THE PROBLEM OF NUCLEON PRODUCTION IN THE QUARK PARTON MODEL

By J. Ranft and Gisela Ranft

Sektion Physik, Karl-Marx-Universität Leipzig*

(Received July 8, 1977)

Quark fragmentation into hadrons, esp. nucleons, is studied fitting empirical fragmentation functions to e^+e^- annihilation data. We find fragmentation functions deviating from counting rule predictions as well as from scaling due to the threshold in kaon and nucleon production. Using these fragmentation functions we study particle production ratios in ep and large $p_\perp$ hadronic reactions. In both cases we find the ratios p/π^+ and $\bar{p}/\pi^-$ to agree roughly in magnitude with the measured ratios. The model is however inconsistent with the $p_\perp^{-12}$ behaviour of large $p_\perp$ proton spectra.

1. Introduction

Hadron production at large transverse momentum is rather successfully described in a model with elastic quark quark scattering [1-4]. The quark distributions and quark fragmentation functions in this model are required to be consistent with hadron production in e^+e^- annihilation and ep reactions. In the present paper we use new data in hadron production in e^+e^- annihilation of the DASP collaboration [5] to determine the quark fragmentation into hadrons, esp. including nucleons. The problems connected with large $p_\perp$ nucleon production in the qq → qq model were not discussed in detail before. In the presently accessible energy range the e^+e^- annihilation data cannot be described with quark fragmentation functions satisfying dimensional counting rules [6]. Also the rise of kaon and nucleon production above threshold leads to non-scaling behaviour of the fragmentation functions. In Section 2 we describe our quark fragmentation functions and the fit to e^+e^- data. With these fragmentation functions we calculate in Section 3 particle production ratios p/π^+ and $\bar{p}/\pi^-$ in deep inelastic electron nucleon reactions and in large $p_\perp$ hadronic reactions. The problem of nucleon production at large $p_\perp$ in the quark quark scattering model is discussed.

* Address: Section Physik, Karl-Marx-Universität, Karl-Marx-Platz 10/11, 701 Leipzig, DDR.

2. The quark fragmentation functions with non-scaling behaviour due to kaon and proton thresholds

The momentum distributions of hadrons C resulting from the fragmentation of quark i are usually defined as

$$\frac{dn_C^i}{dz} = P_C^i(z) = G_C^i(z)/z. \tag{2.1}$$

z is the fraction of parton momentum carried away by the hadron C, $z = \frac{p_C}{p_i}$. In Eq. (2.1) and the following we disregard the transverse momenta of hadrons relative to the direction of the parton. The fragmentation functions $G_C^i(z)$ satisfy the momentum sum rule

$$\sum_C \int_0^1 dz G_C^i(z) = 1. \tag{2.2}$$

According to dimensional counting rules [6] and parton-hadron reciprocity relations [7] the fragmentation functions are expected to behave like

$$G_C^i(z) \sim (1-z)^f \quad \text{for } z \to 1, \quad f = 2n(\bar{i}C)-1. \tag{2.3}$$

$n(\bar{i}C)$ is the number of quarks in the $(\bar{i}C)$ system or the number of the spectator quarks in the fragmentation of i. E.g., in the fragmentation of u quarks one expects according to Eq. (2.3) $f = 1$ for π^+ and K^+ production and $f = 3$ for p production. We shall see that the functions obtained in the fit do not satisfy this behaviour. It is unknown whether the counting rule predictions will be satisfied at high energies. The remarkable feature of the data on π, K and $\bar{p}$ production in e^+e^- annihilation from the DASP collaboration [5] is the approximate scaling in the variable $z_E = 2E_C/\sqrt{s} \approx E_C/p_i$ (p_i is the quark momentum) and the fact that in this variable secondary π, K and N distributions seem to follow one universal curve. As suggested by this z_E scaling we describe the fragmentation of quark i into hadron C by

$$\frac{E_C}{p_C}\frac{dn_C^i}{dz_E} = \hat{P}_C^i(z_E) \tag{2.4}$$

and we choose

$$\hat{P}_C^i(z_E) = \hat{G}_C^i(z_E)/z_E \quad \text{for } m_C/p_i \leqslant z_E \leqslant 1,$$
$$= 0 \quad \text{for } 0 \leqslant z_E < m_C/p_i. \tag{2.5}$$

$\hat{G}_C^i(z_E)$ is parametrized as follows

$$\hat{G}_C^i(z_E) = g_C^i(1-z_E)^{F_C^i}N^i(p_i), \tag{2.6}$$

where $N^i(p_i)$ is an overall normalization to be determined according to the energy sum rule

(2.2). g^i_C and F^i_C are parameters. The quark fragmentation functions defined in the usual way (2.1) in terms of the variable $z = p_C/p_i$ can be obtained from Eqs (2.4) to (2.7) as follows

$$\frac{dn^i_C}{dz} = \frac{G^i_C(z)}{z} = \frac{1}{1+\frac{m_C^2}{z^2 p_i^2}} \frac{\hat{G}^i_C(z_E)}{z_E}, \qquad z_E = \sqrt{z^2 + m_C^2/p_i^2}\,. \tag{2.7}$$

At finite quark momentum these fragmentation functions have an explicit non-scaling behaviour. Asymptotically scaling in z_E and z is obtained. Due to the rise of the production of heavy particles, kaons and nucleons above their thresholds an additional non-scaling is due to the p_i dependence of the normalization factor $N^i(p_i)$

$$\frac{1}{N^i(p_i)} = \sum_C \int\limits_{m_C/p_i}^{1} dz_E z_E \frac{dn}{dz_E}$$

$$= \sum_C \int\limits_{m_C/p_i}^{1} dz_E \sqrt{1 - \frac{m_C^2}{z_E^2 p_i^2}}\, \hat{G}^i_C(z_E). \tag{2.8}$$

In the quark parton model hadron production in e^+e^- annihilation is described by

$$\frac{1}{\sigma}\frac{d\sigma^C}{dz} = \frac{dN_C}{dz} = \frac{\sum\limits_i e_i^2 \frac{1}{z} G^i_C(z)}{\sum\limits_i e_i^2}\,. \tag{2.9}$$

The data of the DASP collaboration is given in terms of z_E

$$\frac{E_C}{p_C}\frac{dN_C}{dz_E} = \frac{\sum\limits_i e_i^2 \frac{1}{z_E} \hat{G}^i_C(z_E)}{\sum\limits_i e_i^2}\,. \tag{2.10}$$

Our definitions are chosen in such a way that this depends only on z_E. We fit these data at $s = 16$ to $25\ \text{GeV}^2$ to our parametrization. Here the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ stays approximately constant. In Table I the parameters of the quark fragmentation functions obtained in this way are given. The parameters for all other cases follow from isospin and charge conjugation and the rule that all disfavoured

TABLE I

Parameters of quark fragmentation into hadrons obtained in the fit

	$u \to \pi^+$	$u \to \pi^-$	$u \to K^+$	$u \to K^-$	$u \to \bar{p}$	$u \to p$	$d \to p$
g^i_C	0.83	0.41	0.42	0.21	0.41	0.35	0.17
F^i_C	2.16	2.66	2.31	2.81	5.0	1.95	2.45

fragmentations into π, K and nucleons are assumed to be the same. We do not consider charmed quarks and heavy leptons. Therefore our fragmentation functions into kaons are probably incorrect. We have quoted them here only because they play a role in the calculation of the threshold factor $N^i(p_i)$. We shall use in the following only the fragmentation functions into nucleons and pions which should not be affected strongly. The quark fragmentation functions into kaons given by Field and Feynman [1] are to be

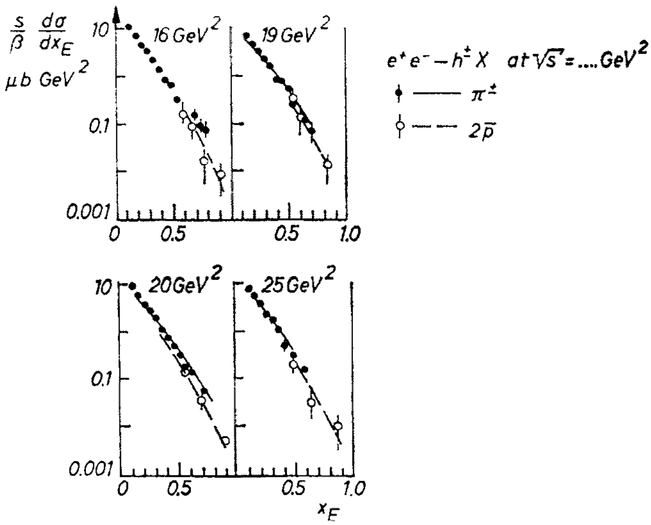

Fig. 1. Comparison of π and $\bar{\text{p}}$ production in e^+e^- annihilation according to our parametrization with data of the DASP collaboration [5]

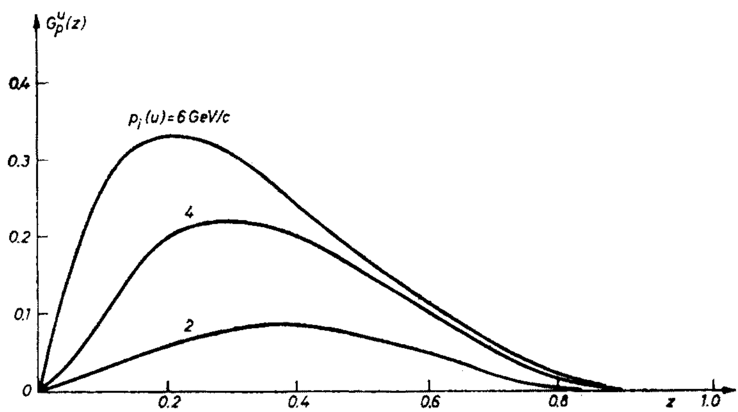

Fig. 2. The quark fragmentation function $G_{\text{p}}^{\text{u}}(z)$ for different energies according to Eq. (2.7)

preferred for calculating K/π ratios at large transverse momentum. We obtain a satisfactory χ^2 in our fit.

In Fig. 1 we compare the result of the fit with the DASP data.

In Fig. 2 we plot the function $G_{\text{p}}^{\text{u}}(z)$ describing the fragmentation of u quarks into protons for different energies to demonstrate the threshold nonscaling behaviour.

3. *Particle production ratios in ep and large $p_\perp$ hadronic reactions*

p/π^+ and $\bar{p}/\pi^-$ ratios in deep inelastic ep scattering were measured by Martin et al. [8]. The data is for s in the range 15 to 31 GeV^2. The hadrons have transverse momenta in the range $0 \leqslant p_\perp^2 \leqslant 0.64$ (GeV/c)^2 and are averaged over the interval $0.4 \leqslant z \leqslant 0.85$. Using quark distribution functions according to Ref. [9] and the quark fragmentation functions

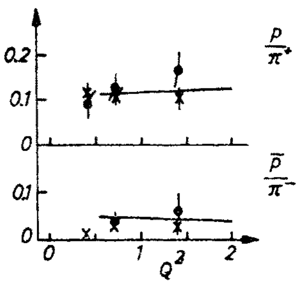

Fig. 3. Comparison of quark model predictions for the p/π^+ and $\bar{p}/\pi^-$ ratios with data from electro-production on protons by Martin et al. [8]

described in Section 2 we calculate the same particle production ratios averaging over the range in the kinematical variables as indicated.

In Fig. 3 these quark model predictions are compared to the data of Ref. [8]. We find a satisfactory agreement. This indicates that the quark parton model with our parametrization of the quark fragmentation into nucleons is consistent within the present experimental errors.

Encouraged by this success we use the same quark fragmentation functions in the quark quark elastic scattering model of large $p_\perp$ particle production. The details of the

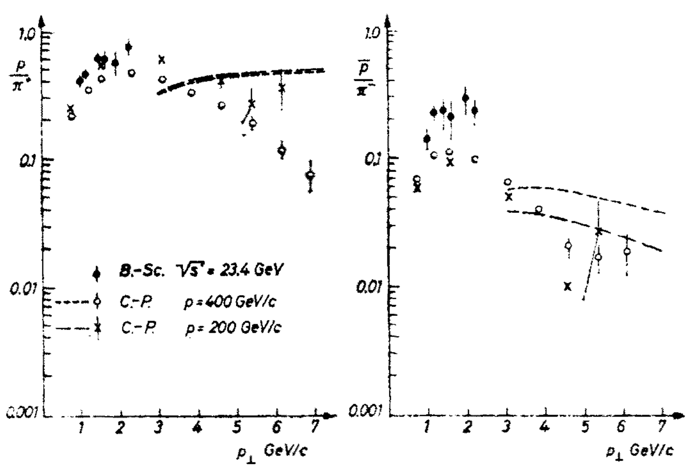

Fig. 4. Comparison of p/π^+ and $\bar{p}/\pi^-$ ratios at large $p_\perp$ predicted by the quark quark scattering model with experimental data of the Chicago-Princeton [10] and British-Scandinavian [11] collaborations

model as we use it are given in earlier papers [3, 4]. In these papers the model is also compared to single particle production and correlation data of secondary pions only. Differences to the model described in Ref. [1] are only in the details of some of the input functions.

In Fig. 4 we compare for $p_\perp > 3$ GeV/c the p/π^+ and $\bar{\text{p}}/\pi^-$ particle production ratios predicted by the model when the quark fragmentation functions of Section 2 are used with data of the Chicago-Princeton [10] and the British-Scandinavian collaborations [11].

The $\bar{\text{p}}/\pi^-$ ratio predicted agrees in magnitude roughly with the data. The $p_\perp$ dependence of large $p_\perp$ π and $\bar{\text{p}}$ production is approximately the same in agreement with the trend of the data. However there is quite some disagreement in the $x_\perp$ dependence. All together we consider the agreement as promising taking into account the error bars of the data and the still unresolved disagreement of the ISR and FNAL data. Furthermore, we stress that the energy range of the e^+e^- experiment from which we extract the quark fragmentation function is lower than the energies of the quarks in the large $p_\perp$ reactions considered. With more and better data we might hope that the p/π^- ratios can be understood in the quark quark scattering model.

The agreement of the predicted p/π^+ ratio with the data is unsatisfactory. Around $p_\perp = 3$ to 4 GeV/c there is good agreement in magnitude. As with the $\bar{\text{p}}/\pi^-$ ratio there is some disagreement in the $x_\perp$ behaviour. The model cannot explain the experimentally found $p_\perp^{-3.26}$ behaviour of the p/π^+ ratio. In fact due to the non-scaling feature of the quark fragmentation functions (Fig. 2) the model gives a p/π^+ ratio like $p_\perp^{0.6}$. The threshold non-scaling behaviour of the quark fragmentation function makes this known discrepancy even worse.

Introducing a suitable non-scaling behaviour in the quark fragmentation into pions and nucleons as e.g. done by Contogouris et al. [12] one might be able to describe the p/π^+ ratios in the large $p_\perp$ reactions alone. In this way, however, no simultaneous consistent description of particle production in e^+e^- and ep reactions can be achieved.

We conclude: The quark quark scattering model seems to provide a consistent description of $\bar{\text{p}}$ production at large transverse momenta. The production of protons cannot be explained in this model. The fact that protons are leading particles suggests that other hard scattering mechanisms [13] are responsible for the bulk of the proton production.

We thank Mr. Discher for assistance with the numerical calculations.

REFERENCES

[1] R. D. Field, R. P. Feynman, preprint, Caltech, CALT 68-565, 1976.
[2] J. Kripfganz, J. Ranft, *Nucl. Phys.* B, to appear.
[3] E.-M. Ilgenfritz, J. Kripfganz, H.-J. Möhring, G. Ranft, J. Ranft, A. Schiller, *Acta Phys. Pol.* **B9**, No 1 (1978).
[4] A. Schiller, E.-M. Ilgenfritz, J. Kripfganz, H.-J. Möhring, G. Ranft, J. Ranft, *Acta Phys. Pol.* **B9**, No 1 (1978)
[5] R. Brandelik et al., DESY preprint 1977 (DASP collaboration), DESY 77-11.
[6] D. Sivers, S. Brodsky, R. Blankenbecler, *Phys. Reports* 23 C (1976).

[7] V. N. Gribov, L. N. Lipatov, *Phys. Lett.* **37B**, 78 (1971).
[8] J. F. Martin et al., *Phys. Lett.* **65B**, 485 (1976).
[9] R. McElhaney, S. F. Tuan, *Phys. Rev.* **D8**, 2267 (1973).
[10] D. Andreasyan et al., *Phys. Rev. Lett.* **38**, 115 (1977).
[11] B. Alper et al., *Nucl. Phys.* **B100**, 237 (1975).
[12] A. P. Contogouris, R. Gaskell, A. Nikolaides, Preprint McGill Univ., 1977.
[13] S. J. Brodsky, *Proc. XVIII Int. Conf. on High Energy Physics*, Tbilisi 1976, p. A4-9.