CALCULATIONS OF DECAY PARAMETERS WITHIN THE KRÓLIKOWSKI-RZEWUSKI THEORY FOR THREE LEE-LIKE MODELS

W. GARCZYŃSKI AND K. URBANOWSKI

Institute of Theoretical Physics, University of Wrocław*

(*Received October 9, 1976*)

Calculations of the quasipotential within simple Lee-like models are performed using the Królikowski–Rzewuski equations for the distinguished component of a state-vector. We consider the two-level quantum system, the one-dimensional Lee model and the usual Lee model. In the last case a comparison of our results with those of Glaser and Källen is made.

1. *Introduction*

In this paper we shall test the Królikowski-Rzewuski equations for a distinguished component of the state-vector of a quantum system by applying them to three simple models [1]. All the considered models are similar in the sense that the distinguished components of the state-vectors are in fact one-dimensional and this circumstance simplifies the calculations [2]. Our aim is to find the quasipotentials governing the time evolution of the considered components of state vectors, and to calculate corresponding decay widths of those components describing unstable systems [3, 4] and [11–17]. In particular, we are inetrested in the V-particle decay problem in the Lee model which was treated long ago by Glaser and Källen using an entirely different approach [5]. Before doing this we consider the one-dimensional Lee model case which may be reduced to a two-level quantum system. The last case may be completely solved and studied in detail [6].

2. *A system with two levels*

Let us consider the simplest nontrivial system having two states only. Any state-vector $\psi(t) \in \mathscr{H}$ may be represented as a linear combination of two basic vectors

$$\psi(t) = \psi_1(t)\, e_1 + \psi_2(t)\, e_2, \tag{2.1}$$

* Address: Instytut Fizyki Teoretycznej, Uniwersytet Wrocławski, Cybulskiego 36, 50-205 Wrocław, Poland.

where e_1, e_2 are orthogonal unit vectors which in the canonical base have the form [6]

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.2}$$

and $\psi_1(t)$, $\psi_2(t)$ are complex functions. Thus the Hilbert space $\mathscr{H}$ splits into the orthogonal sum of two one-dimensional subspaces directed along the vectors e_1, e_2.

$$\mathscr{H} = \mathscr{H}_1 \oplus \mathscr{H}_2, \quad \mathscr{H}_k = \overline{\{\lambda e_k\}}, \quad k = 1, 2.$$

We shall sellect the subspace $\mathscr{H}_1$ as the projecting subspace. The corresponding projector P and its complement Q are

$$Pe_1 = e_1, \quad Pe_2 = 0,$$

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad Q = 1 - P = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2.3}$$

Our task consists of finding the time evolution of the projection

$$\psi_{\|}(t) = P\psi(t) = \psi_1(t) e_1, \tag{2.4}$$

if it is known that the vector $\psi(t)$ evolves according to the Schrödinger equation

$$i \frac{d\psi(t)}{dt} = H\psi(t), \tag{2.5}$$

where H is a Hermitean matrix

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} H_{11} & 0 \\ 0 & H_{22} \end{pmatrix} + \begin{pmatrix} 0 & H_{12} \\ H_{21} & 0 \end{pmatrix} = H_0 + H_1. \tag{2.6}$$

Assuming that $\psi(t_0) \in \mathscr{H}_1$ we will get an equation for $\psi_{\|}(t)$ [1]

$$\left(i \frac{d}{dt} - PHP \right) \psi_{\|}(t) = -i \int_{t_0}^{\infty} d\tau K(t-\tau) \psi_{\|}(\tau), \tag{2.7}$$

where the relevant matrices in this case are

$$PHP = \begin{pmatrix} H_{11} & 0 \\ 0 & 0 \end{pmatrix}, \quad QHQ = \begin{pmatrix} 0 & 0 \\ 0 & H_{22} \end{pmatrix}, \quad PHQ = \begin{pmatrix} 0 & H_{12} \\ 0 & 0 \end{pmatrix}, \tag{2.8}$$

$$K(t) = \Theta(t) PHQ \exp[-itQHQ] QHP$$
$$= \begin{pmatrix} \Theta(t) |H_{12}|^2 \exp(-itH_{22}) & 0 \\ 0 & 0 \end{pmatrix} = K_1(t) P. \tag{2.9}$$

Projecting this equation onto e_1 we obtain

$$\left(i \frac{d}{dt} - H_{11} \right) \psi_1(t) = -i \int_{t_0}^{\infty} d\tau K_1(t-\tau) \psi_1(\tau), \tag{2.10}$$

where, for brevity, we write

$$K_1(t) = \Theta(t)|H_{12}|^2 \exp(-it\, H_{22}). \tag{2.11}$$

Using the retarded solution of the equation

$$\left(i\frac{d}{dt} - H_{11}\right) G_1(t) = \delta(t), \quad G_1(t) = 0 \quad \text{for} \quad t < 0 \tag{2.12}$$

i. e.,

$$G_1(t) = -i\Theta(t)\exp(-itH_{11}), \tag{2.13}$$

we may write for the function $\psi_1(t)$

$$\psi_1(t) = \varphi_1(t) - i\int_{t_0}^{\infty} d\tau \int_{t_0}^{\infty} d\tau' G_1(t-\tau')K_1(\tau'-\tau)\psi_1(\tau), \tag{2.14}$$

where $\varphi_1(t)$ is a solution of the free equation

$$\left(i\frac{d}{dt} - H_{11}\right)\varphi_1(t) = 0, \quad \varphi_1(t) = \varphi_1(t')e^{-i(t-t')H_{11}}. \tag{2.15}$$

The internal integration over τ' may be done and we get the expression

$$\int_{t_0}^{\infty} d\tau' G_1(t-\tau')K_1(\tau'-\tau) = \int_{0}^{\infty} d\tau' G_1(t-\tau-\tau')K_1(\tau')$$

$$= G_1 * K_1(t-\tau) = \Theta(t-\tau)\frac{|H_{12}|^2}{H_{11}-H_{22}}\left[e^{-i(t-\tau)H_{11}} - e^{-i(t-\tau)H_{22}}\right] = L_1(t-\tau). \tag{2.16}$$

We infer from it that the solution $\psi_1(t)$ satisfies the initial condition

$$\psi_1(t_0) = \varphi_1(t_0). \tag{2.17}$$

Iterating the integral equation

$$\psi_1(t) = \varphi_1(t) - i\int_{t_0}^{\infty} d\tau L_1(t-\tau)\psi_1(\tau), \tag{2.18}$$

we find easily, assuming that the modulus of L_1 does not exceed unity

$$\psi_1(t) = \int_{t_0}^{\infty} d\tau\Big[\sum_{n=0}^{\infty}(-i)^n \underbrace{L_1 * \ldots * L_1}_{n}(t-\tau)\Big]\varphi_1(\tau) \tag{2.19}$$

$$= \int_{t_0}^{\infty} d\tau(\mathbf{1}+iL_1)^{-1}(t-\tau)\varphi_1(\tau) \tag{2.20}$$

$$= \int_{t_0}^{\infty} d\tau(\mathbf{1}+iL_1)^{-1}(t-\tau)e^{-i(t-\tau)H_{11}}\varphi_1(t). \tag{2.21}$$

Now, introducing the resolvent kernel by means of the equation

$$(1+iL_1)^{-1}(t-\tau) = (1-iR_1)(t-\tau) = \delta(t-\tau)-iR_1(t-\tau), \tag{2.22}$$

we shall find

$$\psi_1(t) = [1-i\int_{t_0}^{\infty} d\tau R_1(t-\tau)e^{i(t-\tau)H_{11}}]\varphi_1(t), \tag{2.23}$$

where according to the expansion (2.19) we have

$$R_1(t-\tau) = i\sum_{n=1}^{\infty}(-i)^n \underbrace{L_1 * \dots * L_1}_{n}(t-\tau) = (1+iL_1)^{-1} * L_1(t-\tau). \tag{2.24}$$

An explicit expression for R, may be easily found by means of the one side Laplace transform and its inverse. We obtain in this way the formula

$$R_1(t) = -i\Theta(t)\frac{|H_{12}|^2}{\lambda_1-\lambda_2}(e^{t\lambda_1}-e^{t\lambda_2}), \tag{2.25}$$

where the complex numbers λ_1, λ_2 are

$$\lambda_{1,2} = -\frac{i}{2}(H_{11}+H_{22})\pm\frac{i}{2}\sqrt{(H_{11}-H_{22})^2+4|H_{12}|^2}\,. \tag{2.26}$$

Introducing the notation

$$A_1(t, t_0) = \int_{t_0}^{\infty} d\tau R_1(t-\tau)e^{i(t-\tau)H_{11}}$$

$$= -i\frac{|H_{12}|^2}{\lambda_1-\lambda_2}\left[\frac{e^{(t-t_0)(\lambda_1+iH_{11})}-1}{\lambda_1+iH_{11}}-\frac{e^{(t-t_0)(\lambda_2+iH_{11})}-1}{\lambda_2+iH_{22}}\right] \quad \text{for} \quad t > t_0 \tag{2.27}$$

we will get for the function $\psi_1(t)$ the formula

$$\psi_1(t) = [1-iA_1(t, t_0)]\,\varphi_1(t). \tag{2.28}$$

The function $A_1(t, t_0)$ vanishes for $t = t_0$ and depends on these variables only via the difference $t-t_0$.

A purely differential equation for the wave function $\psi_1(t)$ follows from the last formula

$$\left[i\frac{d}{dt}-H_{11}-\frac{\dot{A}_1(t, t_0)}{1-iA_1(t, t_0)}\right]\psi_1(t) = 0 \tag{2.29}$$

valid for $t > t_0$. The last term in brackets determines what is called the quasipotential $V(t, t_0)$ [1, 2] where for the derivative $\dot{A}_1(t, t_0)$ we have from (2.27) at $t > t_0$

$$\dot{A}_1(t, t_0) = \frac{-i|H_{12}|^2}{\lambda_1-\lambda_2}[e^{(t-t_0)(\lambda_1+iH_{11})}-e^{(t-t_0)(\lambda_2+iH_{22})}] \tag{2.30}$$

$$= R_1(t-t_0)e^{i(t-t_0)H_{11}}. \tag{2.31}$$

Therefore, we get for the quasipotential

$$V_1(t, t_0) = \frac{\dot{A}_1(t, t_0)}{1-iA_1(t, t_0)} = \frac{R_1(t-t_0)e^{i(t-t_0)H_{11}}}{1-iA_1(t, t_0)}. \tag{2.32}$$

The same quantity may be found also from the formulae (cf. e. g. [1] formula (2.17) and [2] formulae (13)–(15))

$$V_1(t, t_0)\,[1-iA_1(t, t_0)] = -i\int_{t_0}^{\infty} d\tau K_1 * (1-iR_1)\,(t-\tau)e^{i(t-\tau)H_{11}} \tag{2.33}$$

$$= -i\int_{t_0}^{\infty} d\tau K_1(t-\tau)e^{i(t-\tau)H_{11}}[1-iA_1(\tau, t_0)]. \tag{2.34}$$

They are useful in performing the limiting procedure when $t_0 \to -\infty$. Namely, we have in this limit the following basic formulae

$$V_1 = \lim_{t_0\to-\infty} V_1(t, t_0) \tag{2.35}$$

$$= \lim_{T\to\infty} \frac{\dot{A}_1(T)}{1-iA_1(T)} \tag{2.36}$$

$$= \lim_{T\to\infty} \frac{R_1(T)e^{iTH_{11}}}{1-iA_1(T)} \tag{2.37}$$

$$= -i\lim_{t_0\to-\infty}\int_{t_0}^{\infty} d\tau K_1(t-\tau)e^{i(t-\tau)H_{11}}\,\frac{1-iA_1(\tau, t_0)}{1-iA_1(t, t_0)} \tag{2.38}$$

$$= -i\frac{\int_{-\infty}^{\infty} d\tau K_1 * (1-iR_1)\,(\tau)e^{i\tau H_{11}}}{1-i\int_0^{\infty} d\tau R_1(\tau)e^{i\tau H_{11}}}. \tag{2.39}$$

The wave function $\psi_1(t)$ satisfies the conditions in this limit

$$\left(i\frac{d}{dt} - H_{11} - V_1\right)\psi_1(t) = 0, \tag{2.40}$$

$$\lim_{t\to-\infty}[\psi_1(t) - \varphi_1(t)] = 0. \tag{2.41}$$

The last condition, which replaces the initial condition (2.17) tells us that the quasipotential V_1 should be switched off at minus infinity (adiabatic hypothesis). The easiest way to get the quasipotential V_1 in the quadratic approximation is to drop the functions A in formula (2.38) when the above conditions are fulfilled

$$QH_1Q = PH_0Q = QH_0P = 0. \tag{2.42}$$

We obtain in this way

$$V_1 = -i\int_0^\infty d\tau K_1(\tau)e^{i\tau(H_{11}+i\varepsilon)}+O(|H_{12}|^4) = \frac{|H_{12}|^2}{H_{11}-H_{22}+i\varepsilon}+O(|H_{12}|^4). \tag{2.43}$$

Solving equation (2.40) with this quasipotential one finds

$$\psi_1(t) = \psi_1(0)\exp\left\{-it\left(H_{11}+\mathscr{P}\frac{|H_{12}|^2}{H_{11}-H_{22}}\right)-t\pi|H_{12}|^2\delta(H_{11}-H_{22})+\ldots\right\}. \tag{2.44}$$

The decay is absent when energies of both states are different.

In general, the denominators in the formulae (2.36)–(2.38) should be elliminated, by expansion in the geometric series, before the limit $\tau \to \infty$ is taken.

3. *The one-dimensional Lee-like model*

The next simple model we shall consider by the same method is offered by the Lee-like Hamiltonian describing an interaction of the fermions V and N interacting with Θ-bosons. We shall assume that the relevant operators (creation and destruction) do not depend upon the space variables so the particles do not move and are located at the origin. The Hamiltonian of the system reads

$$H = H_0+H_1, \tag{3.1}$$

where

$$H_0 = m_V V^+V+m_N N^+N+m_\Theta\Theta^+\Theta, \tag{3.2}$$

and

$$H_1 = g(V^+N\Theta+N^+\Theta^+V). \tag{3.3}$$

The nontrivial commutation relations are

$$[\Theta,\Theta^+] = \{V, V^+\} = \{N, N^+\} = 1,$$

$$[V,\Theta] = [N,\Theta] = \{V, N\} = 0. \tag{3.4}$$

We construct the Hilbert space of states in the standard way. Let $|0\rangle$ be a normalized state — a vacuum — defined as usually

$$V|0\rangle = N|0\rangle = \Theta|0\rangle = 0. \tag{3.5}$$

It is clear that it is also an eigenstate for the total Hamiltonian to the zero eigenvalue, so it is a physical vacuum. The following "charges" are conserved in the model

$$Q_1 = V^+V+N^+N, \tag{3.6}$$

$$Q_2 = V^+V+\Theta^+\Theta, \tag{3.7}$$

$$[Q_1, H] = [Q_2, H] = [Q_1, Q_2] = 0. \tag{3.8}$$

Hence, the Hilbert space of states may be decomposed into the direct sum of sectors having definite values of Q_1 and Q_2

$$\mathscr{H} = \bigoplus_{q_1=0}^{2} \bigoplus_{q_2=0}^{\infty} \mathscr{H}(q_1, q_2), \tag{3.9}$$

where $\mathscr{H}(q_1, q_2)$ consists of vectors of the form

$$\mathscr{H}(q_1, q_2) \ni |q_1, q_2; \psi\rangle = \sum_{p=\max(0,q_1-1)}^{\min(1,q_1)} \psi(p, q_1-p, q_2-p)$$

$$\times \frac{1}{\sqrt{p!(q_1-p)!(q_2-p)!}} (V^+)^p (N^+)^{q_1-p} (\Theta^+)^{q_2-p}|0\rangle. \tag{3.10}$$

The first nontrivial sector is $\mathscr{H}(1,1)$ sector to which we shall confine our attention in what follows. A typical element from this sector is

$$\mathscr{H}(1, 1) = \mathscr{H}_V \oplus \mathscr{H}_{N\Theta} \ni |\psi_V, \psi_{N\Theta}\rangle$$

$$= \psi_V V^+|0\rangle + \psi_{N\Theta} N^+ \Theta^+|0\rangle = \psi_V|V\rangle + \psi_{N\Theta}|N\Theta\rangle. \tag{3.11}$$

We are interested in the time evolution of the V-particle state $V^+|0\rangle$. Therefore, we introduce the projector P acting in the sector $\mathscr{H}(1,1)$ as follows

$$P = |V\rangle\langle V| = V^+|0\rangle\langle 0|V, \; Q = 1-P, \tag{3.12}$$

$$P|\psi_V, \psi_{N\Theta}\rangle = |\psi_V, 0\rangle = \psi_{\|}, \tag{3.13}$$

$$Q|\psi_V, \psi_{N\Theta}\rangle = |0, \psi_{N\Theta}\rangle = \psi_{\perp}. \tag{3.14}$$

One easily finds, using the above definitions, that the operator $K(t)$ is given by

$$K(t) = \Theta(t) g^2 \exp[-it(m_N+m_\Theta)]P = K_V(t)\cdot P. \tag{3.15}$$

So the equation for the wave function ψ_V takes on the form

$$\left(i\frac{d}{dt} - m_V\right)\psi_V(t) = -i\int_{t_0}^{\infty} d\tau K_V(t-\tau)\psi_V(\tau). \tag{3.16}$$

Comparing the functions $K_V(t)$ and $K_1(t)$ of the previous section one sees that they coincide upon the following identifications

$$g^2 = |H_{12}|^2, \quad m_N+m_\Theta = H_{22}, \quad m_V = H_{11}. \tag{3.17}$$

Hence, it is possible to use the result (2.43), for the quasipotential in the second order approximation

$$V_V = g^2 V_V^{(2)} + O(g^4) = \frac{g^2}{m_V - m_N - m_\Theta + i\varepsilon} + O(g^4)$$

$$= \mathscr{P}\frac{g^2}{m_V - m_N - m_\Theta} - ig^2\pi\delta(m_V - m_N - m_\Theta) + O(g^4). \tag{3.18}$$

It may be verified, using the asymptotic formulae [10]

$$\lim_{t\to\infty} \frac{\sin xt}{x} = \pi\delta(x), \quad \lim_{t\to\infty} \frac{\cos xt}{x} = 0, \tag{3.19}$$

that the $V_V^{(4)}$ correction to the quasipotential is simply

$$V_V^{(4)} = \frac{1}{(m_V - m_N - m_\Theta + i\varepsilon)^3} \tag{3.20}$$

$$= \tfrac{1}{2} \frac{\partial^2}{\partial \xi^2} \left[\mathscr{P} \frac{1}{\xi} - i\pi\delta(\xi) \right] \Bigg|_{\xi = m_v - m_N - m_\theta} \tag{3.21}$$

After the above preparations we are in a position to discuss the usual Lee model which provides a more physical picture of a decay phenomenon.

4. *The Lee model*

This model describes two spinor particles V and N interacting through spinless boson Θ-particles according to the Hamiltonian [7–9]

$$H = H_0 + H_1, \tag{4.1}$$

where

$$H_0 = m_V \int d^3\vec{p}\, V^+(\vec{p}) V(\vec{p}) + m_N \int d^3\vec{p}\, N^+(\vec{p}) N(p) + \int d^3\vec{k}\, \omega(\vec{k}) a^+(\vec{k}) a(\vec{k}) \tag{4.2}$$

and

$$H_1 = \frac{\lambda}{(2\pi)^{3/2}} \int d^3\vec{k}\, \frac{f[\omega(\vec{k})]}{\sqrt{2\omega(\vec{k})}} \int d^3\vec{p} [V^+(\vec{p}) N(\vec{p} - \vec{k}) a(\vec{k}) + \text{h.c.}]. \tag{4.3}$$

Spinor particles V and N are static while the Θ-particles, associated with the a^+, a operators, are relativistic one with energies

$$\omega(\vec{k}) = \sqrt{\vec{k}^2 + \mu^2}. \tag{4.4}$$

The real cut-off function $f[\omega(\vec{k})]$ and the coupling constant λ together with bare masses m_V, m_N and μ are parameters of the model. The nontrivial commutation relations are

$$\begin{aligned} [a(\vec{k}), a^+(\vec{k}')] &= \delta(\vec{k} - \vec{k}'), \\ \{N(\vec{p}), N^+(\vec{p}')\} &= \delta(\vec{p} - \vec{p}'), \\ \{V(\vec{p}), V^+(\vec{p}')\} &= \delta(\vec{p} - \vec{p}') \\ \{N(\vec{p}), V(\vec{p}')\} &= 0. \end{aligned} \tag{4.5}$$

Similarly as in the one-dimensional Lee model considered before we are interested in the sector $\mathscr{H}$ (1,1) formed by the vectors

$$|\psi_V, \psi_{N\Theta}\rangle = \int d^3\vec{p}\psi_V(\vec{p})V^+(\vec{p})\,|0\rangle$$

$$+ \int d^3\vec{p}d^3\vec{q}\,\psi_{N\Theta}(\vec{p}, \vec{q})N^+(\vec{p})a^+(\vec{q})\,|0\rangle = |\psi_V, 0\rangle + |0, \psi_{N\Theta}\rangle, \tag{4.6}$$

where $\psi_V(\vec{p})$ and $\psi_{N\Theta}(\vec{p}, \vec{q})$ are both square integrable with respect to $\vec{p}$ and $\vec{p}$, $\vec{q}$ respectively, which are complex functions depending on time. This sector is determined by the conserved charges Q_1, Q_2

$$\hat{Q}_1 = \hat{n}_N + \hat{n}_V, \tag{4.7}$$

$$\hat{Q}_2 = \hat{n}_\Theta + \hat{n}_V, \tag{4.8}$$

where $\hat{n}_V$ is the V-particle number operator, $\hat{n}_\Theta$ is the Θ-particle number operator, and so on. Both charges have the value 1 on the vectors (4.6) from the sector $\mathscr{H}(1.1)$. One sees from the formula (4.6) that this sector splits into the orthogonal sum of two subspaces

$$\mathscr{H}(1, 1) = \mathscr{H}_V \oplus \mathscr{H}_{N\Theta}, \tag{4.9}$$

with both parts having the obvious meaning. Therefore, it is reasonable to select the subspace $\mathscr{H}_V$ as the projecting subspace when treating the V-particle decay problem. Hence, we define the projector P in the following way

$$P|\psi_V, \psi_{N\Theta}\rangle = |\psi_V, 0\rangle, \tag{4.10}$$

$$Q|\psi_V, \psi_{N\Theta}\rangle = (1-P)\,|\psi_V, \psi_{N\Theta}\rangle = |0, \psi_{N\Theta}\rangle. \tag{4.11}$$

The kernel $K(t)$ (for the definition see (2.9)) may be found rather easily and reads

$$K(t) = \Theta(t) \int d\Omega(\vec{k}) e^{-it[m_N + \omega(\vec{k})]} P \equiv K_V(t) \cdot P, \tag{4.12}$$

where the measure $d\Omega(\vec{k})$ is given by the formula

$$d\Omega(\vec{k}) = \frac{\lambda^2}{(2\pi)^3} \frac{f^2[\omega(\vec{k})]}{2\omega(\vec{k})} d^3\vec{k}. \tag{4.13}$$

Therefore, the problem is quite similar, in principle, to those considered before. New complications in comparison with previously considered cases come from integration over the momenta. We get the following integro-differential equation for the wave function $\psi_V(t, \vec{p})$

$$\left(i\frac{d}{dt} - m_V\right)\psi_V(t, \vec{p}) = -i\int_{t_0}^{\infty} d\tau K_V(t-\tau)\psi_V(\tau, \vec{p}). \tag{4.14}$$

Having $K_V(t)$ we may calculate the quasipotential V_V in the second order of approximation in the coupling constant λ

$$V_V = -i \int_0^\infty d\tau K_V(\tau) e^{i\tau(m_V+i\varepsilon)} + O(\lambda^4)$$

$$= \int \frac{d\Omega(\vec{k})}{m_V - m_N - \omega(\vec{k}) + i\varepsilon} + O(\lambda^4)$$

$$= \mathrm{Re}\, V_V - \frac{i}{2} \Gamma_V. \tag{4.15}$$

For the decay parameter Γ_V we get after performing the integration over $\vec{k}$

$$\Gamma_V = \frac{\lambda^2}{2\pi} f^2(\omega_0)\Theta(\omega_0-\mu)\sqrt{\omega_0^2-\mu^2}+O(\lambda^4), \quad \omega_0 = m_V - m_N. \tag{4.16}$$

This result coincides, in the approximation considered, with that derived by Glaser and Källen using different methods (cf. [5] formulae (13a) and (27a)).

In a future paper we are going to undertake the investigation of radiation damping in atomic systems using the methods described here.

REFERENCES

[1] W. Królikowski, J. Rzewuski, *Nuovo Cimento* **25B** 739 (1975); see also *Nuovo Cimento* **2**, 203 (1955), **3**, 260 (1956), **4**, 975 (1956), **4**, 1212 (1956).

[2] W. Garczyński, K. Urbanowski, *On a Quasipotential Governing the Time Evolution of a Projection of State — Vector Onto One — Dimensional Subspace.* Preprint No 380, ITP University of Wrocław, 1976.

[3] W. Garczyński, *Phys. Lett.* **56A**, 434 (1976).

[4] W. Garczyński, *An Approach to a Problem of Description of Unstable Quantum Systems*, Lecture at XIII Winter School of Theoretical Physics in Karpacz, 1976 Proceedings.

[5] V. Glaser, G. Källen, *Nucl. Phys.*, **2**, 706 (1956/57).

[6] W. Garczyński, *Nonhermicity of a Hamiltonian and Regeneration Effect in a Theory of Unstable Quantum Systems* , Preprint No 348, ITP, University of Wrocław 1975.

[7] T. D. Lee, *Phys. Rev.* **95**, 1329 (1954).

[8] G. Källen, W. Pauli, *Dan. Mat. Fys. Medd.* **30** No 7 (1955).

[9] S. Schweber, *An Introduction to Relativistic Quantum Field Theory*, New York 1961.

[10] V. S. Vladimirov, *Uravnenija matematicheskoj fiziki*, Moscow 1971 (in Russian).

[11] J. Jersak, *Sov. Tourn. of Nucl. Phys.* **9**, 458 (1969).

[12] L. P. Horwitz, T. P. Marchand, *Rocky Mountain J. of Math.* **1**, 225 (1974).

[13] V. F. Weisskopf, E. P. Wigner, *Z. Phys.* **63**, 54 (1930), **65**, 18 (1930).

[14] H. Ekstein, A. T. Siegert, *Ann. Phys.* **68**, 521 (1971).

[15] L. Fonda et al., *Nuovo Cimento* **15A**, 689 (1973).

[16] R. Zwanzig, *Physica* **30**, 1109 (1964).

[17] H. Mori, *Prog. Theor. Phys.* **33**, 423 (1965), **34**, 399 (1965).