ANALYSIS OF THE $^9Be(d, \alpha_0)^7Li_{g.s.}$ AND $^9Be(d, \alpha_1)^7Li^*$ (470 keV) REACTIONS IN TERMS OF THE TWO-NUCLEON DISTORTED-WAVE BORN APPROXIMATION

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Absolute differential and total cross sections for the $^9Be(d, \alpha)^7Li$ reactions leading to the ground and first excited (470 keV) states of ^7Li have been measured for bomdarding energies of 2.25 to 3.1 MeV. An analysis of these reactions for energies of 0.9 to 3.1 MeV has been performed in terms of the distorted-wave Born approximation (DWBA) for the two-nucleon transfer. The possible reasons of the discrepancies between theory and experiment are discussed.

1. Introduction

Analysis of the differential cross sections and excitation curves of (d, α) reactions on light nuclei aimed to determine the reaction mechanism at low deuteron energies has not hitherto given unique results. The occurence of different direct reaction mechanisms such as pick-up, heavy particle pick-up, consecutive transfer of two nucleons (see Refs [1–4]) has been suggested in addition to the traditionally considered compound nucleus

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formation. Shapiro's dispersion theoretic approach [5] allows to make some general conclusions on the relative importance of the direct processes based on the location of their kinematical singularities.

In the present work the angular distributions for the ^{9}Be(d, α_0)^{7}Li and ^{9}Be(d, α_1)^{7}Li* (470 keV) reactions are described in the energy range from 2.25 to 3.1 MeV. This extends our previous work [6] in which the interval from 0.9 to 2.2 MeV was covered. The singularities corresponding to the two-nucleon pick-up and heavy particle pick-up lie closest to the physical region for these two reactions. We will limit ourselves to the DWBA analysis of the two-nucleon pick-up in the present work. There exists only a few (d, α) works in which the DWBA predictions for the direct pick-up have been compared with the experimental data in the MeV range. A strong argument that such an analysis is worthwhile is that the shape of the angular distribution depends not only on the angular momentum transfer but also on the phase relationships between the wave functions of the transferred pair. The lack of a universally determined shape, as opposed to the single nucleon transfer, compels to make a complete DWBA analysis to judge whether the direct interaction picture is valid.

The angular distributions of ^{9}Be(d, α)^{7}Li have been also measured by Friendland et al. [7] in the energy range from 0.6 to 2.2 MeV. No attempts to compare them with the DWBA predictions have been made, however.

2. Experimental results

Details of the experimental arrangement used in this work were presented in our previous publication [6], thus only the most salient features will be repeated here.

Deuterons have been accelerated in the 3.5 MeV Van de Graaff accelerator of the Institute of Nuclear Research. The bombarding energy has been established with a calibrated magnetic analyser at the accelerator exit with a precision not worse than ±5 keV. The measurements of the angular distributions were carried out with the 30 cm scattering chamber and two silicon surface-barrier detectors. One of them was rotated to measure the angular distributions while the second one served for monitoring.

Targets were prepared by vacuum evaporation of spectroscopically pure beryllium metal on thin organic backings. The average thickness of the beryllium layer was 40 μg/cm². The absolute values of the reaction cross sections have been established using the known (Ref. [8]) ^{9}Be (d, d) ^{9}Be elastic scattering cross sections. For this purpose a complete spectrum at 150° containing the elastic scattering peaks has been measured at each energy.

The angular distributions in the center-of-mass system are presented in Figs 1 and 2. The solid lines passing through the experimental points represent the Legendre-polynomial fits. The total cross sections, presented in Fig. 3, are calculated from the coefficient of the order zero. The data of other authors [9, 10] are presented in Fig. 3 for comparison.

Only the statistical errors of the measurements are plotted in the figures. They do not exceed 3% in most cases. The error of absolute normalization is 5%. This is the error of the absolute values of the ^{9}Be(d, d)^{9}Be cross sections taken from Ref. [8].

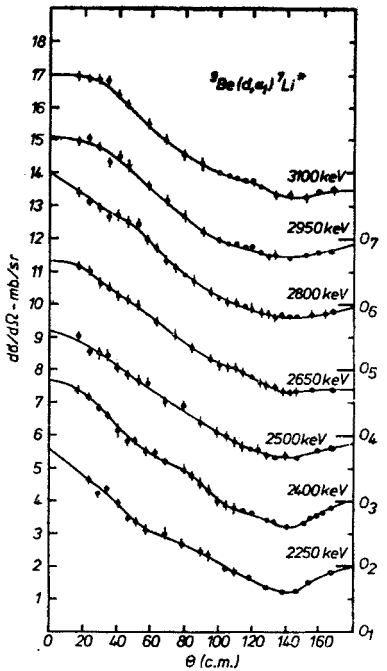

Fig. 1

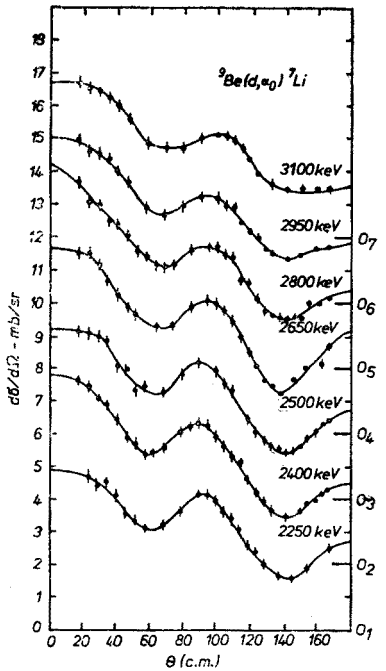

Fig. 2

Fig. 1. Angular distributions for the $^9Be(d, \alpha_0)^7Li$ (g. s.) reaction in the 2.25–3.1 MeV energy range. Solid lines passing through the experimental points represent the Legendre polynomial fits

Fig. 2. Angular distributions for $^9Be(d, \alpha_1)^7Li$(470 keV) reaction in the 2.25–3.1 MeV energy range. Solid lines passing through the experimental points represent the Legendre polynomial fits

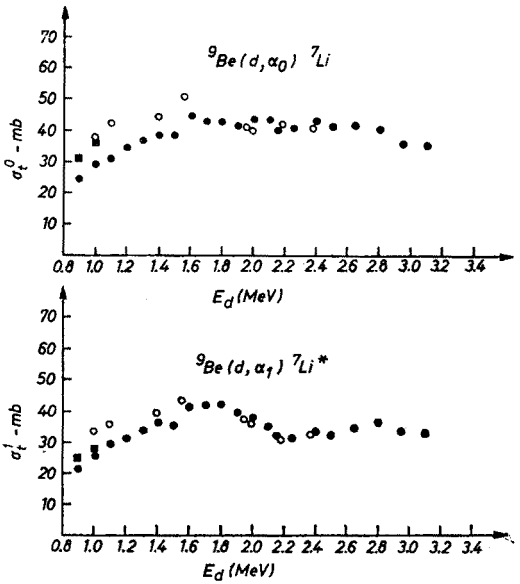

Fig. 3. Total $^9Be(d, \alpha_0)^7Li$(g. s.) and $^9Be(d, \alpha_1)^7Li$(470 keV) cross sections in the 0.9–3.1 MeV energy range (● our results, ○ Biggerstaff et al. [9], ■ Bertrand et al. [10]. Statistical errors do not exceed the dimension of data points

3. Theoretical analysis

The calculations of the DWBA cross sections were made in the zero-range approximation as outlined by Bassel et al. in Ref. [11].

Assuming that in the reaction $A(\mathrm{d}, \alpha)B$ a pair of nucleons with the total spin $S = 1$ and total isospin $T = 0$ is picked-up from the nucleus A, we write the differential cross section in the form:

$$\frac{d\sigma}{d\Omega} \propto \sum_{LJ} \sum_{M} \left| \sum_{l_a l_b} \Gamma^{LM}_{l_a l_b} f^{LJ}_{l_a l_b} P^{M}_{l_b}(\vartheta) \right|^2, \tag{1}$$

where L, J are the orbital and total momenta of the transferred pair; l_a, l_b are the orbital momenta of the distorted waves in the entrance (deuteron) and exit (alpha) channel, respectively. Spin-independent distortion in the entrance channel is assumed. $P^{M}_{l_b}$ is the associated Legendre function. The coefficients $\Gamma^{LM}_{l_a l_b}$ are defined by Eq. (16) of Ref. [11], $f^{LJ}_{l_a l_b}$ denotes the radial matrix element:

$$f^{LJ}_{l_a l_b} = \frac{k_a}{k_b} \int \chi_{l_b}(k_b r) F_{LJ}(r) \chi_{l_a}\left(k_a \frac{M_B}{M_A} r\right) dr. \tag{2}$$

This contains the radial parts of the wave functions describing the relative motion in the entrance $\chi_{l_a}(r)$ and exit channel $\chi_{l_b}(r)$; k_a, k_b are the corresponding wave vectors; M_A, M_B are the masses fo the target and residual nuclei, respectively. The two-nucleon form factor $F_{LJ}(r)$ was taken in the form suggested by Glendenning [12]:

$$F_{LJ}(r) = \sum_{N} G_{NLJ} u_{NL}(2\nu r^2), \tag{3}$$

where $u_{NL}(r)$ are the radial functions of the relative motion of the pair and the residual nucleus. In Glendenning's approach these are the harmonic oscillator wave functions with the principal quantum number N and orbital momentum L. The frequency parameter $\nu = 0.517\ \mathrm{fm}^{-2}$. This corresponds to the widely accepted prescription $\hbar\omega = 41\, A^{-1/3}$ for the oscillator frequency parameter ω. To improve the asymptotics the oscillator wave functions were matched to the Coulomb wave functions corresponding to the actual binding energy ($B_{\mathrm{d}} = 16.68$ MeV) of the deuteron on ^{9}Be nucleus. The functions thus constructed were subsequently normalized to unity. In the asymptotic region, the functions u_{NL} have sings $(-1)^{N+1}$. Thus whether the superposition of three of them yields cancellation or addition in the important for the reaction surface region of the nucleus depends on the magnitudes and signs of the structure amplitudes G_{NLJ}. This differs the two-nucleon transfer from the single-nucleon one, since in the latter the kinematic and structure factors are disentangled.

The structure amplitudes G_{NLJ} have been calculated according to Towner and Hardy [13]:

$$G_{NLJ} = C_2 S^{1/2}_{AB}(l_1 l_2; LSJT) \langle n0, NL; L | n_1 l_1, n_2 l_2; L \rangle, \tag{4}$$

where C_2 is a constant independent of the NLJ quantum numbers. Talmi's coefficient [14] $\langle n0, NL; L|n_1l_1, n_2l_2; L\rangle$ transforms from the quantum numbers n_1l_1, n_2l_2 describing the shell model orbitals occupied by the two nucleons in the nucleus A to the quantum numbers $n\lambda$, NL characterising the relative and center-of-mass motion of the pair. Only the relative s-state ($\lambda = 0$) is taken into account. The energy and angular momentum conservation rules implied by the Talmi bracket for two $1p$-shell nucleons allow the following N, L combinations: $N = 1, L = 0$ (when $n = 2$), $N = 2$, $L = 0$ and $N = 1$, $L = 2$ (when $n = 1$).

The parentage overlap fator $S_{AB}^{1/2}$ measures the extent to which the ground state of A looks like B in a given final state plus two nucleons on the orbitals $n_1\, l_1$, n_2l_2 (coupled to LSJ). Its square is proportional to the two-nucleon spectroscopic factor. We have used the wave functions for the $1p$ shell nuclei given by Boyarkina [15] in order to calculate the $S_{AB}^{1/2}$ factors. These wave functions are given as linear combinations of the $L-S$ basis: $[\,]L_x^{2S_x+1,2T_x+1}$. The bracket [ ] denotes the permutational symmetry (Young pattern) of the wave function; L_x, S_x and T_x are the total orbital momentum, total spin and total isospin of a given nucleus ($x = A, B$).

The particular wave functions used by us had the following form

$$^7\text{Li(g.s.)}\ (\tfrac{3}{2}^-)$$

$$0.986\ [3]\ ^2\text{P} - 0.022\ [21]\ ^2\text{P} - 0.134\ [21]\ ^4\text{P} \qquad (5a)$$
$$-0.063\ [21]\ ^2\text{D} - 0.067\ [21]\ ^4\text{D},$$

$$^7\text{Li (470 keV)}\ (\tfrac{1}{2}^-)$$

$$0.983\ [3]\ ^2\text{P} + 0.153\ [21]\ ^2\text{P} - 0.083\ [21]\ ^4\text{P} \qquad (5b)$$
$$-0.065\ [21]\ ^4\text{D} - 0.014\ [111]\ ^2\text{S},$$

$$^9\text{Be(g.s.)}\ (\tfrac{3}{2}^-)$$

$$0.899\ [41]\ ^2\text{P} - 0.387\ [41]\ ^2\text{D} + 0.121\ [32]\ ^4\text{P} \qquad (5c)$$
$$+0.111\ [32]\ ^2\text{D}.$$

The calculated values of the G_{NLJ} factors for both reactions are listed in Table I. The distorted-wave cross section Eq. (1) was calculated with the Copengahen code GAP-2 and GIER computer. The optical-model potential used to calculate the distorted waves had the form:

$$V(r) = V_C(r) - V_r f(r, r_r, a_r) - i4a_i W_i (d/dr) f(r, r_i, a_i), \qquad (6a)$$

where

$$f(r, r_r, a_r) = \{1 + \exp[(r - r_r A^{1/3})/a_r]\}^{-1}. \qquad (6b)$$

The Coulomb potential $V_C(r)$ is that of a uniformly charged sphere with the radius $1.3\, A^{1/3}$ fm.

TABLE I

Structure amplitudes G_{NLJ}

Reaction	J	L	$N = 1$	$N = 2$
^{9}Be(d, α_0)^{7}Li g.s.	1	0	0.01	0.07
	1	2	0.11	
	2	2	0.12	
	3	2	−0.45	
^{9}Be(d, α_1)^{7}Li*	1	0	−0.02	−0.20
	1	2	0.15	
	2	2	−0.25	

TABLE II

Optical model parameters

Particle	Type	V_r [MeV]	r_r [fm]	a_r [fm]	W_i [MeV]	r_i [fm]	a_i [fm]	Ref.
Deuteron	A	114.2	0.869	1.01	16	2.16	0.323	16
	B	95.78	1.15	0.81	14.95	1.856	0.370	16
α particale	a	228.4	0.869	1.01	16	2.160	0.323	17
	b	75	2.165	0.50	10	2.165	0.20	18
	c	200	1.970	0.60	4	1.970	0.30	19

The parameters of the optical potentials (6) used in the calculations are collected in Table II. The deuteron potentials have been established by us previously. The potential of type "A" is an average potential giving an overall fit [16] to the ^{9}Be(d, d)^{9}Be scattering in the energy range from 0.9 to 2.5 MeV. The parameters of type "B" give best fit to the elastic scattering angular distribution at 2.5 MeV.

The real depth of the alpha type "a" potential was taken as twice the real depth of the deuteron type "A" potential. All the other parameters were left unchanged. Similar procedure has given good results in another kinematically unmatched case that of the ^{50,52,54}Cr (^{3}He, α)^{49,51,53}Cr reactions investigated by Stock et al. [17]. The potential of type "b" results from the parameter searches on ^{6}Li(α, α)^{6}Li scattering at $E_\alpha = 12.5$ MeV (Blieden et al. [18]) while that of type "c" from ^{12}C(α, α)^{12}C at $E_\alpha = 11.0$ MeV (Carter et al. [19]).

4. Discussion

The combinations of all pairs of parameters listed in Table 2 have been tested. The conclusion is that the shapes and magnitudes of the cross sections depend sensitively on the choice of the α-potential. This is because the large binding energy of the deuteron

makes the interior contributions to the reaction matrix elements relatively important. The most satisfactory results were obtained with the (b, B) pair of potentials. These are shown in Fig. 4. The calculated angular distributions are normalized to the data with a unique normalization factor Λ (shown in the figure) for all energies. For the (a, A) and (c, C) combinations a strong increase of the calculated cross sections towards backward

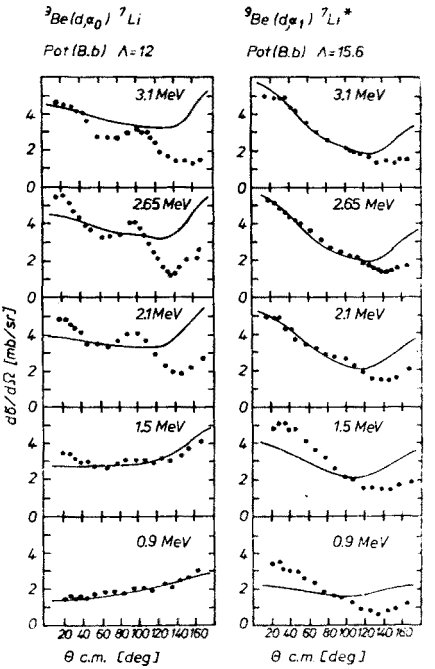

Fig. 4. Comparison of the experimental and DWBA angular distributions for the $^9Be(d, \alpha_0)^7Li$ and $^9Be(d, \alpha_1)^7Li$ reactions (for the parameters of (B, b) potential see Table II in the text)

angles is observed. One may hope to improve the agreement for $^9Be(d, \alpha_1)^7Li$ reaction at lowest energies when better α-potentials are available.

What differs the experimental angular distributions of these two reactions is the presence of a bump around an angle of 95° in the angular distributions of $^9Be(d, \alpha_0)^7Li$ above 1.5 MeV. It would be tempting to associate these differences with different proportions of $L = 0$ and $L = 2$ transfer, which are evident from Table I. The calculated angular distributions for $^9Be(d, \alpha_0)^7Li$ do not show any indication of a maximum, however, being even more flat than those for $^9Be(d, \alpha_1)^7Li$.

5. *Conclusions*

The angular distributions for the $^9Be(d, \alpha_0)^7Li$ and $^9Be(d, \alpha_1)^7Li$ reactions in the energy range from 0.9 to 3.1 MeV have been compared with the DWBA theory for the two-nucleon pick-up. The wave functions of Boyarkina [15] have been used to calculate the structure amplitudes. A satisfactory agreement between the experiment and theory

has been obtained over a limited range of energies for the ^{9}Be(d, α_1)^{7}Li reaction. This can be connected with the insufficient knowledge of the α-potentials in the domain of low energies and light nuclei. Since the angular distributions depend sensitively on the choice of the distorting potentials, better α-potentials are needed to correct for this defficiency. The characteristic differences between the angular distributions for the ^{9}Be(d, α_0)^{7}Li and ^{9}Be(d, α_1)^7 Li reactions have not been reproduced. The origin of the discrepancies may lay in both structure factors and reaction mechanism. Evidence is put forward [20] on substantial separation of clusters in ^{9}Be and ^{7}Li, so that the shell model picture on which our wave functions are based may be of limited validity. As to the reaction mechanism the origin of the discrepancies may be twofold. The reaction ^{9}Be(d, α)^{7}Li is asymmetric with respect to the binding energies of neutron (B_n = 1.67 MeV) and proton (B_p = 16.87 MeV) in ^{9}Be. It has been argued by Bang, Neudachin et al. [21] that the successive transfer may be a preferred reaction mechanism when the wave function of the two nucleons poorly overlap due to large differences in their binding energies.

Finally, evidence has been put forward in our previous publication [22] on the participation of the giant dipole T = 1/2 resonance in the ^{9}Be(d, $p_{0,\,1}$)^{10}Be and ^{9}Be(d, t_0)^{8}Be reactions. The resonance was suggested to be centered around the energy E_d = 1.85 MeV (E_x(^{11}B*) = 17.3 MeV) where the maximum of the ^{9}Be(d, γ)^{11}B capture strength was observed [23]. It would be worth-while to extend the present measurements to higher bombarding energies. The discrepancies between DWBA and experiment should decrease with increasing energy in a way correlated with the decreasing ^{9}Be(d, γ)^{11}B capture strength if the T = 1/2 giant-dipole resonance is responsible for them also in the ^{9}Be(d, α)^{7}Li case.

REFERENCES

[1] M. N. H. Comsan, M. A. Farouk, A. A. El-Kamhawy, M. S. M. El-Tahawy, A. N. Lvov, *Atomkernenergie* **13**, 415 (1968).

[2] J. Nalda, T. Barriuso, J. Casanova, *Nucl. Phys.* **A149**, 350 (1970).

[3] I. I. Zalubovskij, A. P. Lobkovskij, G. L. Vysockij, *Izv. Akad. Nauk SSSR, Ser. Fiz.* **33**, 2057 (1969).

[4] S. W. Cosper, B. T. Lucas, O. E. Johnson, *Phys. Rev.* **139**, B763 (1965).

[5] I. S. Shapiro, in *Selected Topics in Nuclear Theory*, ed. International Atomic Energy Agency, Vienna 1963.

[6] A. Saganek, I. Śledzińska, A. Turos, Z. Wilhelmi, B. Zwięgliński, *Acta Phys. Pol.* **B2**, 473 (1971).

[7] E. Friedland, H. W. Alberts, J. C. von Staden, *Z. Phys.* **267**, 97 (1974).

[8] F. Machali, Z. A. Saleh, A. T. Baranik, F. Asfour, I. Boundouk, V. E. Storizhko, *Atomkernenergie* **13**, 29 (1968),

[9] J. A. Biggerstaff, R. F. Hood, H. Scott, T. McEllistrem, *Nucl. Phys.* **36**, 631 (1962).

[10] F. Bertrand, G. Grenier, J. Pornet, Report CEA-R-3504 (1968).

[11] R. H. Bassel, R. M. Drisko, G. R. Satchler, Report ORNL-3240 (1969).

[12] N. K. Glendenning, *Phys. Rev.* **137**, B102 (1965).

[13] I. S. Towner, J. S. Hardy, *Adv. Phys.* **18**, 401 (1969).

[14] T. A. Brody, M. Moshinsky, *Tables of Transformation Brackets*, Monografies del Instituto de Fisica, Mexico 1960.

[15] A. N. Boyarkina, *Izv. Akad. Nauk SSSR*, Ser. Fiz. **28**, 337 (1964).

[16] B. Zwięgliński, J. Piotrowski, A. Saganek, I. Śledzińska, *Nucl. Phys.* **A209**, 348 (1973).

[17] R. Stock, R. Bock, P. David, H. M. Duhm, T. Tamura, *Nucl. Phys.* **A104**, 136 (1967).
[18] H. R. Blieden, G. M. Temmer, K. L. Warsh, *Nucl. Phys.* **49**, 209 (1963).
[19] E. B. Carter, G. E. Mitchell, H. Davis, *Phys. Rev.* **133**, B1427 (1964).
[20] J. A. Kudeyanov, Z. Matthies, W. G. Nieudatchin, J. F. Smirnov, *Nucl. Phys.* **65**, 529 (1965).
[21] Iens Bang, N. S. Zelenskaya, E. Zh. Magzumov, V. G. Neudachin, *Yad. Fiz.* **4**, 962 (1968).
[22] B. Zwięgliński, A. Saganek, I. Śledzińska, Z. Wilhelmi, *Nucl. Phys.* **A250**, 93 (1975).
[23] W. Del Bianco, S. Kundu, B. Rouben, *Nucl. Phys.* **A232**, 333 (1974).