CHARGE SYMMETRY BREAKING NUCLEAR FORCES IN TWO-BODY AND MANY-BODY NUCLEAR SYSTEMS*

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A simple model of charge symmetry breaking nuclear forces is related directly to charge asymmetry in the low energy $^1S_0$ nucleon-nucleon scattering parameters. Using this model charge asymmetry in the effective range theory parameters is related to the charge asymmetric term in the nuclear binding energies of heavy nuclei, $E_a(N-Z)$. The role of the short-range two-body correlations induced by the dominating charge independent nucleon-nucleon potential is studied and is found to be very important when calculating E_a. The value of E_a is found to be very sensitive to the charge asymmetry in the effective range while being quite insensitive to charge asymmetry in the scattering length. The use of experimental value of E_a in the study of the charge symmetry breaking component of nuclear forces is proposed.

1. *Introduction*

The knowledge of the magnitude of the charge symmetry breaking (CSB) nuclear forces has fundamental importance to nuclear physics. In the study of possible CSB component of nuclear forces one considers the n−n and p−p interactions, switching off "external" electromagnetic effects (Coulomb p−p force, vacuum polarization, interaction of magnetic moments). The CSB of the N−N interaction, purified of the "external" electromagnetic effects, may then follow from "internal" electromagnetic effects, which are related to the electromagnetic properties of mesons, mediating the N−N interaction. The most important of those "internal" electromagnetic effects, leading to CSB, is electromagnetic mixing of the isoscalar and isovector mesons of the same spin and parity [1]. Other "internal" electromagnetic effects, such as radiative corrections at the nucleon-nucleon vertices and the pion-photon exchange potentials, are expected to be an order

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of magnitude smaller than those resulting from meson mixing [2, 3]. Generally, the CSB potentials derived from electromagnetic meson mixing are strongly dependent on assumptions concerning various quantities entering the theory (e.g., coupling constants) and are thus rather uncertain [1, 4–8].

Direct experimental investigations of CSB nuclear forces relate to the $^1S_0$ low-energy N$-$N scattering. The recent value of the n$-$n scattering length, which can be used with some confidence is $a_{nn} = -(16.2 \pm 0.6)$ fm [9, 10]. The corresponding, "purged" of external electromagnetic effects value for the p$-$p system is $a^N_{pp} = -(17.1 \pm 0.2)$ fm [9, 11]. The $^1S_0$ effective ranges are much more difficult to measure than the scattering lengths. The "purged" value $r^N_{pp} = (2.84 \pm 0.03)$ fm [9–11]. As yet, measurements of r_{nn} do not have an accuracy sufficient for a really meaningful comparison with r^N_{pp}. Generally, the value of r_{nn} resulting from the three-body analyses of scattering experiments (e.g., $n+d \to 2n+p$) is much more sensitive to the model used than a_{nn} is [9, 13]. Thus, the value $r_{nn} = (2.4 \pm 0.4)$ fm quoted in Ref. [9] should be treated with caution, being in fact the mean value of several results, obtained from the three-body analyses of $n+d \to$ $\to 2n+p$ experiments. The differences between these results (e.g., $r_{nn} = (3.15 \pm 0.7)$ fm [12] and $r_{nn} = (2.13 \pm 0.4)$ fm [13]) reflect the model dependence of the extracted r_{nn} value.

Summarizing we may say that the recent results concerning the $^1S_0$ scattering lengths suggest $a_{nn} > a_{pp}$, with $\Delta a = a_{nn} - a^N_{pp} = (0.9 \pm 0.8)$ fm. On the other hand, the present experimental results for r_{nn} are not conclusive and cannot yield reliable information about the sign and the magnitude of $\Delta r = r_{nn} - r^N_{pp}$.

The comparison of low-energy n$-$n and "purged" p$-$p scattering parameters seems to be the most sensitive and the most reliable method for studying the CSB nuclear forces (cf. however Refs [14, 15]). Due to the difficulties in the extraction of the low-energy n$-$n scattering parameters, however, it is believed, that the most reliable (but indirect) evidence for the CSB in nuclear physics comes from the theoretical analyses of the Coulomb energy differences of $T = 1/2$ mirror nuclei [16–25]. All these analyses suggest, that the "overall" n$-$n interaction is slightly more attractive than the p$-$p one and phenomenological CSB nuclear forces were introduced to account for the observed Nolen–Schiffer anomaly [26] in the Coulomb energy differences of mirror nuclei [16–20, 21–25]. In most cases [21–25] (with exception of ^{3}H—^{3}He pair where the calculations were performed using variational techniques with realistic trial wave functions [16–18] or Faddeev equations with simple separable N$-$N interactions [19, 20]) the contribution to the binding energy from the CSB nuclear force has been calculated with single-particle wave functions which lead to uncorrelated twobody wave functions at small internucleon distances. Thus, the phenomenological CSB potentials of Refs [21–25] should be considered rather as effective CSB nuclear forces, which may be quite different from the bare CSB N$-$N interaction. The latter is expected to contain a very short-range component resulting from the electromagnetic $\varrho^0 - \omega^0 - \phi^0$ mixing.

In the present paper the relation between charge asymmetry in low-energy N$-$N scattering parameters and charge asymmetry in the nuclear binding energies of heavy nuclei is studied using a simple model of CSB nuclear force. The procedure leading to the determination of our model of the CSB potential is presented in Section 2. In Section 3 we describe the calculation of the charge asymmetric term in the nuclear binding energies of heavy nuclei, $E_a(N-Z)$, and relate Δa and Δr to E_a. Section 4 contains discussion and conclusions.

2. *CSB nuclear forces and charge asymmetry of low energy N−N scattering parameters*

The CSB component of nuclear forces has the usual form [2]

$$V_{\text{CSB}} = [\hat{\tau}_z^{(1)} + \hat{\tau}_z^{(2)}]U_{\text{CSB}}(r) \tag{1}$$

where $\hat{\tau}_z|p\rangle = |p\rangle$ and $\hat{\tau}_z|n\rangle = -|n\rangle$. Our phenomenological U_{CSB} is spin independent (Wigner force). The nuclear n−n and p−p interactions are of the form

$$V_{\text{nn}} = v - 2U_{\text{CSB}},$$

$$V_{\text{pp}} = v + 2U_{\text{CSB}}, \tag{2}$$

where v is the charge independent conponent of the N−N interaction. The function U_{CSB} may be related directly to the charge asymmetry in the $^1\text{S}_0$ low energy scattering parameters. Treating the CSB component as a small perturbation of v we obtain, to first order in U_{CSB} [27]

$$\Delta a = a_{\text{nn}} - a_{\text{pp}}^{\text{N}} = -4\frac{M}{\hbar^2}a^2\int_0^\infty U_{\text{CSB}}(r)u_0^2(r)dr, \tag{3}$$

$$\Delta r = r_{\text{nn}} - r_{\text{pp}}^{\text{N}} = -16\frac{M}{\hbar^2}\int_0^\infty U_{\text{CSB}}(r)u_0(r)u_1(r)dr, \tag{4}$$

where u_0, u_1 are calculated from the $l = 0$ radial wave function, generated by the $^1\text{S}_0$ charge independent potential $v^{^1S_0} = V_0$,

$$\frac{\partial^2 u_0}{\partial r^2} + k^2 u = \frac{M}{\hbar^2}V_0 u, \tag{5}$$

$$u_0(r) = u(k^2, r)|_{k^2=0}, \quad u_1(r) = \frac{\partial}{\partial(k^2)}u(k^2, r)|_{k^2=0}.$$

In Eq. (3) a is the scattering length generated by V_0.

Results of theoretical investigations in which CSB nuclear forces are derived from the electromagnetic meson mixing suggest that U_{CSB} consists of two parts. The short--range part results from the vector meson mixing ($\varrho^0-\omega^0-\phi^0$) while the long-range part follows from the mixing of the pseudoscalar ($\pi^0-\eta^0$) mesons. We shall incorporate these features into our phenomenological CSB force, defining

$$U_{\text{CSB}} = V_s \exp(-n_s\mu r)/n_s\mu r + V_l \exp(-n_l\mu r)/n_l\mu r, \tag{6}$$

where typical values of n_s and n_l are $n_s = 6$ and $n_l = 1$ and $\mu = m_{\pi^0}/\hbar c = 0.685\ \mathrm{fm}^{-1}$. On the other hand, the theoretical knowledge of the strengths of these two U_{CSB} components (even of their signs) is very uncertain [4–8]. Hence, V_l and V_s will be treated in the subsequent considerations as phenomenological parameters. To put the strengths of the long and short-range components on a comparable basis, we shall use the representation

$$V_s = \alpha_s(n_s\mu)^3 v_0, \qquad V_l = \alpha_l(n_l\mu)^3 v_0, \tag{7}$$

where v_0 will be fixed arbitrarily at the value $v_0 = 1\ \mathrm{MeV\,fm}^3$. Using Eqs (3) and (4) we obtain one-to-one correspondence between the charge asymmetry Δa, Δr and the CSB force (6) (for fixed n_l and n_s). Namely, the parameters α_s and α_l may be calculated for fixed Δa and Δr from the system of two linear equations

$$\Delta a = -4\frac{M}{\hbar^2}a^2 v_0 \sum_{i=s,l} \alpha_i(n_i\mu)^2 \int_0^\infty \frac{e^{-n_i\mu r}}{r} u_0^2 dr,$$

$$\Delta r = -16\frac{M}{\hbar^2} v_0 \sum_{i=s,l} \alpha_i(n_i\mu)^2 \int_0^\infty \frac{e^{-n_i\mu r}}{r} u_0 u_1 dr. \tag{8}$$

Using experimental values of Δa and Δr one would be able to determine unambiguously the spin independent U_{CSB}, Eq. (6). As discussed in the Introduction, the experimental knowledge of Δa and Δr is, unfortunately, rather uncertain. We shall use (cf. Introduction)

$$\Delta a = (0.9 \pm 0.8)\ \mathrm{fm}. \tag{9}$$

This proposed interval for the charge asymmetry in a should be, however, taken with a grain of salt because of the model dependence of the analyses which yield the experimental value of a_{nn}. In the case of Δr the experimental situation is much worse. In view of the very strong model dependence of the experimental results for r_{nn} (cf. Introduction) we shall tentatively investigate $-0.4\ \mathrm{fm} \leqslant \Delta r \leqslant 0.4\ \mathrm{fm}$.

In our calculation the functions u_0 and u_1 have been generated by the $^1\mathbf{S}_0$ Reid soft core N−N potential [28]. The results for α_l and α_s are shown in Fig. 1. Generally, the α's depend very strongly on Δr, while their dependence on Δa is quite weak. This characteristic feature is due to big a^2 factor appearing in Eq. (3).

Precision of formulae (3) and (4) has been checked numerically for each U_{CSB}. The examples illustrating the precision of the approximate formulae (3) and (4) are given in Table I. In Table I several input values of Δa and Δr, used in the calculation of the strength parameters α_s and α_l (we assume $n_s = 6$ and $n_l = 1$) are given in the first column. The calculated values of α_s and α_l are shown in the second column. The CSB potentials, determined by α_l, α_s and n_l, n_s using Eq. (8), yield exact values of Δa and Δr shown in the third column. Generally, the precision of the approximate formulae (3), (4) is better than 2% for $|\Delta r| < 0.3$ fm.

Eqs (8) imply (for fixed n_s and n_l) linear dependence of α's on Δa and Δr. We have (cf. Ref. [34])

$$\alpha_i = c_{1i}\Delta r + c_{2i}\Delta a, \tag{10}$$

where

$$c_{1i} = \left(\frac{\partial\alpha_i}{\partial\Delta r}\right)_{\substack{\Delta r=0\\ \Delta a=0}}, \quad c_{2i} = \left(\frac{\partial\alpha_i}{\partial\Delta a}\right)_{\substack{\Delta r=0\\ \Delta a=0}}, \tag{11}$$

and $i = l, s$. For $n_s = 6$ and $n_l = 1$ we obtain

$$c_{1s} = -15.947\ \mathrm{fm}^{-1}, \quad c_{2s} = 0.098\ \mathrm{fm}^{-1},$$
$$c_{1l} = 11.512\ \mathrm{fm}^{-1}, \quad c_{2l} = -0.233\ \mathrm{fm}^{-1}. \tag{12}$$

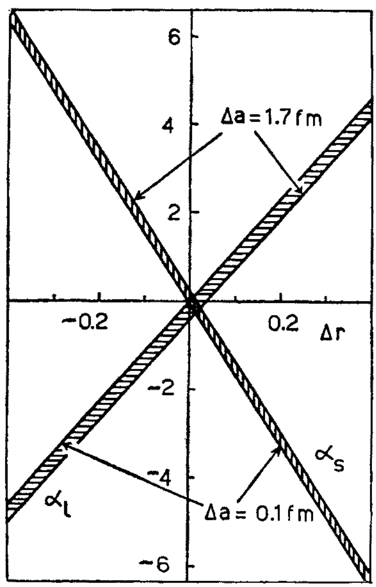

Fig. 1. The dimensionless strenght parameters α_s and α_l (for fixed Δa) versus Δr. The hatched areas include all $\alpha_i(\Delta r)$ $(i = l, s)$ straigth lines corresponding to 0.1 fm $<\Delta a<$ 1.7 fm. The plots correspond to $n_s = 6$ and $n_l = 1$

TABLE I

Strength parameters α_s and α_l (dimensionless) of the phenomenological CSB potentials ($n_s = 6$ and $n_l = 1$) and the values of Δa and Δr (in fm) and $\Delta\delta_0$ (in degrees) calculated from these potentials. For the explanation see the text

Input		α_s	α_l	Exact		$\Delta\delta_0$		
Δa	Δr			Δa	Δr	100 MeV	200 MeV	300 MeV
0.90	0.05	−0.710	0.365	0.90	0.05	−0.3	−0.4	−0.4
0.90	−0.40	6.466	−4.815	0.88	−0.40	−3.3	−3.5	−3.7
1.70	0.20	−3.024	1.905	1.73	0.20	1.8	1.8	1.9
1.70	−0.10	1.760	−1.549	1.71	−0.10	−0.7	−0.8	−0.9

The insensitivity of α's to the value of Δa is reflected by the fact that $|c_{1s}/c_{2s}| \sim 150$ and $|c_{1l}/c_{2l}| \sim 50$.

In order to discuss the charge asymmetry in high energy N–N scattering, implied by our CSB force, we calculated the $^1S_0$ phase shift, δ_0, at several laboratory energies, using $V = v + V_{CSB}$, where v was the $^1S_0$ soft core potential of Reid. The charge asymmetry in δ_0 is defined as

$$\Delta\delta_0 = \delta_{0nn} - \delta^N_{0pp}. \tag{13}$$

The values of $\Delta\delta_0$ for several V_{CSB} and at $E_{lab} = 100$ MeV, 200 MeV and 300 MeV are shown in the last three columns of Table I. Because of the lack of direct n–n scattering experiments no possibility of experimental estimate of the magnitude of $\Delta\delta_0$ at medium and high energies exists. The charge asymmetry $\Delta\delta_0$ is bigger than the experimental error bar in δ^N_{0pp}, quoted in Ref. [29], for $|\Delta r| \gtrsim 0.1$ fm. On the other hand, $\Delta\delta_0$ corresponding to $|\Delta r| < 0.1$ fm is smaller than the experimental error bar in δ^N_{0pp} [29].

Finally, let us mention that the phenomenological CSB potentials, usually assumed to be monotonic functions of r of a rather long range ($\sim 1/\mu$) yield the Δa values which are inconsistent with present estimates of this quantity. For example, the CSB potential of Shlomo and Bertsch [23] introduced in their study of the Coulomb energy differences of the ground states of ^{18}Ne–^{18}O and ^{42}Ti–^{42}Ca mirror nuclei yields $\Delta a = -1.77$ fm and $\Delta r = -0.01$ fm.

3. Charge asymmetry in the nuclear binding energies and role of the short-range correlations

The Liquid Drop Model mass formula gives (in the limit of $A, Z \to \infty$, with simultaneous switching-off of the Coulomb interaction between protons) the "experimental formula" for the binding energy in nuclear matter at normal nuclear density ($\varrho = \varrho_0 = 0.17$ nucleons/fm^3),

$$E/A = E_0 + E_s\alpha^2, \tag{14}$$

where the neutron excess parameter, $\alpha = (\varrho_n - \varrho_p)/\varrho = (N-Z)/A$ is assumed to be small, so that powers of α higher than quadratic are neglected. Extensive semiempirical analysis based on the Liquid Drop Model yields $E_0 = -15.68$ MeV and $E_s = 28.06$ MeV [30]. However, the functional form of E, Eq. (14), corresponds to strictly charge symmetric nuclear hamiltonian. The presence of (small) CSB components in nuclear hamiltonian may imply existence of an additional charge asymmetric term in E/A, of the form $E_a\alpha$, studied recently by the present author using theoretical CSB nuclear forces in nuclear matter calculations [31]. Up to now the term of this form has not been included in the semiempirical mass formulae, but existing results obtained with the use of "charge symmetric" parametrizations seem to indicate that E_a should be small. The calculations where (very uncertain) theoretical CSB potentials have been used lead to the value $E_a \approx -0.2$ MeV [31].

The charge asymmetry parameter in the nuclear binding energy, E_a, seems to be the simplest manifestation of the possible CSB effects in the nuclear many-body ($A \gg 1$) systems.

Our phenomenological CSB potentials will be used for the calculation of E_a within the framework of the Brueckner-Bethe-Goldstone theory of nuclear matter [32, 33]. With our simplifying assumption concerning the spin independence of U_{CSB}, we shall thus obtain a direct link between the CSB in low energy N−N scattering parameters and the CSB effects in many-nucleon systems.

We shall calculate E_a to the first order in U_{CSB}, accounting for the short range correlations which result from the strong CS N−N interaction v. Considering only the CS interaction v, we calculate the CS reaction matrix K^0 in nuclear matter,

$$K^0 = v+v\frac{Q}{e}K^0. \tag{15}$$

The full reaction matrix $K = K^0+K_{CSB}$ is the solution of

$$K = v+V_{CSB}+(v+V_{CSB})\frac{Q}{e}K. \tag{16}$$

We introduce the correlated (in the presence of v) two-body wave function Ψ and the defect wave function $\zeta = \Phi-\Psi$ where Φ is the wave function of the free two-body motion. Using the identity

$$\langle\Phi|v|\Psi\rangle = \langle\Phi|K^0|\Psi\rangle \tag{17}$$

we obtain, to first order in V_{CSB} [21, 31]

$$\langle\Phi|K_{CSB}|\Phi\rangle = \langle\Phi|V_{CSB}|\Phi\rangle+\langle\zeta|V_{CSB}|\zeta\rangle-2\langle\Phi|V_{CSB}|\zeta\rangle. \tag{18}$$

The first term in the right-hand side of Eq. (18) is the first Born approximation to K_{CSB}, while the second and third terms give correction resulting from the short range two-body correlations induced by v.

At the density ϱ related to the average Fermi momentum k_F by $\varrho = 2k_F^3/3\pi^2$ we obtain

$$E_a = -3\varrho\int_0^\infty dr r^2 F(r)U_{CSB}(r), \tag{19}$$

where the first Born approximation to F reads (cf. Ref. [31])

$$F^B(r) = 2\pi\int_0^1 d\tilde{k}\tilde{k}^2(1-\tilde{k})\,[2-j_0(2kr)] = \tfrac{1}{3}\pi\left[1-\frac{3}{2k_Fr}j_0(k_Fr)j_1(k_Fr)\right], \tag{20}$$

and $\tilde{k} = k/k_F$, k being the nucleon momentum (divided by $\hbar$) in the CM system of a nucleon pair. Using the partial wave expansions for Φ and Ψ we obtain for remaining (correlated) part of F

$$F(r)-F^B(r) = 2\pi\int_0^1 d\tilde{k}\tilde{k}^2(1-\tilde{k})\sum_{Jll's}[1+(-1)^{l+s}]$$
$$\times(2J+1)\,[\chi_{ll'}^{Js}(k,r)-2j_l(kr)\delta_{ll'}]\chi_{ll'}^{Js}(k,r). \tag{21}$$

Our partial defect wave functions, $\chi^{Js}_{ll'}$, differ from those defined in [33] by a factor of r, otherwise the notation follows that of Ref. [33].

The calculation has been performed self-consistently at the normal nuclear density $\varrho_0 = 0.17$ nucleons/fm^3 ($k_F = 1.36\ \mathrm{fm}^{-1}$) using standard approximations. The soft core potential of Reid [28] has been used as the charge independent N–N interaction v in $J \leqslant 2$ partial waves. The contribution to F from $J > 2$ states has been neglected. Our

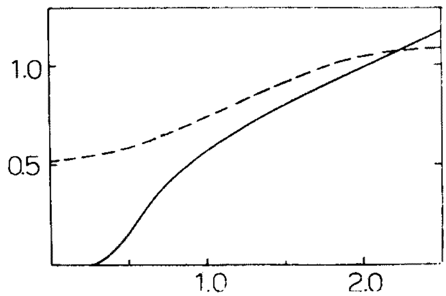

Fig. 2. The function $F(r)$ (solid line) and its first Born approximation $F^B(r)$ (dashed line). For explanation see the text

results for F are shown in Fig. 2, where for the sake of comparison the first Born approximation to F, F^B, is also shown. The healing property of Ψ implies $F \approx F^B$ at sufficiently large r. On the other hand, at small distances F is very strongly supressed as compared to F^B and practically vanishes for $r \gtrsim 0.3$ fm: this reflects the existence of the strong short-range correlations induced by the strong, repulsive short-range component of v. This effect may be very important when calculating E_a, because U_{CSB} contains a very short ranged component of the range $\hbar c/m_\varrho{}^0 \approx 0.2$ fm.

The charge asymmetry parameter E_a has been calculated with CSB potentials discussed in Section 2. The parameter E_a depends very strongly on Δr, but is quite insensitive to Δa.

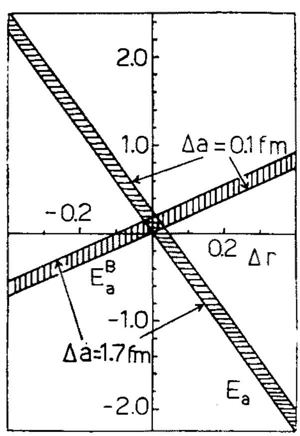

Fig. 3. The charge asymmetry parameter E_a and its first Born approximation E^B_a (in MeV) versus Δr (for fixed Δa). The hatched areas include all the $E_a(\Delta r)$ and $E^B_a(\Delta r)$ straight lines corresponding to 0.1 fm$<\Delta a<$ 1.7 fm. The calculations have been done for $n_s = 6$ and $n_l = 1$

Let us mention that the sensitivity of the binding energy to the value of effective range (for charge independent nuclear forces) is well known in the case of the nuclear three-body calculations (see, e.g., Ref. [36]).

Our results for E_a are shown in Fig. 3. The role of the short-range correlations induced by v is very important; inclusion of these correlations, when passing from F^B to F, yields the change of sign of E_a for $|\Delta r| > 0.1$ fm. Let us recall that in the case of monotonic, long-ranged phenomenological CSB potentials of Refs [21–23] the corrections induced by the short range correlations were negligible [31].

In order to investigate the dependence of E_a on Δr and Δa we notice that formulae (12) for α_t and α_s imply (cf. Fig. 3)

$$E_a(\Delta a, \Delta r) = a_1 \Delta r + a_2 \Delta a, \tag{22}$$

where

$$a_1 = \left(\frac{\partial E_a}{\partial \Delta r}\right)_{\substack{\Delta r=0\\ \Delta a=0}} = -5.64\ \mathrm{MeV\,fm}^{-1}, \tag{23}$$

$$a_2 = \left(\frac{\partial E_a}{\partial \Delta a}\right)_{\substack{\Delta r=0\\ \Delta a=0}} = 0.16\ \mathrm{MeV\,fm}^{-1}. \tag{24}$$

The weak dependence of E_a on Δa is reflected by the fact that $|a_1/a_2| \approx 40$.

4. *Discussion and conclusions*

Our simplified semiphenomenological model relates the charge asymmetry in the low energy $^1S_0$ N–N scattering parameters (Δa, Δr) to the (hypothetical) charge asymmetry term in the nuclear mass formula, $E_a(N-Z)$. The charge asymmetry parameter, E_a, is found to be very sensitive to the charge asymmetry in the effective range, Δr. The present measurements of r_{nn} do not have an accuracy sufficient for a meaningful comparison with r^N_{pp}. Generally, it may be expected that the nuclear binding energy systematics will exclude such a high values of Δr as $|\Delta r| \gtrsim 0.2$. E.g., for $\Delta a = 0.9$ fm and $\Delta r = 0.3$ fm we would have $E_a = -1.5$ MeV. In the case of the nucleus ^{208}Pb the contribution to the nuclear binding energy resulting from CSB component of nuclear forces would then be $\delta E_{CSB} = E_a(N-Z) = -66$ MeV. This contribution would constitute 25% (!) of the conventional volume symmetry energy term $\delta E_{sym} = E_s(N-Z)^2/A = 260$ MeV.

In the nuclear matter limit the existence of δE_{CSB} would have an obvious consequence: at a fixed total density $\varrho = \varrho_n + \varrho_p$ the minimum of the nuclear binding energy would be reached at $\varrho_n \neq \varrho_p$, namely, at

$$\alpha_0 = (\varrho_n - \varrho_p)/\varrho = -\tfrac{1}{2} E_a/E_s.$$

The charge asymmetry $\Delta a = 0.9$ fm, $\Delta r = 0.3$ fm yields $\alpha_0 = 2.6\%$.

The parameter E_a enters the formula determining the β stability line. The modification implied by the CSB effects will consist in replacing $c^2(M_n - M_p)$ by $c^2(M_n - M_p) + 2E_a$.

According to results presented in Section 3 the quantity $c^2(M_n - M_p) + 2E_a$ becomes negative for $\Delta r \lesssim 0.15$ fm.

Presented above discussion leads to the conclusion that a reasonable value of E_a may be expected to fulfill the inequality $|E_a| \gtrsim 0.5$ MeV. This would correspond to $|r_{nn} - r_{pp}^N| \gtrsim 0.1$ fm. The negative sign of E_a would be reasonable in the view of the Nolen-Schiffer anomaly in the Coulomb energy differences of mirror nuclei [26]. The sign of E_a is (in our model) directly related to the sign of Δr only for $|\Delta r| \simeq 0.05$ fm, the negative E_a corresponding then to positive Δr ($r_{nn} > r_{pp}^N$).

The introduction of the spin dependence of U_{CSB} would spoil the one-to-one correspondence between Δr, Δa and E_a. The spin structure of U_{CSB} derived from the electromagnetic mixing of the isoscalar and isovector mesons seems to indicate that this spin dependence may be rather weak [4,18] (cf. Section 7 of Ref. [35]).

Systematic investigation of the charge asymmetry in the α-dependence of the smooth volume part of the binding energies of heavy nuclei may represent quite a sensitive method for studying the CSB component of nuclear interaction. In contrast to the theoretical analyses of the Coulomb energy differences in $T = 1/2$ mirror nuclei, the proposed study of the volume (Liquid Drop Model) part of the nuclear binding energies of heavy nuclei seems to be relatively simple.

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