LOW ENERGY HADRON DYNAMICS WITH MEDIUM-MASS QUARKS

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The possibility of understanding low energy hadron dynamics in terms of point-like medium-mass quarks is discussed. After introducing the basic assumptions of the model and cataloguing some empirical knowledge of quark wave functions, the ideas of the model are applied to the study of several phenomena, including baryon magnetic moments, radiative meson decays, G_A/G_V, the charge radii of the neutron and the K^0, and K_{l_3} decay.

1. *Introduction*

Although only a few years ago any discussion with a quark mass appearing in it was highly suspect, we are now so accustomed to the idea that these days we find any discussion without a quark mass appearing in it *passé*. There is, for example, such universal agreement that the charm quark has a mass around 1.5 GeV that no one even coughs anymore when this value is mentioned. The utility of potential models for understanding the charmonium spectrum is also widely accepted.

In deep inelastic scattering we have established without much doubt the presence of the quarks which behave as though they are elementary Dirac particles with the expected charges. We have even established that the dominant Foch-space states are just those anticipated by quark spectroscopy. Most recently, we have begun to understand many aspects of the short distance behaviour of hadron dynamics in terms of the interactions between these same apparently structureless quarks.

There has been a certain reluctance, however, to apply these new ideas to low energy hadron dynamics. There are a number of good reasons for this, of course, the most outstanding being the anticipation that the simplicity of the dynamics at short distances will not persist at large distances. However, I can see no compelling reason for this pessimistic attitude, and what I propose to do in these lectures is discuss some non-spectroscopic consequences of taking the optimistic view that low energy hadron dynamics can be de-

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scribed in terms of the same structureless spin-$\frac{1}{2}$ quarks which have been so successful in describing short distance behaviour.

In particular, I will assume that the known hadrons are composed of the usual four colourful, effectively structureless spin-$\frac{1}{2}$ quarks u, d, s, and c with masses $m_u \simeq m_d \simeq 0.34$ GeV, $m_s \simeq 0.48$ GeV, and $m_c \simeq 1.5$ GeV. I further assume that these quarks are universally coupled to coloured gluons which give rise to some (complicated) effective interquark potential including a long-range confining term.

To these fundamental assumptions we must add one further element in order to actually make any computations: We must specify how we intend to make the connection between the non-relativistic quark model and the calculation of relativistically invariant matrix elements. A variety of methods for making this connection have been proposed and used; fortunately most results are fairly insensitive to this point. As an illustration of the difficulty, however, consider the usual two-body Schrödinger Hamiltonian in relative coordinates

$$H = \frac{P^2}{2M} + H_{\rm rel}, \tag{1}$$

where

$$H_{\rm rel} = \frac{\pi^2}{2\mu} + V(\varrho, \ldots), \tag{2}$$

and where

$$M = m_1 + m_2, \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}, \tag{3}$$

$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M}, \qquad \vec{\varrho} = \vec{r}_1 - \vec{r}_2, \tag{4}$$

$$\vec{P} = M \frac{d\vec{R}}{dt}, \qquad \vec{\pi} = \mu \frac{d\vec{\varrho}}{dt}, \tag{5}$$

and where $V(\varrho, \ldots)$ is the potential. This Hamiltonian cannot be correct since it is certainly not true that the total mass $M = m_1 + m_2$ for mesons. Of course, even for strong binding it is possible (as I shall try to demonstrate shortly) that the internal motion is non-relativistic [1] so that the ground-state wave function of the internal coordinates can be described by (2), but in a transition like $\omega \to \pi\gamma$ in which there is a large change in M a non-relativistic "atomic transition" picture breaks down.

The most commonly used prescription is to simply ignore this change in M, or alternatively, to calculate transition amplitudes in the SU(6) limit, where $M_\omega = M_\pi$ for example. One then assumes that SU(6)-breaking does not significantly change the matrix elements in question. I prefer a related but somewhat more general prescription, which I call the "mock hadron hypothesis":

> Consider the T-matrix for any process involving a hadron H. After contracting out all non-hadronic fields, the T-matrix will contain one or more hadronic matrix

elements M. (For example, in $H \to H'l\nu$, $M \propto \langle H' | J^{\mu}(0) | H \rangle$.) Decompose M into Lorentz amplitudes $A_i(v_1, v_2, \ldots)$ where the v's are variables like momentum transfer. Now define a *mock hadron* $\tilde{H}$ to be a collection of quarks with the wave function of the quarks in H, but with all binding turned off. Since $\tilde{H}$ has the same quantum number structure as H, one can in many cases[1] unambiguously define a mock matrix element $\tilde{M}$ with mock amplitudes $\tilde{A}_i(\tilde{v}_1, \tilde{v}_2, \ldots)$. We hypothesize that $A_i = \tilde{A}_i$ when $v = \tilde{v}$.

Besides being a Lorentz-invariant correspondence this prescription has several other desirable features that recommend it over the usual approaches, including the fact that it allows us to calculate a number of SU (6)-breaking effects. Still, with the exception of some processes involving the pion (for which the $M \neq m_1 + m_2$ problem is most severe), any reasonable method will give more or less the same results.

2. *What we know about wave-functions*

The first item of business here is to convince you that even for medium-mass quarks a non-relativistic description is possible. It is easy to get this wrong if one concentrates on the hadron spectrum; the most relevant information, on the contrary, is obviously our knowledge of hadron radii.

From "direct" measurements of electromagnetic form factors [2] we know only that

$$\langle r^2 \rangle_p^{1/2} = 0.81 \text{ fm},^{2} \tag{6}$$

$$\langle r^2 \rangle_{\pi^\pm}^{1/2} = 0.7 \pm 0.1 \text{ fm}, \tag{7}$$

although information on $\langle r^2 \rangle_{K^\pm}^{1/2}$ should soon be available. On the other hand, from the Chou-Yang hypothesis [4] which relates $\frac{d\sigma}{dt}$ (AB → AB) to $[F_A(t)]^2[F_B(t)]^2$ and shielding corrections, one can conclude that

$$\langle r^2 \rangle_p^{1/2} = 0.80 \pm 0.04 \text{ fm}, \tag{8}$$

$$\langle r^2 \rangle_{\pi^\pm}^{1/2} = 0.66 \pm 0.10 \text{ fm}, \tag{9}$$

$$\langle r^2 \rangle_{K^\pm}^{1/2} = 0.62 \pm 0.10 \text{ fm}, \tag{10}$$

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[1] The hadronic part M of the T-matrix will in general depend on some subset of the dynamical variables associated with the whole T-matrix. If there is some constraint on these dynamical variables which applies to M but not to $\tilde{M}$ (for example, if M involves all the 4-momenta, then one would have four-momentum conservation in M but not in $\tilde{M}$) then there may not be a unique correspondence between the A_i and the $\tilde{A}_i$ and the mock hadron hypothesis may be ambiguous. In a process like $H \to H'l\nu$ there is no problem since in both M and $\tilde{M}$ there are two independent momenta. In $\pi^0(k) \to \gamma(q_1)\gamma(q_2)$, on the other hand, there are only two independent momenta, say q_1 and q_2, in M while in $\tilde{M}$ there are three independent momenta $\tilde{k}$, $\tilde{q}_1$, and $\tilde{q}_2$. (Fortunately, it is possible to avoid ambiguity in this particular case; in fact we consider this process in Section 5.)

[2] This value comes from the dipole fit and may be about 10% too low [3].

$$\langle r^2\rangle_{\varrho,\omega} \simeq 0.62\pm0.15 \text{ fm}, \tag{11}$$

$$\langle r^2\rangle_{\phi}^{1/2} \simeq 0.37^{+0.20}_{-0.37} \text{ fm}, \tag{12}$$

$$\langle r^2\rangle_{\psi}^{1/2} \leqslant 0.20 \text{ fm}, \tag{13}$$

which values are in accord with the direct measurements (6) and (7) and also in rough agreement with the expectation that

$$\langle r^2\rangle_{\pi^\pm}^{1/2} \simeq \langle r^2\rangle_{\varrho}^{1/2} \simeq \langle r^2\rangle_{K^\pm}^{1/2} > \langle r^2\rangle_{\phi}^{1/2} > \langle r^2\rangle_{\psi}^{1/2}.$$

We can now consider the implications of these values for the non-relativistic character of the quark motion. Since my intention here is only to illustrate that a non-relativistic bound state is possible, I can avoid the impression that something is being hidden behind a screen of Airy functions by simply imagining that the confining potential is harmonic. Then with

$$H = \frac{\pi^2}{2\mu} + \tfrac{1}{2} k\varrho^2 \tag{14}$$

the ground state is

$$\psi = (2\pi\varrho_0^2)^{-3/4} e^{-\varrho^2/4\varrho_0{}^2}, \tag{15}$$

where

$$\varrho_0 = (4k\mu)^{-1/4}. \tag{16}$$

Since

$$\langle r^2\rangle^{1/2} = \tfrac{1}{2}\langle \varrho^2\rangle^{1/2} = \frac{\sqrt{3}}{2}\varrho_0, \tag{17}$$

we must take $\varrho_0^2 \simeq 0.5$ fm^2 for the ϱ and the π; it then follows that

$$\left\langle \frac{p^2}{2m}\right\rangle = \tfrac{1}{2}\left\langle \frac{\pi^2}{2\mu}\right\rangle = \frac{3}{16\mu\varrho_0^2} \simeq 85 \text{ MeV}. \tag{18}$$

The first neglected term in the expansion of $\sqrt{p^2+m^2}$ is therefore only $\sim 6\%$, and we conclude that a non-relativistic description *is* possible, at least for the low-lying states. We can, however, also see that relativistic effects are not entirely negligible, so that calculations cannot be expected to be more than $\sim 10\%$ accurate.

What else do we know about wave-functions? We also know a great deal about $\psi(0)$, the wave-function of the quark-antiquark pair in mesons at zero spatial separation. If we define

$$\langle 0|A^{\mu}_{1+i2}(0)\,|\pi^-(k)\rangle \equiv \frac{if_\pi M_\pi k^\mu}{(2\pi)^{3/2}\sqrt{2M_\pi}}, \tag{19}$$

$$\langle 0|A^{\mu}_{4+i5}(0)\,|K^-(k)\rangle \equiv \frac{if_K M_K k^\mu}{(2\pi)^{3/2}\sqrt{2M_K}}, \tag{20}$$

$$\langle 0|\,j^{\mu}_{em}(0)\,|V(k\lambda)\rangle \equiv \frac{\varepsilon^\mu(k\lambda) f_V M_V^2}{(2\pi)^{3/2}\sqrt{2M_V}}, \tag{21}$$

then by calculating these matrix elements (all of which involve quark-antiquark annihilation) with mock mesons one finds that

$$|\psi_\pi(0)| = \left(\frac{1}{\sqrt{3}}\right)\tfrac{1}{2} f_\pi M_\pi \tilde{M}_\pi^{1/2}, \tag{22}$$

$$|\psi_K(0)| = \left(\frac{1}{\sqrt{3}}\right)\tfrac{1}{2} f_K M_K \tilde{M}_K^{1/2}, \tag{23}$$

$$|\psi_\varrho(0)| = \left(\frac{1}{\sqrt{3}}\right)\frac{1}{\sqrt{2}} f_\varrho M_\varrho \tilde{M}_\varrho^{1/2}, \tag{24}$$

$$|\psi_\omega(0)| = \left(\frac{1}{\sqrt{3}}\right)\frac{3}{\sqrt{2}} f_\omega M_\omega \tilde{M}_\omega^{1/2}, \tag{25}$$

$$|\psi_\phi(0)| = \left(\frac{1}{\sqrt{3}}\right)\tfrac{3}{2} f_\phi M_\phi \tilde{M}_\phi^{1/2}, \tag{26}$$

$$|\psi_\psi(0)| = \left(\frac{1}{\sqrt{3}}\right)\tfrac{3}{4} f_\psi M_\psi \tilde{M}_\psi^{1/2}, \tag{27}$$

$$|\psi_{\psi'}(0)| = \left(\frac{1}{\sqrt{3}}\right)\tfrac{3}{4} f_{\psi'} M_{\psi'} \tilde{M}_{\psi'}^{1/2}, \tag{28}$$

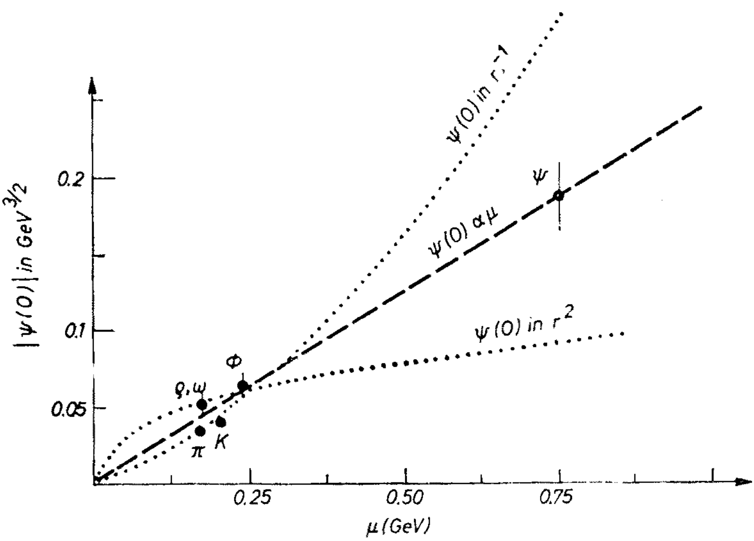

Fig. 1. $|\psi_i(0)|$ vs the reduced mass μ of the $q\bar{q}$ pair

where $\tilde{M}_i$ is the mock meson mass and where we have explicitly separated out the factor of $(1/\sqrt{3})$ that is expected from colour. These values are plotted for the ground states against the reduced mass of the quark-antiquark system in Figure 1 which indicates that [5]

$$|\psi_i(0)| \sim \mu_i \tag{29}$$

intermediate between the Coulombic result ($\mu^{3/2}$) and the results for confining potentials ($\mu^{3/8}$ for $\frac{1}{2}kr^2$ and $\mu^{1/2}$ for br). This is not the relation suggested by van Royen and Weisskopf [6], partly because the mock hadron hypothesis has resolved their $\pi-K$ "paradox", partly because they did not have the ψ, and partly because they took $g_A \simeq \frac{3}{4} g_V$ at the quark level, contrary to our assumptions. We discuss the G_A/G_V "problem" in Section 6.

Notice that we have already learned something from the behaviour of $\psi(0)$: it *is* consistent with the presence of a short range Coulomb and long range confining potential in mesons as expected in quantum chromodynamics. By looking at these values for $\psi(0)$ a little harder, we can make another test of QCD and the picture we are using.

From the large $\varrho-\pi$ mass splitting and the strong departure of $\psi_\varrho(0)/\psi_\pi(0)$ from unity, we can see that the spin-spin interaction in this system is strong. Since

$$\langle \vec{S}_1 \cdot \vec{S}_2 \rangle_{S=0} = -\tfrac{3}{4}, \quad \langle \vec{S}_1 \cdot \vec{S}_2 \rangle_{S=1} = +\tfrac{1}{4}, \tag{30),(31}$$

we can "remove" this effect (to expose the characteristics of the confining potential alone) by considering the appropriate quantities; crudely speaking, we can pay attention to $\frac{1}{4}(3\varrho+\pi)$.

Very roughly then we expect that if we were to turn off the spin-spin interactions we would find that $\langle r^2 \rangle_\pi^{1/2} = \langle r^2 \rangle_\varrho^{1/2} \simeq (0.63 \pm 0.10)$ fm and $|\psi_\pi(0)| = |\psi_\varrho(0)| \simeq (0.049 \pm 0.007)$ GeV$^{3/2}$. But these two quantities are not independent, since $\int d^3r\, |\psi|^2 = 1$, so we can make a test of the consistency of this whole approach. In fact we have already determined that in a harmonic potential

$$\psi = (2\pi\varrho_0^2)^{-3/4} e^{-\varrho^2/4\varrho_0^2} \tag{32}$$

where $\varrho_0^2 \simeq 0.5$ fm^2 to give the correct value of $\langle r^2 \rangle^{1/2}$. This would then give

$$\psi(0) = (2\pi\varrho_0^2)^{-3/4} \simeq 0.037 \text{ GeV}^{3/2}. \tag{33}$$

Since the presence of a Coulomb-type term will tend to increase $\psi(0)$ from this value somewhat, the agreement with the value of $\psi(0)$ extracted just above from $\pi \to \mu\nu$ and $\varrho \to l^+l^-$ is really very good. Our picture has thus been able to relate the size of a meson to its leptonic decay rate, confirming that this general approach, with the inclusion of colour, is a viable one.

Armed with this knowledge of wave functions, and encouraged by this evidence for the consistency of the model, we discuss other of its dynamical consequences in the following sections.

3. *Magnetic moments of baryons*

The predictions of the baryon magnetic moments was one of the earliest quark model results [7]. Here we have a new feature: since for structureless quarks $\mu_q = \dfrac{e_q}{2m_q}$ and

TABLE I

Baryon magnetic moments

Moment	Formula	Theory (μ_N)	Experiment (μ_N)
μ_p	$\frac{e}{2m_d}[1]$	$+2.8$	$+2.79$
μ_n	$\frac{e}{2m_d}[-\frac{2}{3}]$	-1.9	-1.91
μ_Λ	$\frac{e}{2m_d}\left[-\frac{1}{3}\frac{m_d}{m_s}\right]$	-0.65	-0.67 ± 0.06
μ_{Σ^+}	$\frac{e}{2m_d}\left[\frac{8}{9}+\frac{1}{9}\frac{m_d}{m_s}\right]$	$+2.7$	$+2.62\pm0.41$
μ_{Σ^-}	$\frac{e}{2m_d}\left[-\frac{4}{9}+\frac{1}{9}\frac{m_d}{m_s}\right]$	-1.0	-1.48 ± 0.37
μ_{Ξ^0}	$\frac{e}{2m_d}\left[-\frac{2}{9}-\frac{4}{9}\frac{m_d}{m_s}\right]$	-1.5	—
μ_{Ξ^-}	$\frac{e}{2m_d}\left[\frac{1}{9}-\frac{4}{9}\frac{m_d}{m_s}\right]$	-0.6	-1.85 ± 0.75

since we "know" the quark masses, we can make absolute predictions which include SU (3) breaking [5, 8]. The results are given in Table I.

It is remarkable that the values $m_d \simeq 0.34$ and $m_s \simeq 0.48$ which are natural in the spectroscopy of hadrons [8, 9] are also successful here, even to predicting SU (3)-breaking effects. For example, the symmetric quark model would predict that $\mu_\Lambda \simeq -0.93\ \mu_N$, in strong disagreement with experiment.

4. *Magnetic dipole decays of mesons*

The decays

$$V \to P\gamma, \tag{34}$$

$$P \to V\gamma \tag{35}$$

of the *S*-wave mesons are magnetic dipole decays determined by the matrix elements

$$\langle P(k')|\, j^\mu_{em}(0)\, |V(k\lambda)\rangle \propto \mu_{PV}. \tag{36}$$

The transition magnetic moments μ_{PV} can be easily calculated using mock mesons [10]. The calculation is quite similar to the one of the previous section except for the presence of an overlap integral

$$I_{PV} = \int d^3\varrho\, \psi^*_P(\vec{\varrho})\psi_V(\vec{\varrho}) \tag{37}$$

between the initial and final meson wave functions. Since we know that spin-spin effects are significant in the S-wave states (e.g., $\psi_\pi(0) \neq \psi_\varrho(0)$) we know that $I_{PV} < 1$. We shall in fact choose

$$I^2_{P_iV_j} = 0.6 \pm 0.1 \tag{38}$$

to fit the data, but note that it is a value consistent with actual model calculations. (The indicated theoretical uncertainty of ± 0.1 is to allow for fluctuations in $I^2{}_{P_iV_j}$ as a function of i and j). With

$$x \equiv \frac{m_d}{m_s} \tag{39}$$

and

$$\phi_{P(V)} \equiv \theta_{P(V)} - \arctan(1/\sqrt{2}), \tag{40}$$

where $\theta_{P(V)}$ is the quadratic nonet mixing angle of the pseudoscalar (vector) mesons, we obtain the results of Table II.

TABLE II

M1 decays of S-wave mesons

Decay	$\dfrac{\mu_{PV}}{\mu_P I_{PV}}$	Theory (keV)	Experiment (keV)
$\varrho \to \pi\gamma$	$\frac{1}{3}$	75 ± 12	55 ± 25 [a]
$\varrho \to \eta\gamma$	$\sin\phi_P$	46 ± 7	50 ± 13
$\eta' \to \varrho\gamma$	$\cos\phi_P$	95 ± 16	$(0.30 \pm 0.02)\Gamma_{\eta'}$
$\omega \to \pi\gamma$	$\cos\phi_V$	720 ± 120	870 ± 80
$\omega \to \eta\gamma$	$\frac{1}{3}\cos\phi_V \sin\phi_P + \frac{2}{3}x \sin\phi_V \cos\phi_P$	4.9 ± 0.8	$3.0^{+2.5}_{-1.8}$
$\eta' \to \omega\gamma$	$\frac{1}{3}\cos\phi_V \cos\phi_P - \frac{2}{3}x \sin\phi_V \sin\phi_P$	10 ± 2	$(0.030 \pm 0.008)\Gamma_{\eta'}$
$\phi \to \pi\gamma$	$\sin\phi_V$	6.9 ± 4.0	5.9 ± 2.1
$\phi \to \eta\gamma$	$\frac{1}{3}\sin\phi_V \sin\phi_P - \frac{2}{3}x \cos\phi_V \cos\phi_P$	70 ± 17	65 ± 15
$\phi \to \eta'\gamma$	$\frac{1}{3}\sin\phi_V \cos\phi_P + \frac{2}{3}x \cos\phi_V \sin\phi_P$	0.27 ± 0.06	—
$K^{*0} \to K^0\gamma$	$-\frac{2}{3}\left(\frac{1+x}{2}\right)$	120 ± 25	75 ± 35
$K^{*+} \to K^+\gamma$	$\frac{1}{3}(2-x)$	75 ± 17	<80

[a] This is not the value quoted by the experimenters; see references [10] and [11]. There is a rather extensive literature on this controversial measurement which cannot, as reported, be reconciled with very general models [12].

We note that the results lend strong support once again to the notion that $\dfrac{m_d}{m_s} \simeq \dfrac{0.34}{0.48} \simeq 0.7$; for example, $\phi \to \eta\gamma$ would be 140 KeV in the symmetric model.

5. *Two photon decays of the pseudoscalar mesons*

The T-matrix for the decay $\mathrm{P}(k) \to \gamma(q_1\lambda_1)\gamma(q_2\lambda_2)$ is proportional to

$$T^{\mu\nu} = \int dz e^{i(q_1-q_2)\frac{z}{2}} \langle 0|T\left[j^\mu\left(\frac{z}{2}\right)j^\nu\left(-\frac{z}{2}\right)\right]|\mathrm{P}(k)\rangle. \tag{41}$$

As mentioned in Section 1, there would seem to be a difficulty in applying the mock hadron hypothesis to this process since $T^{\mu\nu}$ must be constructed out of q_1 and q_2 only (since $k = q_1+q_2$) while $\tilde{T}^{\mu\nu}$ may depend on $\tilde{q}_1$, $\tilde{q}_2$, and $\tilde{k} \neq \tilde{q}_1+\tilde{q}_2$. Fortunately, however, the expression (41) for $T^{\mu\nu}$ (which is unique in that it respects Bose symmetry even when $k \neq q_1+q_2$) leads to a single Lorentz amplitude of the form

$$T^{\mu\nu} = A\varepsilon^{\mu\nu\varrho\sigma}(q_1-q_2)_\varrho k_\sigma \tag{42}$$

so that the method remains applicable. We can therefore insert a mock meson for **P** in terms of which the decay occurs via the simple annihilation diagram $q\bar{q} \to \gamma\gamma$ of Figure 2.

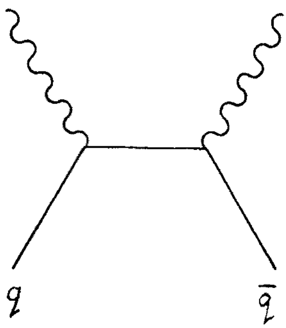

Fig. 2. The decay $\mathrm{P} \to \gamma\gamma$

After a bit of algebra one then finds that [5]

$$\Gamma(P \to \gamma\gamma) = \frac{16\pi\alpha^2}{M_\mathrm{P}^2}\left|\sum_i \psi_i(0)F_i\right|^2, \tag{43}$$

where

$$F_i = a_i\left(\frac{e_i}{e}\right)^2\left(\frac{M_\mathrm{P}}{\tilde{M}_{\mathrm{P}_i}}\right)^{1/2}\left\langle\frac{M_\mathrm{P}}{4p}\ln\left[\frac{m_i^2+(p+\frac{1}{2}M_\mathrm{P})^2}{m_i^2+(p-\frac{1}{2}M_\mathrm{P})^2}\right]\right\rangle_{wfi}, \tag{44}$$

and where

$$\langle f(p)\rangle_{wfi} \equiv \frac{\int d^3p\phi_i(p)f(p)}{\int d^3p\phi_i(p)},$$

with $\phi_i(p)$ the momentum wave function of the quark-antiquark pair of type i found with amplitude a_i in the pseudoscalar meson **P**.

If $p \ll m_i \simeq \frac{1}{2} M_P$ then $F \simeq 1$ and formula (42) gives the decay rate of parapositronium if one puts $M_P \simeq 2m_e$ and $|\psi(0)|^2 = \frac{m_e^3 \alpha^3}{8\pi}$. In parapositronium the rate is sensitive only to $|\psi(0)|$ because the virtual electron line propagates only a distance $d \sim O(1/m_e)$ while the wave function has a size on the order of $a_0 = \frac{1}{m_e \alpha}$. In a pseudoscalar meson, on the other hand, the virtual line propagates through a distance of the order of 1 fm so the rate is sensitive to the wave function in a large volume about the origin (i.e., $F \neq 1$). Since we know $\psi_P(0)$ either directly or by interpolation in Figure 1, and since we know $\langle r^2 \rangle_P^{1/2}$ with some accuracy, it is not difficult to estimate F, albeit with a rather large theoretical uncertainty in some cases, to obtain the results of Table III [3].

TABLE III

Two photon decays

Decay	Theory	Experiment
$\pi^0 \to \gamma\gamma$	13 ± 7 eV	7.8 ± 0.9 eV
$\eta \to \gamma\gamma$	0.7 ± 0.4 keV	0.32 ± 0.06 keV
$\eta' \to \gamma\gamma$	6 ± 2 keV	$(0.020 \pm 0.003)\Gamma_{\eta'}$
$\eta_c \to \gamma\gamma$	7 ± 2 keV	—

Note that by referring back to Table II we can perform a consistency check on η' decays. From the Tables

$$\left.\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\eta' \to \varrho\gamma)}\right|_{\text{theory}} = \frac{6 \pm 2}{95 \pm 16} = 0.06^{+0.04}_{-0.02}, \qquad (45)$$

while experimentally this ratio has the value 0.066 ± 0.014 in good agreement.

We reserve our comments on the relation of this method of calculating $\pi^0 \to \gamma\gamma$ to those relying on the triangle anomaly for Section 9.

6. *G_A/G_V and the neutron charge radius* [4]

One argument that has often been made against the use of structureless quarks as hadronic constituents is the bad result one obtains for G_A/G_V, the ratio of the axial vector to the vector coupling constant in neutron beta-decay. In the SU(6) quark model with

[3] The result for $\eta_c \to \gamma\gamma$ in reference [5] suffers from an arithmetical error and should be multiplied by 4/9.

[4] Much of the work discussed in this section was done in collaboration with G. Karl [13]. For some related studies, see references [14] and [15]. Reference [14] also invokes the spin-spin interaction as the origin of the neutron charge radius.

structureless quarks one would have $G_A/G_V = 5/3$ compared to the observed value $G_A/G_V|_{exp} = 1.25 \pm 0.01$.

Although this *may* be a sign that constituent quarks are not simple, there is another possibility: it may mean that the SU(6) assumption used in deriving the result is rather seriously violated. Of course we know that SU (6) is broken in the nucleon system, since

$$M_\Delta - M_N \simeq 300 \text{ MeV}, \tag{46}$$

which implies that there is a strong repulsive force between parallel spins. For example, there might be a potential

$$\Sigma = \sum_{i<j} a(r_{ij})\vec{S}_i \cdot \vec{S}_j, \tag{47}$$

which in lowest order perturbation would give

$$M_N = M_0 - \tfrac{3}{4}\, a, \tag{48}$$

$$M_\Delta = M_0 + \tfrac{3}{4}\, a, \tag{49}$$

where a is the expectation value of $a(r)$ in the unperturbed wave-functions. Is there any way such a potential could change the "bad" result for G_A/G_V?

Yes, and a very simple way, too. The SU(6) wave functions of a proton and neutron are, schematically,

$$|\mathrm{p}\uparrow\rangle \sim |(\mathrm{ud})_{S=0}\mathrm{u}\uparrow\rangle, \tag{50}$$

$$|\mathrm{n}\uparrow\rangle \sim |(\mathrm{ud})_{S=0}\mathrm{d}\uparrow\rangle, \tag{51}$$

where $(\mathrm{ud})_{S=0}$ denotes a u and d quark in a spin-zero state (which we call in what follows the "core" quarks) and where the spin of nucleon is being carried entirely by the non-core quark (which we call in what follows the "outer" quark). In this configuration the outer quark experiences no net spin-spin force from the core quarks, while the core quarks are *attracted* to each other by Σ. *We can therefore expect the core quarks to lie closer to the nucleon CM than the outer quark* [5]. This will break SU (6); in particular in the matrix element

$$\langle \mathrm{p}|A^\mu|\mathrm{n}\rangle \tag{52}$$

one will have d → u transitions that take a quark from being a core quark to being an outer quark (or vice-versa), a transition that will have a spatial overlap integral less than one.

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[5] This picture might at first (via the uncertainty principle) seem to contradict the condition $d(x)/u(x) \to 0$ as $x \to 1$ (where u and d are the quark structure functions of the proton) [17]. That this is not (necessarily) true is easily demonstrated. Although the d quark in the proton is on the average at smaller distances from the centre-of-mass than the outer u quark, this only tells us that the Fourier transform of the d-quark spatial wave function is *broader* than that of the u-quark. It does not tell us how the Fourier transform behaves as $p \to \infty$, corresponding to the limit $x \to 1$. See also reference [18] and the lectures by D. M. Scott at this School (this issue p. 1061).

A priori, this effect could either increase or decrease G_A/G_V, while of course the strength of the effect will depend on the nature of Σ. In fact, however, it is easy to show that

$$\frac{G_A}{G_V} = \frac{5}{3}\left[\frac{1-\frac{3}{5}(1-I_2)}{1-\frac{1}{3}(1-I_2)}\right], \tag{53}$$

where I_2 is an overlap between two different core quark-outer quark configurations, so that independent of the details of Σ we have a mechanism that will reduce G_A/G_V. Given, moreover, that Σ causes an SU(6)-breaking of $\sim 30\%$ in masses, it is not unreasonable to expect $5/3 \to 1.25$ by this effect.

We now momentarily turn our attention to another consequence of these ideas which is both immediate and very significant: the neutron will not be (locally) neutral! Since the core and outer quarks are displaced from one another, the neutron will have a positive centre. More quantitatively, let $\vec{r}_0 = \vec{r}_1 - \vec{r}_2$ be the separation of the core quarks and let $\vec{R} = \vec{r}_3 - \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$ be the vector from the core centre-of-mass to the outer quark. Then in the SU (6) limit we will have

$$\langle r_0^2\rangle^{1/2} = \frac{2}{\sqrt{3}}\langle R^2\rangle^{1/2}, \tag{54}$$

but here we expect that

$$\langle r_0^2\rangle^{1/2} = \frac{2}{\sqrt{3}}\xi\langle R^2\rangle^{1/2}, \tag{55}$$

where ξ is of course related to I_2 defined earlier, and where

$$0 \leqslant \xi \leqslant 1. \tag{56}$$

In terms of the parameter ξ it can be shown that for small values of I_2

$$\langle r^2\rangle_n \simeq \left[\frac{-1+\xi^2}{3+\xi^2}\right]\langle r^2\rangle_p. \tag{57}$$

In fact the neutron charge radius *is* negative [16]:

$$\langle r^2\rangle_n^{\text{experimental}} = -0.13 \pm 0.02\ \text{fm}^2 \tag{58}$$

indicating a very strong distortion away from SU (6) symmetry. We can see then that the spin-spin interaction mechanism will: 1) reduce G_A/G_V toward its observed value, 2) give the neutron a positive centre, and 3) produce effects of comparable strength, as required, in both domains.

Of course finally one must turn to specific models for Σ to see if the effect is strong enough to give the observed values. We have already argued that the Δ-N mass splitting is qualitatively similar in strength to the effects needed here; while specific models are not compelling, they support this conclusion [13, 14]. Confirmation that the spin-spin interaction is responsible for these effects will, however, require more detailed studies.

Finally, we should mention some flies in the ointment: breaking SU (6) has other effects besides these two, not all of which are welcome. For example, the old SU (6) result that $\mu_p/\mu_n = -3/2$ is increased by about 10%. But then we would argue that that result was always too good anyway!

7. *The charge radius of the K^0*

Speaking of charge radii of neutral particles, what about the K^0? Since $m_s > m_d$, the anti-s quark in the K^0 will lie closer to the meson centre-of-mass than the d quark, so the kaon will, like the neutron, have a positive core.

More precisely, if $\vec{\varrho}$ is the relative coordinate of the $d\bar{s}$ pair, we will have

$$\langle r^2 \rangle_{K^0} = -\tfrac{1}{3}\left(\frac{m_s - m_d}{m_s + m_d}\right)\langle \varrho^2 \rangle_{K^0} \tag{59}$$

$$= -\left(\frac{1-x^2}{2+x^2}\right)\langle r^2 \rangle_{K^\pm} \tag{60}$$

$$\simeq -0.08 \pm 0.03\ \mathrm{fm}^2. \tag{61}$$

Although vector meson dominance also predicts a small negative charge radius for the K^0 [19], experiments [20] are at present slightly positive, albeit with large errors. A new FNAL result should shed light on this question soon.

8. *Form factors in K_{l_3} decay*

The matrix element governing, for example, the decay

$$K^0 \to \pi^- \bar{l} \nu_l \tag{62}$$

is

$$\langle \pi^-(k') \,|V^\mu_{4-i5}(0)\,| K^0(k)\rangle \equiv \frac{1}{(2\pi)^3}\,[f_+(t)\,(k+k')^\mu + f_-(t)\,(k-k')^\mu]. \tag{63}$$

In the SU(3) limit, $f_+(t) = f_\pi(t)$ (the pion electromagnetic form factor) and $f_-(t) = 0$. The form factors $f_\pm(t)$, which in this decay are measured only over a small range in t, are usually parametrized in the form

$$f_\pm(t) = 1 + \frac{\lambda_\pm t}{M_\pi^2}. \tag{64}$$

Using mock mesons one can show that [21]

$$f_+(0) = \left(\frac{1+x}{2x}\right)^{1/2}\left[\left(\frac{3+x}{4}\right) I_{\pi K} + \left(\frac{1-x}{4}\right) B_{\pi K}\right], \tag{65}$$

$$f_-(0) = -\tfrac{3}{4}\left(\frac{1+x}{2x}\right)^{1/2}\left[(1-x)I_{\pi K}+\left(\frac{1+3x}{3}\right)B_{\pi K}\right], \tag{66}$$

$$\lambda_\pm = \frac{M_\pi^2 x}{12(1+x)}\left[\frac{J_{\pi K}-\left(\frac{1+x\mp 2x}{4}\right)(J_{\pi K}-C_{\pi K})}{I_{\pi K}-\left(\frac{1+x\mp 2x}{4}\right)(I_{\pi K}-B_{\pi K})}\right], \tag{67}$$

where $x = \dfrac{m_d}{m_s}$ as usual and where I, J, B and C are various (weighted) overlap integrals:

$$I_{\pi K} \equiv \int d^3\varrho\,\psi_\pi\psi_K, \tag{68}$$

$$J_{\pi K} \equiv \int d^3\varrho\,\varrho^2\psi_\pi\psi_K, \tag{69}$$

$$B_{\pi K} \equiv \tfrac{1}{3}\int d^3\varrho\,\varrho\left[\psi_\pi\frac{\partial\psi_K}{\partial\varrho}-\frac{\partial\psi_\pi}{\partial\varrho}\psi_K\right], \tag{70}$$

$$C_{\pi K} \equiv \tfrac{1}{5}\int d^3\varrho\,\varrho^3\left[\psi_\pi\frac{\partial\psi_K}{\partial\varrho}-\frac{\partial\psi_\pi}{\partial\varrho}\psi_K\right]. \tag{71}$$

TABLE IV

K_{l3} form factors for $I_{\pi K} = 1$

Quantity	Theory	Experiment
$f_+(0)$	$\left(\frac{1+x}{2x}\right)^{1/2}\left(\frac{3+x}{4}\right) = 1.02$	0.97 ± 0.04
$\xi(0) \equiv \frac{f_-(0)}{f_+(0)}$	$\frac{3}{4}\left(\frac{1+x}{2x}\right)^{1/2}(1-x) = -0.23$	-0.17 ± 0.05
λ_+	$\frac{M_\pi^2 x}{3(1+x)}\langle r^2\rangle_\pi = 0.033 \pm 0.006$	0.029 ± 0.002

If $I_{\pi K} = 1$, then $J_{\pi K} = \langle\varrho^2\rangle_\pi = 4\langle r^2\rangle_{\pi^\pm}$ and $B_{\pi K} = C_{\pi K} = 0$ giving the results of Table IV which are clearly in excellent agreement with experiments. If $I_{\pi K} < 1$ (of course we expect it to be slightly less than unity), then this makes all of $f_+(0)$, $\xi(0)$, and λ_+ smaller, in magnitude, thereby improving the agreement.

Once again the mock hadrons seem capable of characterizing a property of real hadrons with accuracy.

9. *Comments on the relation to VMD and PCAC*

One of the real puzzles of the approach I have outlined is its relation to the usual methods of low energy phenomenology like vector meson dominance and PCAC. Consider, for example, the pion. In this picture, the π and the ϱ are entirely similar structures which happen to have a large mass difference arising from the spin-spin interaction of their constituent quarks. The connection to the picture of the pion as the Goldstone boson of broken chiral symmetry is certainly not apparent. The quark model picture thus led us in Section 5 to a calculation of $\pi^0 \to \gamma\gamma$ in which the triangle anomaly played no role and in which π^0, η, η', and η_c decays were all treated on an equal footing. I can only comment that it seems to me that the triangle anomaly calculation, besides being slightly schizophrenic in its treatment of the four apparently similar pseudoscalars, is open to the criticism that, via PCAC, it depends on the association of the spatially extended pion with the pointlike axial vector current. Such an association may only be viable so long as the pion is *effectively* pointlike in size; the triangle anomaly, on the contrary, depends upon the pointlike character of the axial vector current and so requires a *literally* pointlike pion, in contradiction to the quark model picture.

The calculation of the K_{l_3} form factors offers another case in point. Vector meson dominance relates λ_+ to the mass of the K^* resonance:

$$\lambda_+^{\mathrm{VMD}} = \frac{M_\pi^2}{M_{K^*}^2} = 0.024, \tag{72}$$

which is in reasonable agreement with experiment, while in the previous section we have related λ_+ to the physical size of the pion and kaon! What is more, our calculation of $\xi(0)$ extrapolates to the Callan-Treiman point, though the relation to PCAC is once again, to say the least, obscure. In a similar vein, we have already noted that vector meson dominance, which relates the K^0 charge radius to the $\phi - \varrho$ mass difference, gives a result very similar to the prediction of Section 7, namely

$$\langle r^2 \rangle_{K^0}^{\mathrm{VMD}} \simeq -2\left[\frac{1}{M_\varrho^2} - \frac{1}{M_\phi^2}\right] \simeq -0.06\ \mathrm{fm}^2. \tag{73}$$

Once again the physical basis of the connection between these two calculations is not apparent. I could continue this list but I am sure by now you have heard the question.

10. Conclusion

I have tried in this very brief account to demonstrate to you that although there are reasons why the use of pointlike constituent quarks *might* have failed to describe the low energy dynamics of hadrons, a more optimistic view is warranted. The case for this general approach is not, however, conclusive; further study, both theoretical and experimental, is needed befote that stage is reached. Perhaps in these lectures I will even have managed to convince some of you that this possibility is sufficiently promising that you will yourself be tempted to think about some of the many questions that remain to be answered.

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