ON THE POSSIBILITY OF UNIVERSAL DESCRIPTION OF ANGULAR DISTRIBUTION IN INTERACTIONS OF HADRONS WITH NUCLEI

By B. Furmańska, R. Hołyński, S. Krzywdziński and W. Wolter

Institute of Nuclear Physics, Cracow*

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The validity of the hypothesis that the angular distribution of shower particles produced in hadron-nucleus interactions can be parametrized by the number of slow particles emitted from the struck nucleus is tested. Relation between the mean number of slow particles and the mass number A of the target nucleus or the corresponding average number of collisions inside the target nucleus is found.

It was shown in papers [1] and [2] that the angular distribution of shower particles n_s ($\beta \geqslant 0.7$) produced in proton-emulsion interactions at 67 GeV, 200 GeV and of the order of few thousands GeV can be parametrized using the number of heavy ionizing particles N_h ($\beta < 0.7$) emitted from the struck nucleus. Our parametrization of inclusive distribution turned out to be:

$$\frac{1}{N}\frac{dn_s}{d\eta} = a(\eta, E)+b(\eta, E)N_h, \tag{1}$$

where E stands for energy of the incoming proton and $\eta = -\ln \tan \frac{\theta_L}{2}$. It also turns out that pion-emulsion interactions at 60 GeV and 200 GeV obey the same linear parametrization but with different coefficients. Thus generally the angular distribution of shower particles produced in hadron-emulsion interactions can be parametrized by the following formula:

$$\frac{1}{N}\frac{dn_s}{d\eta} = a_I(\eta, E)+b_I(\eta, E)N_h, \tag{2}$$

where I denotes the incoming particle.

* Address: Instytut Fizyki Jądrowej, Zakład V, Kawiory 26a, 30-055 Kraków, Poland.

TABLE I

Values of a_I and b_I per unit η interval

η interval	200 GeV protons			200 GeV π^-		
	a_p	b_p	χ^2 NDF = 3	a_π	b_π	χ^2 NDF = 3
$-2 < \eta \leqslant -1$	.006 ±.003	.003 ±.001	.01	.003 ±.008	.002 ±.001	1.76
$-1 < \eta \leqslant 0$	.021 ±.006	.020 ±.002	.96	.019 ±.010	.021 ±.003	5.32
$0 < \eta \leqslant 1$	.322 ±.025	.112 ±.006	6.74	.192 ±.029	.106 ±.007	7.82
$1 < \eta \leqslant 2$	1.097 ±.050	.222 ±.009	.22	.681 ±.057	.210 ±.013	11.03
$2 < \eta \leqslant 3$	1.753 ±.064	.219 ±.011	7.72	1.482 ±.087	.158 ±.015	7.58
$3 < \eta \leqslant 4$	2.066 ±.063	.122 ±.009	7.31	1.815 ±.094	.109 ±.014	6.06
$4 < \eta \leqslant 5$	1.828 ±.047	.033 ±.006	6.82	1.918 ±.073	.023 ±.010	2.78
$5 < \eta \leqslant 6$	.928 ±.027	−.007 ±.003	.29	1.139 ±.045	−.022 ±.006	.89
$6 < \eta \leqslant 7$	.309 ±.015	−.008 ±.002	6.71	.325 ±.025	−.011 ±.003	4.78
$7 < \eta \leqslant 8$	.062 ±.008	−.001 ±.001	3.71	.092 ±.013	−.004 ±.001	5.92

In Table I there are listed the numerical values of the coefficients a_I and b_I of formula (2) for interactions of protons and pions at the energy of 200 GeV. Linear fits were done for $N_h \leqslant 20$. From formula (2) it follows immediately the well known relation between the average multiplicity $\overline{n_s}$ and the number of N_h particles:

$$\overline{n_s} = A_I(E) + B_I(E)N_h, \tag{3}$$

where

$$A_I(E) = \int a_I(\eta, E)d\eta, \qquad B_I(E) = \int b_I(\eta, E)d\eta.$$

Using the parametrization given by Eq. (2) one can reproduce the angular distribution of n_s particles for a given N_h value. Fig. 1 shows as an example the angular distributions of n_s particles in proton-emulsion interactions at 67 GeV and 200 GeV for arbitrarily chosen N_h values. From Fig. 1 one can draw the following conclusions:

1. For a given primary proton energy there exists an angular interval $\eta > \eta_E$ in which, to a good approximation, the angular distribution does not depend on N_h. More

precisely — the number of n_s tracks somewhat decreases with increasing number of N_h tracks.

2. With increasing primary proton energy, η_E moves towards higher values of η as the rapidity of the incident particle ($\Delta\eta_E = \ln(E_2/E_1) = \ln(200/67) = 1.1$).

3. For $\eta < \eta_E$ the number of n_s tracks increases with increasing number of N_h tracks and the maximum of the angular distribution moves towards the smaller values of η.

4. For $\eta \lesssim 1.5$ the angular distribution of n_s tracks does not depend on primary proton energy and is described by the number of N_h tracks.

The main aim of this work is to test the validity of the hypothesis that the angular distribution of shower particles produced in hadron-nucleus interaction can be para-

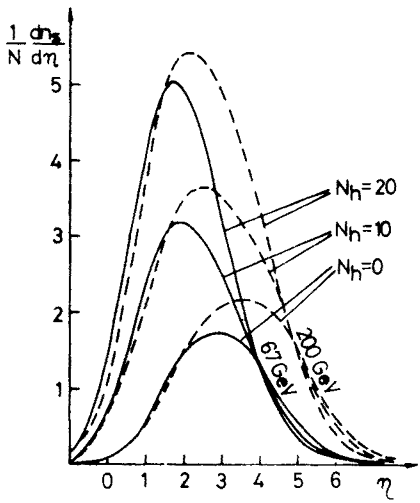


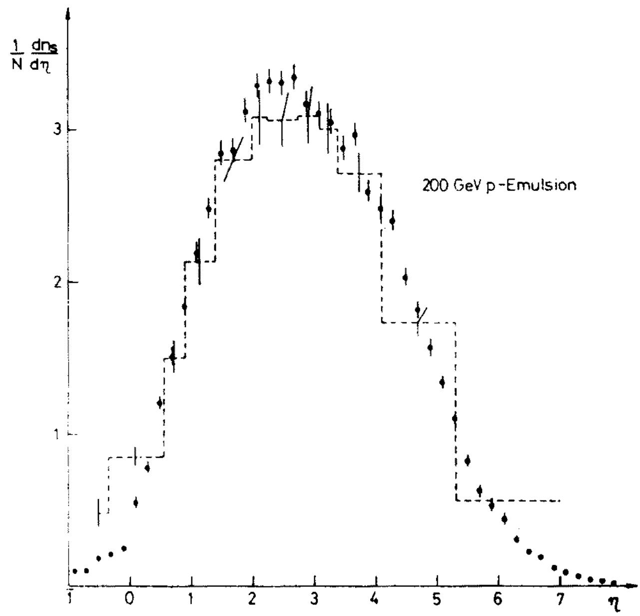
Fig. 1. Pseudorapidity distributions of particles produced in proton-emulsion interactions at 67 GeV — solid lines and 200 GeV — dashed lines. The distributions illustrate the parametrization of our data according to Eq. (1) for N_h = 0, 10, 20

metrized by formula (2) and that the coefficients a_I and b_I do not depend on the mass number A of the target nucleus and are the same as those found in emulsion.

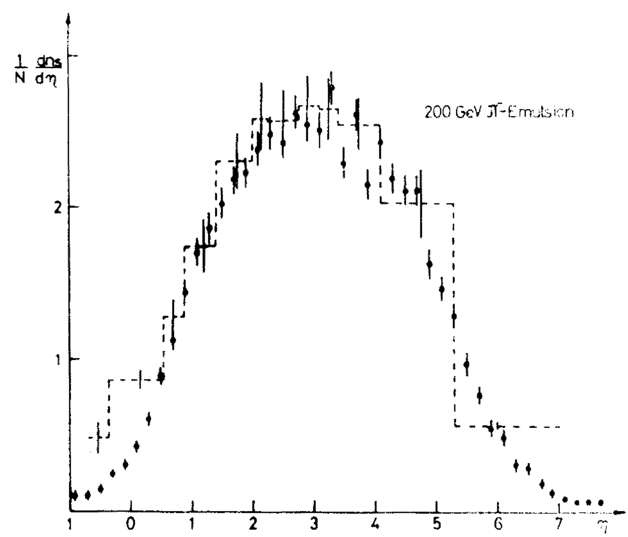
Since formula (2) is linear in n_s and N_h, it remains valid when $\overline{N}_h$ denotes the average number of slow particles emitted from the struck nucleus. Thus we shall test the validity of the formula:

$$\frac{1}{N}\frac{dn_s}{d\eta} = a_I(\eta, E) + b_I(\eta, E)\overline{N}_h. \tag{4}$$

In order to test our hypothesis we shall make use of the counter experiment data obtained by Busza et al. [3]. They analyzed the angular distribution of fast particles produced in hadron interactions with different target nuclei. Among many targets they used also nuclear emulsion. This allows a direct comparison between their and our data. In Fig. 2 we present angular distributions of particles produced by 200 GeV protons and pions



a



b

Fig. 2. Comparison of data obtained on nuclear emulsion as a target — from counter experiment [3] — dashed lines and from emulsion experiment- [1] — dots; 2a) 200 GeV proton interactions, 2b) 200 GeV π^- interactions

in interactions with nuclear emulsion as a target — one distribution is the result of measurements performed in nuclear emulsion, the other obtained in the counter experiment. Except of the both ends of the η distribution the agreement is very good (see Table II). Therefore in our analysis we shall restrict ourselves to the η interval $0.58 \leqslant \eta < 5.28$.

TABLE II

Comparison of data on nuclear emulsion target obtained from our emulsion experiment and from counter experiment [3]

η interval	200 GeV protons $\frac{1}{N} n_s(\eta)$ Our data	200 GeV protons $\frac{1}{N} n_s(\eta)$ Busza's data [3]	200 GeV protons χ^2 NDF = 1	200 GeV π^- $\frac{1}{N} n_s(\eta)$ Our data	200 GeV π^- $\frac{1}{N} n_s(\eta)$ Busza's data [3]	200 GeV π^- χ^2 NDF = 1
$-.67 \leqslant \eta < -.38$	.05 ±.01	.14 ±.03	8.84	.05 ±.01	.11 ±.03	4.05
$-.38 \leqslant \eta < .58$	.53 ±.01	.80 ±.06	18.64	.44 ±.02	.74 ±.06	21.98
$.58 \leqslant \eta < .92$	.57 ±.02	.54 ±.04	.63	.43 ±.02	.46 ±.04	.33
$.92 \leqslant \eta < 1.39$	1.06 ±.02	1.00 ±.07	.59	.82 ±.03	.82 ±.07	.004
$1.39 \leqslant \eta < 1.99$	1.76 ±.03	1.68 ±.09	.80	1.28 ±.04	1.38 ±.10	.87
$1.99 \leqslant \eta < 2.25$	.85 ±.02	.80 ±.05	.89	.62 ±.02	.67 ±.06	.70
$2.25 \leqslant \eta < 2.76$	1.69 ±.03	1.56 ±.09	2.00	1.28 ±.04	1.31 ±.10	.10
$2.76 \leqslant \eta < 3.08$	1.02 ±.02	.99 ±.06	.27	.82 ±.03	.85 ±.06	.22
$3.08 \leqslant \eta < 3.38$	.92 ±.02	.90 ±.05	.14	.82 ±.03	.79 ±.06	.17
$3.38 \leqslant \eta < 4.08$	1.94 ±.03	1.90 ±.09	.20	1.67 ±.04	1.78 ±.12	.74
$4.08 \leqslant \eta < 5.28$	2.20 ±.03	2.08 ±.12	1.01	2.24 ±.05	2.42 ±.28	.39
$5.28 \leqslant \eta < 7.00$	.78 ±.02	.96 ±.03	17.25	.89 ±.03	1.12 ±.04	22.43

We have adopted the following procedure of testing the validity of formula (4). Let us denote by $n(\eta_i)$ the mean number of particles in $\Delta\eta_i$ interval produced in interactions of protons or pions with a given target nucleus (experimental data obtained by Busza et al. [3]) and by $n_s(\eta_i)$ the number of particles given by formula (4) in the same $\Delta\eta_i$ interval. By minimizing the χ^2 function with $\bar{N}_h$ as a free parameter:

$$\chi^2 = \sum_{0.58 \leqslant \eta_i < 5.28} \frac{[n(\eta_i) - n_s(\eta_i)]^2}{\sigma^2(\eta_i) + \sigma_s^2(\eta_i)} \tag{5}$$

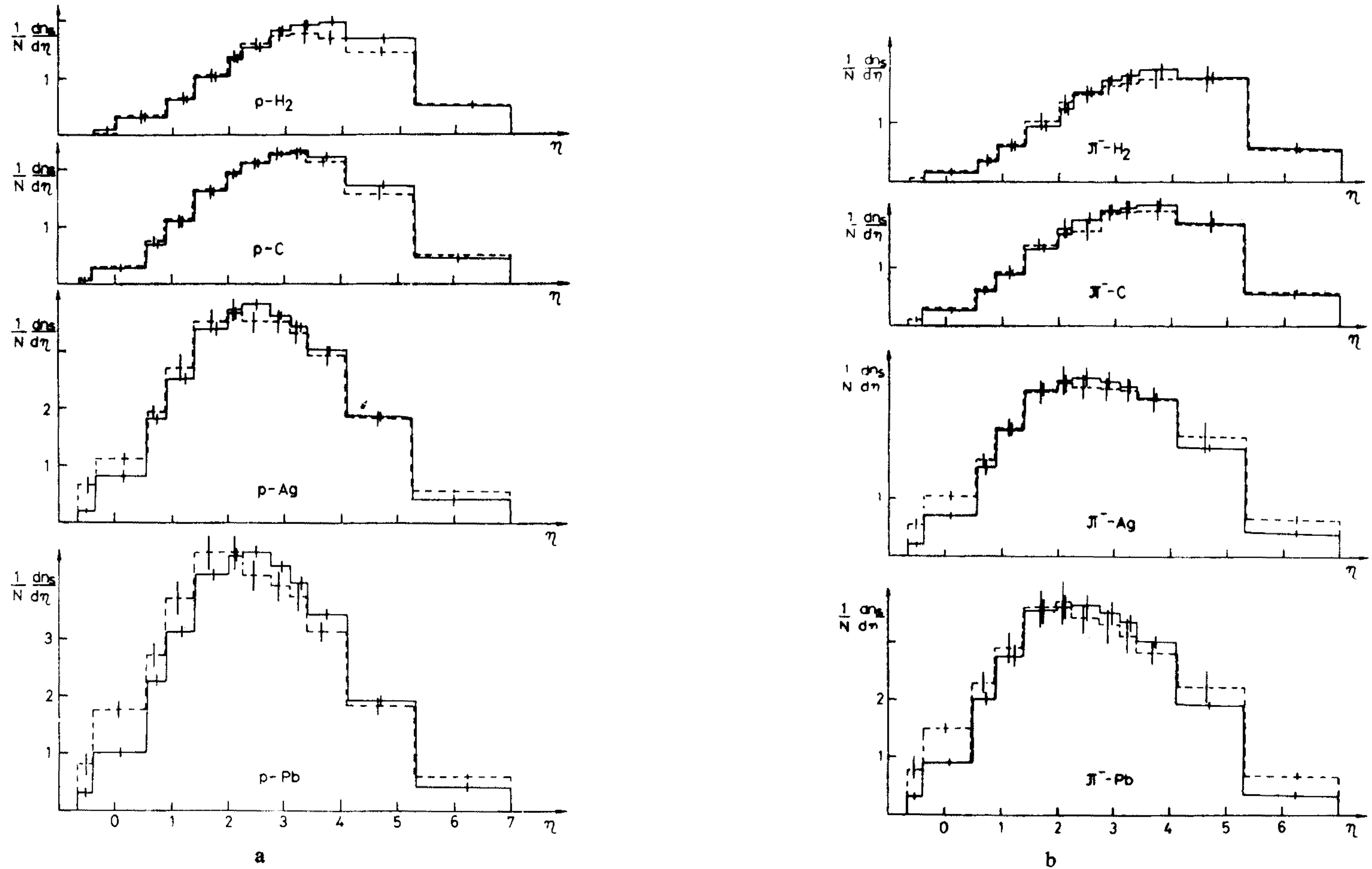


Fig. 3. Pseudorapidity distributions of particles produced by 200 GeV protons (Fig. 3a) and 200 GeV negative pions (Fig. 3b) on H_2, C, Ag and Pb targets. Solid lines — distributions obtained from our emulsion data by a procedure described in the text (cf. Eq. (5)). Dashed lines — counter experiment data [3]

TABLE III

Values of $\overline{N}_h$, $\overline{N}_b$ and $\overline{N}_g$ and the corresponding χ^2 values obtained for 12 nuclear targets by a procedure described in the text

200 GeV protons						target	200 GeV π^-					
$\overline{N}_h$	χ^2 (NDF=9)	$\overline{N}_b$	χ^2 (NDF=9)	$\overline{N}_g$	χ^2 (NDF=9)		$\overline{N}_h$	χ^2 (NDF=9)	$\overline{N}_b$	χ^2 (NDF=9)	$\overline{N}_g$	χ^2 (NDF=9)
−1.12 ± .49	11.2	−1.33 ± .36	16.8	−1.06 ± .21	18.4	H_2	.55 ±.61	5.9	− .69 ± .55	13.1	− .78 ± .31	12.9
.97 ± .40	5.8	.24 ± .30	11.0	− .14 ± .18	9.2	Be	1.93 ± .50	2.4	.53 ± .45	5.7	− .12 ± .25	6.1
1.65 ± .39	4.2	.74 ± .30	8.4	.14 ± .17	5.7	C	2.40 ± .50	3.3	.90 ± .45	6.6	.08 ± .25	6.3
3.93 ± .50	2.6	2.40 ± .37	4.9	1.10 ± .21	1.5	Al	4.47 ± .65	.6	2.55 ± .57	.7	.99 ± .30	1.2
5.80 ± .61	1.9	3.81 ± .46	2.8	1.92 ± .27	.7	Ti	6.23 ± .78	.8	4.00 ± .67	.5	1.78 ± .35	1.2
6.68 ± .61	2.6	4.46 ± .47	2.5	2.30 ± .27	.6	Emulsion	7.13 ± .82	1.0	4.76 ± .71	.5	2.17 ± .37	1.1
6.92 ± .64	2.6	4.64 ± .48	2.4	2.41 ± .28	.7	Cu	7.29 ± .85	1.1	4.89 ± .74	.6	2.25 ± .38	1.2
8.68 ± .75	4.9	5.99 ± .55	2.7	3.21 ± .32	.9	Mo	9.05 ± .97	1.9	6.40 ± .84	1.0	3.07 ± .43	1.9
9.28 ± .78	6.4	6.44 ± .57	3.2	3.47 ± .33	1.3	Ag	9.63 ± .99	2.6	6.88 ± .87	1.3	3.33 ± .44	2.0
11.95 ± .95	17.2	8.52 ± .70	8.9	4.73 ± .42	4.7	W	12.10 ±1.31	6.3	9.08 ±1.10	2.7	4.53 ± .57	3.8
12.67 ±1.02	18.9	9.08 ± .75	10.6	5.07 ± .44	5.9	Pb	12.83 ±1.31	8.1	9.72 ±1.19	4.0	4.87 ± .60	4.6
13.40 ±1.08	22.2	9.68 ± .80	12.9	5.44 ± .48	7.6	U	13.57 ±1.42	9.3	10.39 ±1.27	4.3	5.24 ± .65	5.7

where $\sigma(\eta_i)$ and $\sigma_s(\eta_i)$ denote the errors of $n(\eta_i)$ and $n_s(\eta_i)$ respectively, we have found the mean values of $\bar{N}_h$ for 12 targets exposed to 200 GeV protons and pions by Busza et al. [3]. The reliability of our procedure is illustrated in Fig. 3 where the angular distribution of 200 GeV proton and pion interactions with H_2, C, Ag and Pb nuclei are compared with the distributions calculated from Eq. (4). The corresponding distributions coincide with each other very well (see χ^2 values in Table III). It is very striking that the same parametrization (with the coefficients a_l and b_l extracted from emulsion measurements) is working for nuclei as different as hydrogen and lead.

The average values of $\bar{N}_h$ as a function of the mass number A of the target nucleus obtained from the above procedure are presented in Fig. 4 and Table III. In Fig. 4 we have

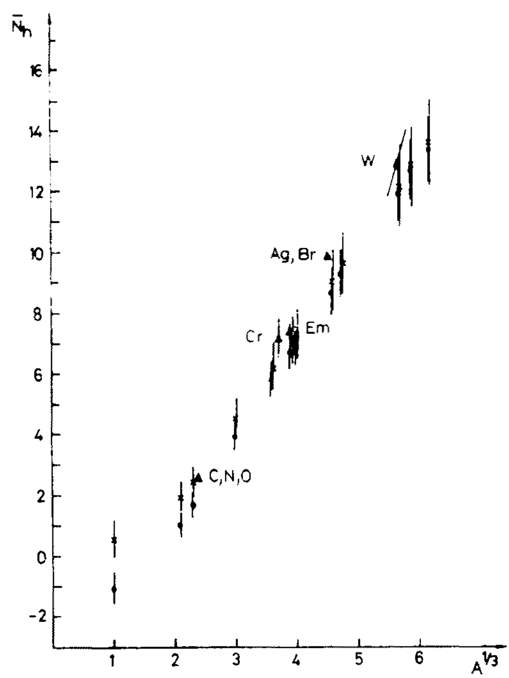


Fig. 4. $\bar{N}_h$ vs A dependence obtained from the minimum χ^2 procedure described in the text (cf. Eq. (5)); ● 200 GeV proton interactions, × 200 GeV π^- interactions, ▲ experimental data for Cr, W nuclei, CNO, AgBr groups and nuclear emulsion

also plotted the known experimental values of the average $\bar{N}_h$ for interactions of protons with Cr, W [4], CNO and AgBr [5] and for interactions of protons and pions with nuclear emulsion [6]. The agreement between the directly measured $\bar{N}_h$ values and those found by the above procedure supports strongly our hypothesis.

The dependence $\bar{N}_h$ on $\bar{\nu}_A$ for interactions of protons and pions, which follows from the already found $\bar{N}_h$ vs A dependence, is presented in Fig. 5. The mean number of collisions $\bar{\nu}_A$ of the incoming particle inside the target nucleus with the mass number A was calculated using the formula $\bar{\nu}_A = \frac{A\sigma_p}{\sigma_A}$, where σ_p and σ_A denote the inelastic cross-section on the proton and the nucleus with mass number A, respectively. One can see from Fig. 5 that there is no unique relation between the mean number of $\bar{N}_h$ and the mean number of collisions. It depends on the nature of the projectile.

We would like to point out that the angular distribution of shower particles can be also parametrized using the number of black tracks N_b or grey tracks N_g emitted from the struck nucleus ($N_h = N_b + N_g$)[1]. The relations $\frac{1}{N}\frac{dn_s}{d\eta}$ vs N_b and $\frac{1}{N}\frac{dn_s}{d\eta}$ vs N_g are in good approximation linear, similarly as it was found for N_h as a parameter (formula (2)).

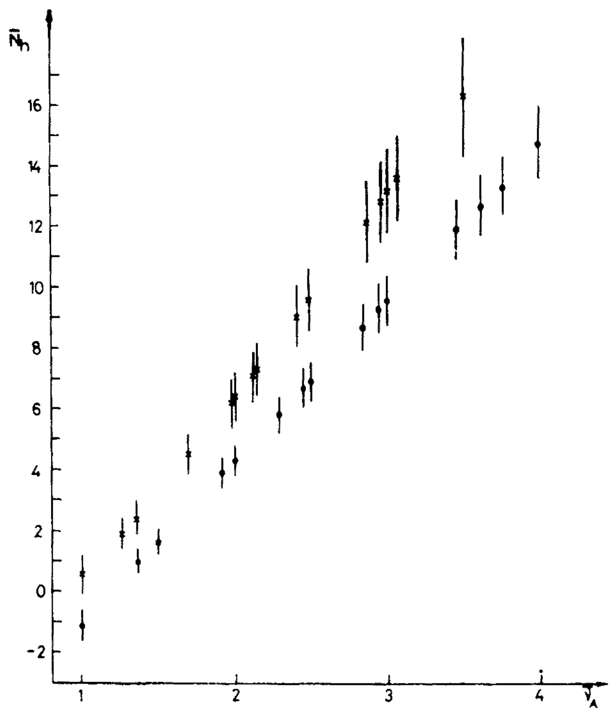


Fig. 5. $\bar{N}_h$ vs $\bar{\nu}_A$ dependence obtained from the minimum χ^2 procedure (cf. Eq. (5)); ● 200 GeV proton interactions, × 200 GeV π^- interactions

By repeating again the same procedure (cf. Eq. (5)) we have obtained the mean values of $\bar{N}_b$ and $\bar{N}_g$ for each target nucleus (see Table III). The relations we have got between the mean values of $\bar{N}_b$ or $\bar{N}_g$ and the mass number A of the target nucleus or the corresponding values of $\bar{\nu}_A$ are presented in Figs. 6 and 7.

[1] N_b — tracks having rang ein emulsion $R \leqslant 3600$ mμ, N_g — tracks with relative ionization >1.4 and $R > 3600$ mμ.

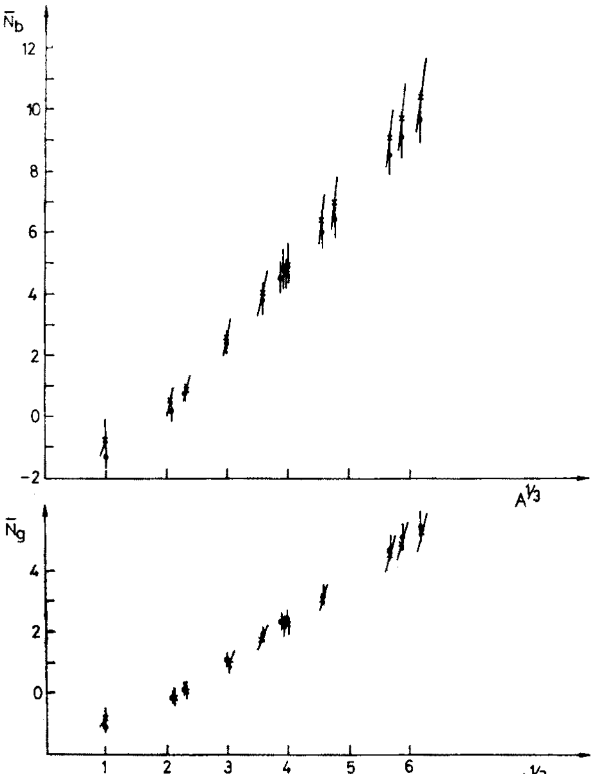


Fig. 6

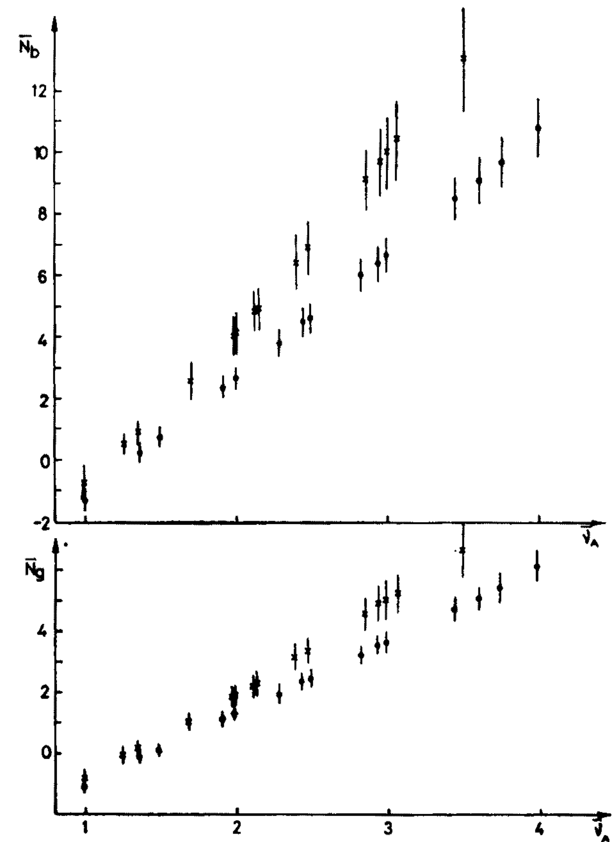


Fig. 7

Fig. 6. The same as Fig. 4 for $\overline{N}_b$ and $\overline{N}_g$
Fig. 7. The same as Fig. 5 for $\overline{N}_b$ and $\overline{N}_g$

We conclude that the angular distribution of produced particles in proton-nucleus or pion-nucleus interactions can be parametrized as follows:

$$\frac{1}{N}\frac{dn_s}{d\eta} = a_I(\eta, E)+b_I(\eta, E)N_{\mathrm{slow}}, \tag{6}$$

where N_{slow} could be N_h, N_b or N_g. The coefficients a_I and b_I for a chosen N_{slow} parameter do not depend on the mass number A of the target nucleus. In other words the angular distribution of shower particles produced in interactions of a given primary particle at a given energy does not depend on the target nucleus provided the same number of slow particles N_{slow} has been emitted. Since formula (6) holds for any nucleus it is also valid for any mixture of nuclei (e.g. nuclear emulsion). The relation between $\overline{N}_{\mathrm{slow}}$ and $\bar{\nu}_A$ is not universal. It depends on the nature of primary particle.

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