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ASCOLI ANALYSIS AND DETERMINATION OF THE FIRE-BALL DIMENSION*

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Grand angular momentum basis is used in three meson partial wave analysis. New assumptions for Ascoli analysis are proposed. By eliminating isobar model approximation we avoid problems of nonunitarity. Ascoli analysis in this basis may be additionally used for the determination of size of the meson systems.

A partial wave analysis is the most valuable source of information about the low-mass three meson systems diffractively produced in the reaction meson + proton → three mesons + proton. All, slightly different versions of the method (known as Ascoli analysis) have used Wick's basis for the description of three meson states and have made several assumptions widely discussed in [1]. The basic assumptions are questioned [2], mainly parametrization of the amplitude in the spirit of the isobar model approximation. The parametrization causes a violation of unitarity as shown in [3].

In this paper a modified version of Ascoli analysis is presented. Instead of Wick's states we use the relativistic grand angular momentum (GAM) basis [4]. In this basis three meson states are labelled by eigenvalues of the so-called togetherness operator Γ^2. They are "partially localized" in a "fire-ball" [5], dimension of which is determined by the eigenvalue of Γ^2 and invariant mass M_{123} of meson 1, 2 and 3. Therefore in the partial wave expansion using the GAM basis we can keep only a finite number of waves with the lowest values of Γ. Other properties of GAM basis enable us to avoid the izobar model approximation and also the problems of nonunitarity.

In the standard partial wave analysis of the reaction $a+b \to 1+2+3+4$ the 3-meson system (123) is described by the state $|\vec{P}_{123} = 0\Lambda[J^P M_{123}]s_n jl\eta\rangle$, where J^P is the spin and intrinsic parity of the (123) system, Λ — helicity of the (123) system, s_n — square of the invariant mass of the two mesons involved in coupling n, j — spin of the di-meson

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system, l — orbital angular momentum of this system (ij) relative to the third meson — k, η — naturality of the exchange.

Using this particular basis we get the density matrix of the (123) system, which varies over the Dalitz plot, namely

$$\varrho_{qq'}(s, t, M_{123}, s_n) = \sum_{\lambda_b, \lambda_4} h^q_{\lambda_b\lambda_4}(h^{q'}_{\lambda_b\lambda_4})^*,$$

with

$$q \equiv \{J^P \Lambda \eta j l n\},$$

$$h^q_{\lambda_b\lambda_4} = \langle \vec{P}_{123}\Lambda[J^P M_{123}]s_n j l \eta; \vec{p}_4\lambda_4|U|\vec{p}_a; \vec{p}_b\lambda_b\rangle.$$

The quantities s, t are, respectively, the square of the center-of-mass energy and the square of the 4-momentum transfer between nucleons. The momenta are denoted $\vec{p}_i$ and baryon's helicities λ_i.

In order to factorize out the dependence of the density matrix on s_n, an important assumption is made. Three meson decay is described by the isobar model, which relies on the idea of a strong final state two-particle interaction. The isobar model approximation is strongly motivated by the empirical observation that the three meson decays proceed through quasi-two-body states. It actually implies that the density matrix does not vary over the Dalitz plot for a given q-state. But it has also an unwanted consequence. We know well that this approximation causes a violation of unitarity [2, 3].

Let us discuss an alternative set of assumptions. Instead of the Wick's three particle states we use the GAM basis vectors $||\vec{P}_{123}\Lambda[J^P M_{123}]\Gamma_n j l \eta\rangle$, which are widely described in [4]. The continuous parameter s_n is replaced in GAM basis by a discrete quantum number which has the following properties. It is connected by the relation:

$$\Gamma^2 \psi_\Gamma = \Gamma(\Gamma+4)\psi_\Gamma,$$

with the eigenvalue of the togetherness operator Γ^2. It is important for further considerations that the quantum number satisfies the condition

$$\Gamma - j - l = 2k \geqslant 0, \tag{1}$$

where k is a non-negative integer. It follows that for a given value of Γ the possible values of j and l are restricted by the inequality: $0 \leqslant j+l \leqslant \Gamma$. This implies also the following restriction $0 \leqslant J \leqslant \Gamma$.

The physical meaning of the quantity Γ is given by the simple semiclassical equality

$$\Gamma^2 = b^2[M^2_{123} - (\sum_{i=1}^{3} M_i)^2], \tag{2}$$

where b is a relativistic analogue of the multiparticle impact parameter. $M_0 = \sum_{i=1}^{3} M_i$ is a sum of the individual rest masses. Therefore, we may say that the GAM basis is "partially localized" [6] in a region of the dimension b.

Now we use the GAM basis to obtain the partial wave expansion of the reaction amplitude $f_{\lambda_b\lambda_4} = \langle \vec{p}_1\vec{p}_2\vec{p}_3;\, \vec{p}_4\lambda_4 | U | \vec{p}_a;\, \vec{p}_b\lambda_b\rangle$. The technique of the derivation is the same as used in [1]. One arrives at the final formula for the amplitude

$$f_{\lambda_b\lambda_4} = \sum_Q G_Q(\varphi, \theta, \gamma;\, s_1, s_2, M_{123})\bar{h}^Q_{\lambda_b\lambda_4}(s, t, M_{123}). \tag{3}$$

The function G_Q, where $Q \equiv \{\Gamma_n J^P\ \Lambda\eta jln\}$ are the known functions of two Dalitz plot variables s_1, s_2 and three Euler angles φ, θ, γ. From [4] one gets the explicit formula for G_Q

$$G_Q = (|\vec{\kappa}_{ij}|/|\vec{p}_{ij}|)^{j+\frac{1}{2}}(|\vec{\kappa}_k|/|\vec{p}_k|)^{l+\frac{1}{2}} P^{(j+\frac{1}{2},\, l+\frac{1}{2})}_{(\Gamma-j-l)/2}(x)\, Y^{jl}_{J^P\Lambda\eta}(\vec{p}_{ij}, \vec{p}_k),$$

where

$$|\vec{\kappa}_{ij}| = [(M_{ij}-M_i-M_j)/(M_{123}-M_0)]^{\frac{1}{2}},$$

$$|\vec{\kappa}_k| = [(M_{123}-M_{ij}-M_k)/(M_{123}-M_0)]^{\frac{1}{2}}.$$

The Jacobi polynomials $P^{(\alpha,\beta)}_k(x)$ are the functions of x

$$x = |\vec{\kappa}_{ij}|^2 - |\vec{\kappa}_k|^2 = (M_{123}-2M_{ij}+M_i+M_j-M_k)/(M_{123}-M_0),$$

and

$$Y^{jl}_{J^P\Lambda\eta} = \sum_{m_1 m_2} \langle jlm_1m_2|J^P\Lambda\eta\rangle\, |\vec{p}_{ij}|^j Y^j_{m_1}(\varphi_1, \theta_1)\, |\vec{p}_k|^l Y^l_{m_2}(\varphi_2, \theta_2).$$

The angles $(\varphi_\alpha, \theta_\alpha)$ have the meaning of the spherical angles of the standard relativistic moments $\vec{p}_{ij}$ and $\vec{p}_k \cdot \vec{p}_{ij}$ is the momentum of one of the decay products of the di-meson system (ij) in the di-meson rest frame, and $\vec{p}_k$ is the momentum of the di-meson system of interest in the (123)-rest frame.

The matrix elements $\bar{h}^Q_{\lambda_b\lambda_3} = \langle \vec{P}_{123} = 0\Lambda[J^P M_{123}]\Gamma_n jl\eta;\, \vec{p}_4\lambda_4|U|\vec{p}_a;\, \vec{p}_b\lambda_b\rangle$ which play the role of unknown expansion coefficients determine the density matrix $\varrho_{QQ'}(s, t, M_{123}) = \sum_{\lambda_b\lambda_4} \bar{h}^Q_{\lambda_b\lambda_4}(\bar{h}^{Q'}_{\lambda_b\lambda_4})^*$. Notice that without assumptions the density matrix does not depend on s_n. However, the above formula is of no practical value as long as it contains an infinite set of such coefficients. Therefore we need several assumptions to reduce the number of the unknown functions.

For this purpose at first we invoke the well known idea of the "centrifugal barrier effects". For finite energy in the partial wave expansion of the GAM type, we can keep only finite number of lowest values of Γ_n. The miximum value $\tilde{\Gamma}$ may be estimated from (2). If we consider, for example, the low-mass three meson systems diffractively produced in the reactions meson+proton → three mesons+proton and take for the dimension of the "fire-ball" [5] $b \approx 1$ fm, we get $\tilde{\Gamma} = 4$.

The possible values of J,j and l are restricted by (1). Using it we can keep in (3) only the terms with $0 \leqslant J \leqslant \Gamma_n$, $0 \leqslant j+l \leqslant \Gamma_n$. Further restriction is imposed by the conservation of parity. Namely, it is not difficult to find from (1) that the parity of the GAM state is $P = (-1)^{\Gamma_n+1}$. Thus the invariance under P reduces the number of possible waves for a factor of one half.

Thus we see that using the GAM basis we replace the first three assumptions of the standard Ascoli analysis by the one discussed above. Notice that in this way we avoid problems of nonunitarity, but on the other hand we have more unknown amplitudes to be determined (at least twice the number needed before).

It is clear that further assumptions have to be made in order to reduce the number of the unknown parameters. In this place the standard Acsoli analysis assumes the factorization of the amplitude in a production amplitude which is independent of l, j and η and a decay parameter which is independent of Λ, λ_b, λ_4 and η. Of course, this assumption may be simply adopted to our approach. But we may try a stronger one, given by the factorization:

$$\bar{h}^{Q}_{\lambda_b\lambda_4}(s, t, M_{123}) = T^{J^P\eta}_{\Lambda\lambda_b\lambda_4}(s, t, M_{123})C^{\Gamma_n J^P jl\eta}(s, t, M_{123}).$$

This reduces the number of parameters to be determined to a realistic level.

We conclude the paper with a remark concerning the applicability of the partial wave expansions derived above for determination of sizes of three-meson systems. Fitting the quantities $\bar{\varrho} = \sum T \cdot T^*$ and C by a maximum likelyhood method one can calculate the number of events in a given $J^P\Lambda\eta$ state and for a given $J^P\Lambda\eta$ state, the number of events produced in different GAM waves Γ_n. We suppose that the contribution of the partial waves with higher Γ will be negligible. The highest significant value of Γ : $\tilde{\Gamma}_n$ and mass of the three-meson system under consideration determine, via formula (2), the size of the three-meson system related to a given decay-mode.

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