Vol. B8 (1977) *ACTA PHYSICA POLONICA* No 11

DUAL UNITARISATION SCHEME: SEMI-ANALYTIC TREATMENT OF THE OXFORD–RUTHERFORD TYPE MODEL

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(Received June 20, 1977)

A method of calculating trajectory parameters in the Oxford–Rutherford type model is presented. The poles, mixing angles and residues of the f and f' trajectories are calculated at different values of momentum transfer t in the SU(3) symmetry breaking model. The contribution of different regions of phase-space can be directly examined. The contribution of configurations with overlapping clusters builds the deviation from planar trajectories for large positive t.

2. *Introduction*

The dual unitarisation scheme [1–4] developed in the last three years has provided a new understanding of the dynamics of hadronic phenomena. So far many results in this approach have been obtained only in the one-dimensional approximation [5, 6].

This approximation, in which one neglects transverse degrees of freedom (or rather assumes them to be integrated over), was thoroughly used in explaining the behaviour of physical processes near the momentum transfer $t \approx 0$ [5], [6]. On the other side in most of the calculations with full t-dependence which have been done by now one neglects t_{min} effects or the j-dependence of the kernel representing the reggeon loop, or both [6, 7].

In the Oxford–Rutherford model [2] both these effects are properly taken into account but the resulting model is complicated and the numerical results obtained from subtracting comparable quantities carry a large degree of uncertainty [8]. The main difficulty lies in determining the parameters of lower lying trajectories. In fact one cannot calculate important parameters even for the trajectory next to the leading one [8]. It would be therefore interesting to have a simplified treatment of the Oxford–Rutherford scheme in which after taking cluster overlap (OVC) and j-dependence of the kernel into account one could also examine where and to what extent no cluster overlap (NOVC) and j-independent kernel approximation is correct. The knowledge of the j-dependent kernel is of course of great interest. Knowing the kernel explicity we could find with good accuracy the poles,

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residues and mixing angles in the SU(3) symmetry breaking case. There will be no problems with lower lying trajectories. Inclusion of baryonic exchanges would also be straightforward.

In this paper such a simplified treatment of the Oxford-Rutherford model is proposed and discussed. We find that for large negative t the approximation neglecting configurations of overlapping clusters is good, whereas for large positive t it is just this region of the s-channel phase-space which gives important corrections to the planar trajectories. At $t = 0$ we are roughly midway between these two extreme situations.

The rest of the paper is organized as follows. In the next section we find a j-dependent kernel in the Oxford-Rutherford type model [2]. Section 3 is devoted to the evaluation of the parameters for f, f' trajectories in the SU(3) symmetry breaking case. Section 4 contains discussion. Summary of the results is presented in the last section.

2. *j-dependent kernel in the Oxford-Rutherford type model*

Let us consider the one loop diagram with twisted reggeon exchanges (Fig. 1) and the corresponding amplitude taken from Ref. [2]

$$\frac{1}{i}\,\mathrm{disc}_s\,\tilde{L} = \frac{1}{N}\int\limits_{\sqrt{s_1}+\sqrt{s_2}<\sqrt{s}} ds_1 ds_2 s_1^{\alpha_1(t)} s_2^{\alpha_2(t)}$$

$$\times \int\limits_{\mathrm{LIPS}(s)} d(1)d(2)\,[G^M_{MM}(t;t_1,t_2)]^2 \left(\frac{s}{s_1 s_2}\right)^{\alpha_L(t_1)+\alpha_L(t_2)} \tag{1}$$

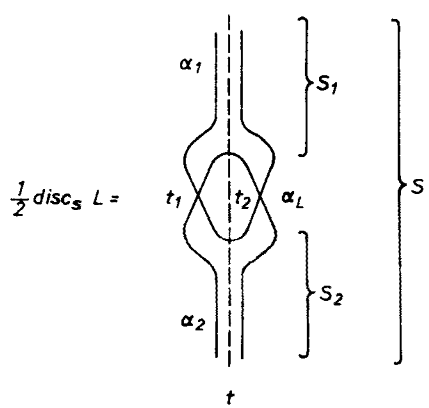

Fig. 1. One loop diagram with twisted reggeon exchanges

The triple-regge vertex function G^M_{MM} is parametrised as in Ref. [1, 2]

$$[G^M_{MM}(t;t_1,t_2)]^2 = G(t)\left(1+R\,\frac{s_1 s_2}{s}\right)^{t_1+t_2}, \tag{2}$$

where $G(t) = \frac{4g^2N}{(2\pi)^4}\exp(-\kappa t)$. In Eq. (1) $\alpha_L(t_i) \equiv \alpha_L^0 + \alpha_L' t$, N = number of flavours and $\alpha_L(t)$ is the trajectory of the reggeons in the loop.

Performing the transverse momentum integration we obtain

$$\frac{1}{i}\,\mathrm{disc}_s\,\tilde{L} = \int\limits_{\sqrt{s_1}+\sqrt{s_2}<\sqrt{s}} ds_1 ds_2 \frac{G(t)}{N}\frac{2\pi}{a\sqrt{s}\,|p|}\exp\left(\frac{a}{2}\left|\frac{k}{p}\right| t\right)$$

$$\times \exp\left(-2a\frac{s_1 s_2}{s}\right)\left(\frac{s}{s_1 s_2}\right)^{2\alpha_L^0} s_1^{\alpha_1(t)} s_2^{\alpha_2(t)}, \tag{3}$$

where $a = \ln\left[R + \frac{s}{s_1 s_2}\right]$, k is the CMS-momentum of the cluster s_1 and p is the CMS-momentum of one of the initial particles. We put $\left|\frac{k}{p}\right| = 1$, $|p| = \frac{\sqrt{s}}{2}$ for simplicity.

Now we introduce new variables x_1, x_2, z

$$s_1 = e^{x_1}, \quad s_2 = e^{x_2}, \quad \frac{s}{s_1 s_2} = e^z = e^{Y-x_1-x_2}, \quad s = e^Y, \tag{4}$$

and from Eq. (3) we get

$$\frac{1}{i}\,\mathrm{disc}_s\,\tilde{L} = \frac{\pi G(t)e^Y}{N}\int\limits_{x_1,x_2>0} dx_1 dx_2 \int\limits_{-\infty}^{+\infty} dz\,\delta(Y-x_1-x_2-z)$$

$$\times \exp\{x_1(\alpha_1(t)-1)+x_2(\alpha_2(t)-1)+(2\alpha_L^0-2)z\}\,\frac{\exp\left\{\left(\frac{t}{2}-2e^{-z}\right)\ln(R+e^z)\right\}}{\ln(R+e^z)}. \tag{5}$$

Writing the delta function in the Fourier representation and performing x_1, x_2 integrations we obtain in j-representation

$$\tilde{L}(j, \alpha_L^0, t) = \frac{\pi G(t)}{N}\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty} dk \frac{1}{j-1-ik}\,\frac{1}{ik+1-\alpha_1(t)}\,\frac{1}{ik+1-\alpha_2(t)}$$

$$\times \int\limits_{-\infty}^{+\infty}\frac{dz}{\ln(R+e^z)}\exp\left\{(2\alpha_L^0-2-ik)z+\left(\frac{t}{2}-2e^{-z}\right)\ln(R+e^z)\right\}. \tag{6}$$

For $z > 0$ we close the k-contour in the lower part of the complex k-plane, whereas for $z < 0$ the k-contour has to be closed in its upper part. It is easily seen that for $z < 0$ the contribution from region $\mathrm{Im}\,k > 0$, $|k| \to \infty$ is zero. Similarly one can show that the only singularities to be taken into account for $z < 0$ are the poles at $ik = \alpha_1(t) - 1$,

$ik = \alpha_2(t)-1$. Factorising off reggeon propagators $(j-\alpha_1)^{-1}$, $(j-\alpha_2)^{-1}$ we have for the kernel the following expression:

$$L(j, \alpha_L^0, t) = \frac{\pi G(t)}{N}\left\{\int_0^\infty \frac{dz}{\ln(R+e^z)} \exp\left[(2\alpha_L^0-1-j)z+\left(\frac{t}{2}-2e^{-z}\right)\ln(R+e^z)\right]\right.$$

$$+\frac{j-\alpha_2(t)}{\alpha_1(t)-\alpha_2(t)}\int_0^\infty \frac{dw}{\ln(R+e^{-w})}\exp\left[(\alpha_1-2\alpha_L^0+1)w+\left(\frac{t}{2}-2e^{w}\right)\ln(R+e^{-w})\right]$$

$$\left.+(\alpha_1 \rightleftarrows \alpha_2)\right\}. \tag{7}$$

Knowing explicitly kernel $L(j, \alpha_L^0, t)$ the problem of what the cylinder influence is on the original EXD poles can be immediately solved. Let us discuss now the importance of various terms in Eq. (7) at $j = \alpha_1(t)$. From Eq. (7) we see that for $j = \alpha_1$ rising the contribution from the configurations of non-overlapping clusters (NOVC) $(z > 0)$ decreases $\left(\text{the essential factor in the integrand behaves like } \exp\left(-\left(\alpha_1^0+\frac{t}{2}\right)z\right)\right)$. On the other side the configurations of overlapping clusters (OVC) are then dominant (the j-dependence of the integrand is like $\exp(jw)$). Needless to say this feature is quite general and does not depend on the specific model we have chosen here. At $t \approx 0$ we are roughly midway between these two extreme situations. For the purposes of the next section we will now introduce a shorthand notation for Eq. (7)

$$K_0(j, \alpha_L^0, t) = \pi G(t)\int_0^\infty \frac{dz}{\ln(R+e^z)}\exp\left[(2\alpha_L^0-1-j)z-\left(2e^{-z}-\frac{t}{2}\right)\ln(R+e^z)\right],$$

$$K_1(j, \alpha_L^0, t) = \pi G(t)\int_0^\infty \frac{dw}{\ln(R+e^{-w})}\exp\left[(j-2\alpha_L^0+1)w-\left(2e^{w}-\frac{t}{2}\right)\ln(R+e^{-w})\right], \tag{8}$$

so that

$$NL(j, \alpha_L^0, t) = K_0(j, \alpha_L^0, t)+\frac{j-\alpha_2(t)}{\alpha_1(t)-\alpha_2(t)}K_1(\alpha_1, \alpha_L^0, t)+(\alpha_1 \rightleftarrows \alpha_2).$$

3. *The cylinder shift for f, f′ trajectories in the SU(3) symmetry breaking case*

The cylinder mixes $|R\rangle = \frac{1}{\sqrt{2}}(|p\bar{p}\rangle+|n\bar{n}\rangle)$ and $|S\rangle = |\lambda\bar{\lambda}\rangle$ states in such a way that at the new poles $j = \alpha_1(t)$, $j = \alpha_2(t)$ we have the states

$$|1\rangle = \cos\chi_1|R\rangle+\sin\chi_1|S\rangle,$$

$$|2\rangle = \cos\chi_2|R\rangle+\sin\chi_2|S\rangle. \tag{9}$$

As the kernel is j-dependent the mixing angles do not fulfill the relation $\chi_2 = \frac{\pi}{2} + \chi_1$.

The equation for the full propagator F is

$$F = P + PLF, \tag{10}$$

where

$$P = \begin{bmatrix} s^{\alpha_R(t)} & 0 \\ 0 & s^{\alpha_S(t)} \end{bmatrix} \tag{11}$$

in the $|R\rangle$, $|S\rangle$ base, and $\alpha_{R,S}(t) = \alpha^0_{R,S} + \alpha' t$. We expect F to have the form

$$F = \begin{bmatrix} \beta(\alpha_1, t)\cos^2\chi_1 s^{\alpha_1} & \beta(\alpha_1, t)\cos\chi_1 \sin\chi_1 s^{\alpha_1} \\ +\beta(\alpha_2, t)\cos^2\chi_2 s^{\alpha_2} & +\beta(\alpha_2, t)\cos\chi_2 \sin\chi_2 s^{\alpha_2} \\ & \\ \beta(\alpha_1, t)\sin\chi_1 \cos\chi_1 s^{\alpha_1} & \beta(\alpha_1, t)\sin^2\chi_1 s^{\alpha_1} \\ +\beta(\alpha_2, t)\sin\chi_2 \cos\chi_2 s^{\alpha_2} & +\beta(\alpha_2, t)\sin^2\chi_2 s^{\alpha_2} \end{bmatrix}. \tag{12}$$

Writing Eq. (10) in the $|R\rangle$, $|S\rangle$ base and using the method described in the previous section after comparing the residues at $j = \alpha_1(t)$ and $j = \alpha_2(t)$ we obtain:

$$\alpha_i - \alpha_R = L_{RR}(\alpha_i) + \operatorname{tg}\chi_i L_{SR}(\alpha_i), \qquad \alpha_i - \alpha_S = L_{SS}(\alpha_i) + \operatorname{ctg}\chi_i L_{SR}(\alpha_i), \tag{13a}$$

where

$$L_{RR}(\alpha) = \tfrac{2}{3}\{K_0(\alpha, \alpha^0_R, t) + K_1(\alpha, \alpha^0_R, t)\},$$

$$L_{SR}(\alpha) = \frac{\sqrt{2}}{3}\{K_0(\alpha, \alpha^0_K, t) + K_1(\alpha, \alpha^0_K, t)\},$$

$$L_{SS}(\alpha) = \tfrac{1}{3}\{K_0(\alpha, \alpha^0_S, t) + K_1(\alpha, \alpha^0_S, t)\}. \tag{13b}$$

In Eq. (13b) $\alpha^0_{R,K,S}$ are the intercepts of the leading planar trajectories $|p\bar{p}\rangle$, $|p\bar{\lambda}\rangle$, $|\lambda\bar{\lambda}\rangle$. The functions K_0, K_1 are defined in Eq. (8).

The position of the new poles can be easily obtained from the equality

$$\alpha_{1,2} = \tfrac{1}{2}\{\alpha_R + \alpha_S + L_{RR} + L_{SS} \pm \sqrt{(\alpha_R + L_{RR} - \alpha_S - L_{SS})^2 + 4L^2_{SR}}\}, \tag{14}$$

where L_{RR}, L_{SR}, L_{SS} are given by Eq. (13b) (with an appropriate α), and the sign $+(-)$ corresponds to the pole $\alpha_1(\alpha_2)$.

The mixing angles are given by

$$\operatorname{tg}\chi_i = \frac{\alpha_i - \alpha_R - L_{RR}(\alpha_i)}{L_{SR}(\alpha_i)}. \tag{15}$$

To calculate residues (in the one-channel version of the Pomeron equation — Eq. (10)) we treat the second part of the kernel which is due to the OVC contribution as a linear in j approximation of the form $K_1(j, \alpha^0_L, t)$. For the matrix case the use of the forms

$K_1(j, \alpha_L^0, t)$, $\alpha_L^0 = \alpha_{R,K,S}^0$ instead of their linear counterparts is rather convenient. This may be done if $\frac{\partial}{\partial j} K_1(j, \alpha_L^0, t)|_{\alpha_i}$ does not differ too much from ($\alpha_M = \alpha_{R,S}$)

$$\frac{K_1(\alpha_i, \alpha_L^0, t) - K_1(\alpha_M, \alpha_L^0, t)}{\alpha_i - \alpha_M}.$$

For not very large j, t where the calculations of regge trajectories and mixing angles are reliable this is indeed the case. Clearly, the replacement of the original OVC-part of the kernel by $K_1(j, \alpha_L^0, t)$ has no influence on regge trajectories and mixing angles, which are calculated without this substitution. Thus the residues can be calculated from

$$\beta^{-1}(j, t) = 1 - \tfrac{1}{2} \left\{ \frac{\partial L_{RR}}{\partial j} + \frac{\partial L_{SS}}{\partial j} \pm \frac{(\alpha_R + L_{RR} - \alpha_S - L_{SS}) \left(\frac{\partial L_{RR}}{\partial j} - \frac{\partial L_{SS}}{\partial j} \right) + 4 L_{SR} \frac{\partial L_{SR}}{\partial j}}{\sqrt{(\alpha_R + L_{RR} - \alpha_S - L_{SS})^2 + 4 L_{SR}^2}} \right\} \tag{16}$$

with the sign $+(-)$ for $\alpha_1(\alpha_2)$ respectively.

The derivatives of the loop matrix elements are

$$\frac{\partial L_{RR}}{\partial j} = \tfrac{2}{3} \left(\frac{\partial}{\partial j} K_0(j, \alpha_R^0, t) + \frac{\partial}{\partial j} K_1(j, \alpha_R^0, t) \right),$$

$$\frac{\partial L_{SR}}{\partial j} = \frac{\sqrt{2}}{3} \left(\frac{\partial}{\partial j} K_0(j, \alpha_K^0, t) + \frac{\partial}{\partial j} K_1(j, \alpha_K^0, t) \right),$$

$$\frac{\partial L_{SS}}{\partial j} = \tfrac{1}{3} \left(\frac{\partial}{\partial j} K_0(j, \alpha_S^0, t) + \frac{\partial}{\partial j} K_1(j, \alpha_S^0, t) \right), \tag{17}$$

where

$$\frac{\partial}{\partial j} K_0(j, \alpha_L^0, t) = -\pi G(t) \int_0^\infty dz \frac{z}{\ln (R + e^z)} \exp \left\{ (2\alpha_L^0 - 1 - j) z - \left(2e^{-z} - \frac{t}{2} \right) \ln (R + e^z) \right\},$$

$$\frac{\partial}{\partial j} K_1(j, \alpha_L^0, t) = \pi G(t) \int_0^\infty dw \frac{w}{\ln (R + e^{-w})} \exp \left\{ (j - 2\alpha_L^0 + 1) w - \left(2e^w - \frac{t}{2} \right) \ln (R + e^{-w}) \right\}. \tag{18}$$

With the help of the formulas (14)—(18) the poles, mixing angles and residues can be directly evaluated on a computer. The important technical point is that one needs to calculate only one-dimensional integrals, and there are no problems connected with subtracting comparable quantities. We have done numerically the calculation for the following values of the parameters:

$$g = 7.0, \quad \kappa = +1.3, \quad N = 3, \quad R = 1.5$$

for f and f' trajectories. The results are shown in Fig. 2—5.

4. *Discussion*

In comparison with the original Oxford–Rutherford model the calculations presented here overestimate slightly phase-space contribution. In Fig. 2 we have visualised the regge trajectories in two cases: a) with and b) without the contribution of overlapping

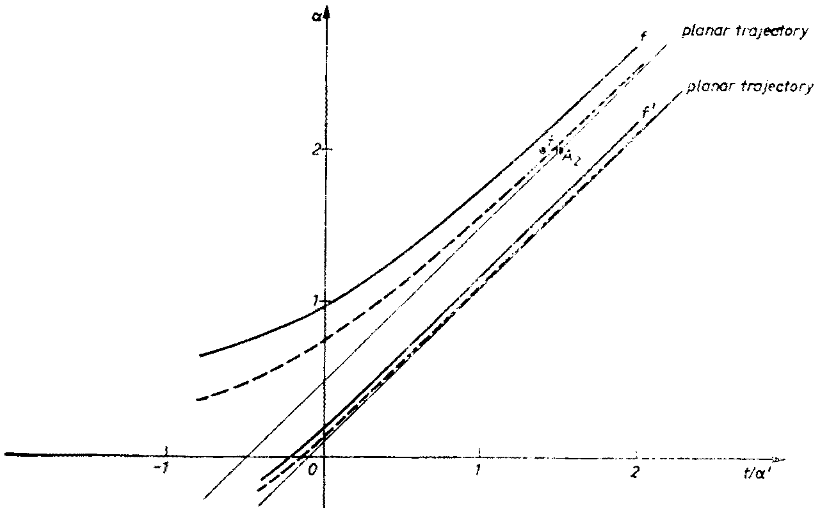

Fig 2. Planar and shifted regge trajectories in the SU(3) symmetry breaking case for the following values of the input parameters: $\alpha_R^0 = 0.5$, $\alpha_K^0 = 0.35$, $\alpha_S^0 = 0.1$, $N = 3$, $g = 7.0$, $\varkappa = 1.3$; solid lines — whole phase-space contribution, dashed lines — only configurations with no overlapping clusters allowed

clusters. It can be seen that the deviation from planarity for large positive t is mainly due to the inclusion of the OVC contribution. The f-trajectory passes near the physical f-meson but, as in Ref. [8], the influence of the cylinder correction is overestimated. This is probably due to the fact that the parametrisation of the triple-regge vertex function was obtained from the planar bootstrap programme only at momentum transfers close to zero. However, the dominance of the OVC contribution at large positive t will not be altered by other forms of $G(t)$. By requiring the planar bootstrap to be fulfilled at each value of t it should be possible to determine the triple-regge vertex function in a more reliable way. (Configurations with overlapping clusters should then of course be considered in the planar bootstrap equation.)

In Fig. 3 the quark content of the bare Pomeron is shown in both cases a) and b). We notice that the OVC configurations are not properly taken into account for $t > 0.5 \div 1.0$, leading to the ill-behaved quark content of the f pole. Similarly in Fig. 4 the amount of nonstrange quarks in the f' pole is visualised. The behaviour of the curve at t smaller

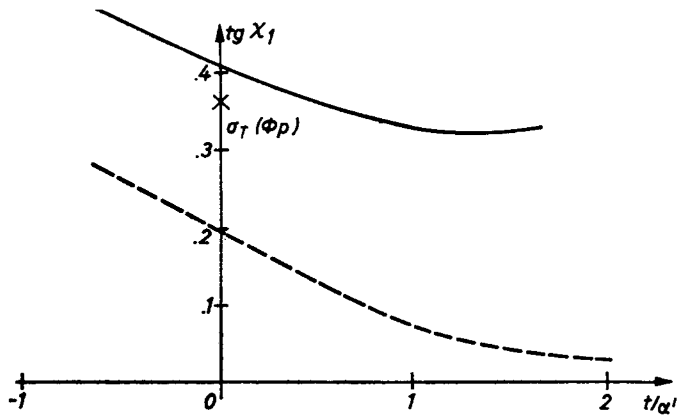

Fig. 3. Mixing angle for the shifted f trajectory; solid line — whole phase-space contribution, dashed line — only configurations with no overlapping clusters allowed

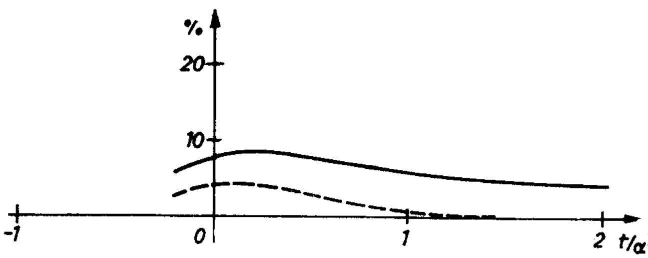

Fig. 4. Nonstrange quark content of the shifted f' pole

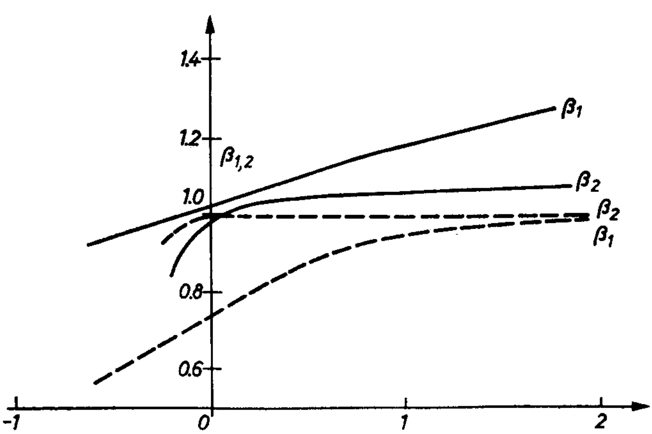

Fig. 5. The residues of the f and f' poles; solid lines — whole phase-space contribution, dashed lines — only configurations with no overlapping clusters allowed

than zero is due to the proximity of the regge-regge cut $\alpha_{RR}(t) = 2\alpha_R^0 - 1 + \frac{t}{2}$ to the shifted f' pole. As the original triple-regge vertex function $G(t)$ in the Oxford–Rutherford model does not vanish at the non-sense points the loop integral L_{RR} becomes divergent at $\alpha_2(t) = \alpha_{RR}(t)$ giving tg $\chi_2 \to -\infty$ and leaving only a strange component in the wave function of f'.

Fig. 5 shows the residues of f and f' trajectories. The rather abrupt fall in the magnitude of the f' residue at t smaller than zero is caused once again for the same reason as the behaviour of tg χ_2 in this region. The rise of the f residue with growing t is due to the rather well-established t-behaviour of the triple-regge vertex function $G(t)$. Indeed, in a simple SU(3) symmetric model we have

$$\beta(j, t)|_{j=\alpha_1} = \left\{\frac{\partial}{\partial j}\left(j-\alpha_R(t)-\frac{g^2(t)}{j-\alpha_{RR}(t)}\right)\Bigg|_{j=\alpha_1}\right\}^{-1} = \left(1+\frac{g^2(t)}{(\alpha_1(t)-\alpha_{RR}(t))^2}\right)^{-1} \tag{19}$$

which decreases when $g(t)$ increases.

We have also done the calculations for ω and φ trajectories (the only difference when compared with the f, f' case is the change of sign of the right hand sides in Eqs (13c) and (17)). The results obtained confirm the well-known fact that the intercept of the ω trajectory without introducing baryon exchange effects lies too low. To get a reasonable picture of ω and φ trajectories a sizeable amount of baryon exchanges must be incorporated into the above scheme. In fact such a calculation has been done in the one-dimensional approximation [9]. In the method presented above inclusion of baryon exchanges is straightforward and does not involve any difficulties, so one should be able to perform the t-dependent analysis of the importance of baryonic effects on the trajectories ω, φ. To get better results than those presented above a triple-regge vertex function with non-sense zeroes extracted explicitly should be used. We have not attempted to repeat our calculations for G_{MM}^M with non-sense zeroes built in as first the planar bootstrap equations for t away from $t \sim 0$ should be solved.

5. *Summary*

We have presented a semi-analytic method of computing reggeon parameters in the Oxford–Rutherford type model. The method allows us to calculate all physically interesting quantities in the SU(3) symmetry breaking case for f, f' trajectories with a straightforward extension to include baryon exchange effects when discussing the $C = -1$ sector.

The configurations of overlapping clusters of the s-channel phase-space are recognised to be very important for large positive momentum transfer t. In fact, had we neglected this contribution, the f-trajectory would have passed almost through the position of the physical A_2 pole. Better knowledge of the triple-regge vertex function is still required. It is conceivable that in the planar reggeon bootstrap equation the contribution from the configurations of overlapping clusters will increase with growing t, thus reducing effectively in this region the triple-regge vertex function.

The author would like to thank J. Kwieciński for stimulating discussions.

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