take these effects as evidence for the existence of quantum statistical correlations.

The Bose-type effects calculated and studied in this paper are mainly of kinematical origin, i.e. are determined by the available phase space. As an interesting result, we found that these "kinematical" correlations alone are of the right order of magnitude to understand the striking features of the discussed like-pion ϕ correlations. One should also say, however, that these data only allow a rough estimate of the quantum statistical contribution. Several questions remain open. This concerns in particular the sensitivity of the Bose effect to the details of the dynamics and the space-time development of the production process. This problem deserves further experimental and theoretical studies. Below we discuss some effects which could help to determine the importance of several mechanisms and to distinguish between them.

i) The only dynamical input of the model discussed in Chapter 2 is the transverse momentum cut-off. It leads to a significant consequence: an increase of the strength of the Bose effect with increasing transverse momentum (compare Eq. (14) and the discussion in Chapter 3). Of course, such an effect may be destroyed by further dynamical structures. This would be the case, e.g., in several cluster models. Moreover, such an effect is not expected in a simple second-order interference scheme. Thus, it provides a sensitive test for the discussed uncorrelated jet model with Bose statistics.

ii) A rather different behaviour of $\gamma\gamma$ correlations is expected in different schemes. For $\pi^0\pi^0$ pairs, any reasonable model will predict quantum statistical correlations similar to those of charged pions. However, the π^0 are not detected directly but are the (dominant) source of the observed γ quanta. Corresponding to the supposed π^0 correlations a $\gamma\gamma$ correlation function can be calculated. This quantity should be compared with data. Experimental deviation from the correlation function computed in this way would be a direct indication of a second-order interference contribution. It builds up after the emission of the γ quanta, i.e. after the π^0 decays. If such a contribution would be dominant, the resulting $\gamma\gamma$ Bose peak would be more narrow than in the case of charged pions. The opposite would happen if quantum statistical correlations would only operate on the level of pions. Only such correlations are taken into account by the uncorrelated jet model discussed in the present paper.

iii) $\pi^+\pi^-$ Bose-type correlations would be a further interesting effect. As we have discussed in Chapter 4 isospin symmetry of strong interactions should lead to some unlike-pion quantum statistical correlations. ψ decay is the prominent candidate to find them. No such effects are expected from pure second-order interference. This is not a question of our ignorance of the isospin group in such a scheme. In fact, there is no room for it. The condition necessary for the appearance of second-order interference is that the two particles are identical in the sense that the detector cannot distinguish between them. But it clearly can distinguish between a π^+ and a π^-.

In our opinion, an experimental study of these problems seems to be relevant and should be done.

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