MULTI TEMPERATURE MODELS OF INCLUSIVE SPECTRA

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It is possible to fit the single particle inclusive spectrum by a suitable temperature distribution. We discuss the resulting two-particle spectrum in this model and show that it contains the observed correlations.

1. *Introduction*

Large p_T inclusive data are generally thought of in terms of hard q–q scattering processes and detailed models are available to fit the rapidly accumulating amount of data. Nevertheless there is still interest in the older and simpler statistical models. Justification for this continued interest would include the following points:

(*i*) they describe many of the general features of inclusive processes, particularly at small p_T, in a simple way.

(*ii*) there is evidence that large p_T and small p_T processes are not really different and that the same model should describe both — certainly there must be a smooth joining between "large" and "small" p_T.

(*iii*) it is important to know what features of the data can be described by purely statistical arguments, together with momentum conservation, and what features require detailed dynamics.

On the other hand we know that statistical models have many serious inadequacies:

(a) they ignore momentum conservation,

(b) they fail completely in the longitudinal direction,

(c) a single temperature distribution gives too small a cross-section for large p_T.

In this work we consider the "multi-temperature" distributions which have been proposed by several authors, e.g. Benecke et al. [1, 2] Gorenstein et al. [3], to solve problem (c). Since, as explained below, any $d\sigma/dp_T$ can be fitted by a suitable temperature

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distribution, such a fit is not significant unless either the parameters can be related to other theoretical ideas (as, for example, in Ref. [4]) or other predictions can be made [2]. We shall concern ourselves here with what can be said about the correlations. The single particle distribution depends on the product of the cross-section at a given temperature and the multiplicity at a given temperature, whereas in the two-particle distribution we require the product of the cross-section and the square of the multiplicity. The single particle distribution does not therefore determine the two particle distribution but, as we shall see, it provides constraints which can in principle be tested experimentally.

In the next section we define the measured quantities and express them in terms of temperature dependent cross-sections and multiplicities, thereby obtaining some general predictions of the model. Section 3 is devoted to a discussion of the correlations in the model of Fröyland [4]. In Section 4 we show how the two particle distribution can be related directly to the single particle distribution if we take a simple form for the multiplicity. In fact a multiplicity which is independent of temperature gives an adequate fit to the available data. The last section summarises the conclusions.

2. *The model*

The one-pion inclusive crosss-section is defined by

$$f(p_T) \equiv E\frac{d^3\sigma}{dp^3} \equiv \frac{d^3\sigma}{p_T dp_T d\phi dy} \equiv \frac{1}{2\pi}\frac{d^2\sigma}{p_T dp_T dy}. \tag{2.1}$$

We shall work at y = 0 and will ignore the y-variable. We shall denote p_T simply by p. The assumption that $f(p)$ is a sum of fixed temperature distributions is expressed by writing

$$f(p) = \int_0^\infty d\lambda \sigma(\lambda) g_\lambda(p), \tag{2.2}$$

where

$$g_\lambda(p) = a(\lambda)e^{-\lambda p}. \tag{2.3}$$

Here λ is the inverse temperature and $\sigma(\lambda)$ is the cross-section for producing a temperature λ^{-1}. We have ignored the pion mass which makes no difference except for very small p. The multiplicity at fixed λ is given by

$$n(\lambda) = \int_0^\infty p dp g_\lambda(p) = \frac{a(\lambda)}{\lambda^2}, \tag{2.4}$$

and the total inelastic cross-section by

$$\sigma^{\text{inel}} = \int_0^\infty \sigma(\lambda) d\lambda. \tag{2.5}$$

Note that (2.4) requires $a(\lambda) = O(\lambda^2)$ for small λ; if $a(\lambda)/\lambda^2$ is constant we have a multiplicity independent of temperature.

From (2.2) and (2.4) we have

$$f(p) = \int_0^\infty d\lambda \sigma(\lambda) a(\lambda) e^{-\lambda p}, \tag{2.6}$$

i.e. $f(p)$ is the Laplace transform of the product $\sigma(\lambda)a(\lambda)$. Given any experimental $f(p)$ we can determine the product $\sigma(\lambda)a(\lambda)$ from (2.6). In general all these quantities will be energy dependent, but we shall work at fixed energy and make no attempt to parameterise the energy dependence.

The two-pion inclusive cross-section is defined by

$$f(p_1, p_2) = \frac{d^6\sigma}{(p_1 dp_1 d\phi_1 dy_1)(p_2 dp_2 d\varphi_2 dy_2)} . \tag{2.7}$$

We shall always consider 1 and 2 to have different charges. At a given value of λ we assume there are no correlations (this is essentially the content of (2.3)), so

$$f(p_1, p_2) = \int_0^\infty d\lambda \sigma(\lambda) f_\lambda(p_1) f_\lambda(p_2) = \int_0^\infty d\sigma \sigma(\lambda) a(\lambda)^2 e^{-\lambda(p_1+p_2)}, \tag{2.8}$$

i.e. $f(p_1, p_2)$ is the Laplace transform of $\sigma(\lambda)a(\lambda)^2$.

Some constraints on $f(p_1, p_2)$ follow immediately. First, we note that it is independent of the azimuthal directions of pions 1 and 2. This is to be expected in a model which

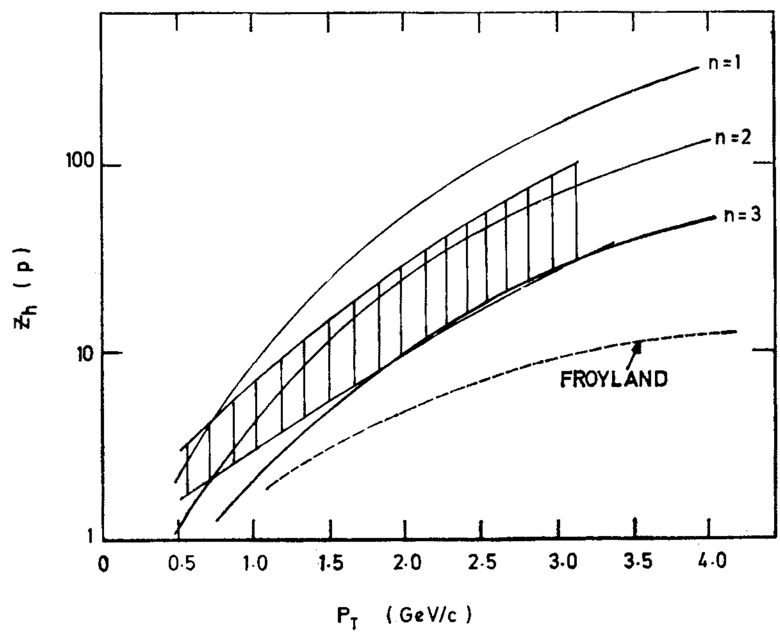

Fig. 1. Showing $Z_h(p)$ the ratio between the conditional inclusive cross-section and the inclusive cross--section. A value of unity means that there in no correlation effect. The lower limit of the shaded region is the experimental value on the same side and the upper limit is the experimental value on the opposite side. The dashed curve is the prediction of the Fröyland model and the curves labelled by values of n are the predictions of the model of Section 4

ignores momentum conservation. Experimentally, this prediction is not verified but is probably a reasonable "first order" statement (see figure). Whether significantly better agreement can be obtained by introducing momentum-recoil is at present being investigated (the effect certainly goes in the right direction).

Secondly, we see that $f(p_1, p_2)$ is a function of (p_1+p_2) only. We are not aware of any data which might enable this to be tested.

Finally, it is clear that (2.8) implies generally *positive* correlation. To see this we define

$$Z(p_1, p_2) = \frac{f(p_1, p_2)\sigma^{\text{inel}}}{f(p_1)f(p_2)} \tag{2.9}$$

$$= \frac{\int_0^\infty d\lambda \sigma(\lambda) a(\lambda)^2 e^{-\lambda(p_1+p_2)} \int_0^\infty d\lambda \sigma(\lambda)}{\int_0^\infty d\lambda \sigma(\lambda) a(\lambda) e^{-\lambda p_1} \int_0^\infty d\lambda \sigma(\lambda) a(\lambda) e^{-\lambda p_2}} . \tag{2.10}$$

Since $\sigma(\lambda)$ and $a(\lambda)$ are positive definite, this quantity is greater than or equal to unity for $p_1 = p_2$. Thus we expect positive correlations at least over some range in the neighbourhood of $p_1 = p_2$. These correlations are due to the fact that more than one value of λ is contributing. If we replace $\sigma(\lambda)$ by $\sigma^{\text{inel}}\ \delta(\lambda-\lambda_0)$, then we obtain $Z \equiv 1$.

The experiments actually measure a somewhat different qunatity in which the trigger is not a particle of fixed momentum p_2, but rather one of momentum $p_2 > h$. Thus we define

$$Z_h(p) = \frac{\int_{p_2>h} p_2 dp_2 dy_2 d\phi_2 f(p, p_2)\sigma^{\text{inel}}}{\int_{p_2>h} p_2 dp_2 dy_2 d\phi_2 f(p_2) f(p)} . \tag{2.11}$$

Again, at least for $p \sim h$, we expect $Z > 1$.

The experimental values of $Z_h(p)$, with $h = 2$ GeV/c and with p in the same (opposite) hemisphere, are given by Büsser [5] et al. In their notation

$$Z_h(p) = \frac{F(h)}{F\,(\text{fully inclusive})}. \tag{2.12}$$

The experimental results are shown in the figure. As we piedict, $Z_h(p)$ is greater than 1. We now turn to more specific models to see if quantitative agreement is possible.

3. *Fröylands' model*

Fröyland [4] assumes that the inverse temperature is a linear function of the impact parameter and, since the cross-section as a function of impact parameter is known from analysis of the elastic differential cross-section [6], is able to obtain a unique form for $\sigma(\lambda)$ and $a(\lambda)$ by fitting the inclusive distribution. He obtains

$$\lambda = b + 10.1 s^{-1/4}, \tag{3.1}$$

where b is the impact parameter in $(\mathrm{GeV}/c)^{-1}$, and

$$a(\lambda) = 0.56\,\pi^{-1}b^2. \tag{3.2}$$

The cross-section is given by Henzi and Valin [6] as

$$\sigma(\lambda) = 2\pi b(0.95)\theta(b)e^{-b^2/4B}, \tag{3.3}$$

with B given as a function of s in figure 2 of Ref. [6].

Some physical justification for the particular form of $a(\lambda)$ is given in Fröyland's paper, where it is shown that the quoted parameters give a good fit to the single particle inclusive data at all p.

We have evaluated the resulting expression for $Z_h(p)$ at $\sqrt{s} = 52.7$ GeV and $h = 3\,\mathrm{GeV}/c$. It is plotted as a dashed line in the figure and we see that the correlation effect is too small to explain the observations. This probably rules out the linear identification of λ with impact parameter. Fröyland (private communication) has made similar calculations and has also made some attempt to include recoil.

4. General treatment

In this section we shall express $f(p_1, p_2)$ and hence $Z_h(p)$ directly in terms of $f(p)$ and a particular parameterised form for $a(\lambda)$. We begin by inverting (2.6)

$$\sigma(\lambda)a(\lambda) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} f(p)e^{\lambda p}dp, \tag{4.1}$$

where we have chosen the contour to go along the real axis. This will normally be possible since $\sigma(\lambda)a(\lambda)$ tends to zero as $\lambda \to \infty$.

Now consider $f(p_1, p_2)$ given by (2.8) which we can write as

$$f(p_1, p_2) = \int_0^{\infty} d\lambda a(\lambda)e^{-\lambda(p_1+p_2)}\int_{-i\infty}^{+i\infty}\frac{dq}{2\pi i}f(q)e^{\lambda q}. \tag{4.2}$$

For p_1 and p_2 positive we can reverse the order of integration to obtain

$$f(p_1, p_2) = \int_{-i\infty}^{+i\infty}\frac{dq}{2\pi i}f(q)\int_0^{\infty} d\lambda a(\lambda)e^{-\lambda(p_1+p_2-q)}. \tag{4.3}$$

We choose to parameterise $a(\lambda)$ in the form

$$a(\lambda) = \sum a_n\lambda^n. \tag{4.4}$$

Then,

$$f(p_1, p_2) = \int_{-i\infty}^{+i\infty}\frac{dq}{2\pi i}f(q)\sum_n\frac{a_n n!}{(p_1+p_2-q)^{n+1}}\,. \tag{4.5}$$

Closing the contour to the right yields

$$f(p_1, p_2) = \sum (-1)^n a_n f^{(n)}(p_1+p_2) \tag{4.6}$$

which gives a simple relation between $f(p_1, p_2)$ and the n^{th} order derivatives $f^{(n)}$ of the single particle inclusive cross-section.

To evaluate $Z_h(p)$ we need also

$$\int_h^\infty p_2 dp_2 f(p_1, p_2) = \sum a_n (-1)^{n-1} [h f^{(n-1)}(h+p_1) - f^{(n-2)}(h+p_1)] \tag{4.7}$$

where we have extended the notation $f^{(n)}$ to $f^{-1}(h) = -\int_h^\infty f(p)dp$, etc. Then,

$$Z_h(p) = \frac{\sigma^{\text{inel}} \sum_n a_n (-1)^n [h f^{(n-1)}(h+p) - f^{(n-2)}(h+p)]}{f(p)\, [h f^{(-1)}(h) - f^{(-2)}(h)]}\,. \tag{4.8}$$

Rather than evaluating this directly from the data we shall use the 3-parameter fit to $f(p)$ of Vanryckeghem [7]

$$f(p) = A e^{-k[p^2+\nu^2]^{1/4}}, \tag{4.9}$$

with

$$A = (186 \pm 69) \cdot 10^4 s^{-1/2} \text{ mb (GeV)}^{-3/2}, \tag{4.10}$$

$$k = [(16.50 \pm 0.13) - (1.37 \pm 0.04) \log s^{1/2}] \text{ GeV}^{-1/2}, \tag{4.11}$$

and

$$\nu = (0.29 \pm 0.04) \text{ GeV}. \tag{4.12}$$

If we ignore terms which are $O\left(\frac{1}{k\sqrt{h}}\right)$ compared to 1, we can approximate

$$f^{(n)}(h) \simeq \left(\frac{-k}{2\sqrt[4]{h^2+\nu^2}}\right)^n f(h). \tag{4.13}$$

Then

$$Z_h(p) = \frac{\sigma^{\text{inel}}}{A} \sum_n a_n \left(\frac{k}{2\sqrt[4]{(h+p)^2+\nu^2}}\right)^n \left(\frac{(h+p)^2+\nu^2}{h^2+\nu^2}\right)^{1/4}$$
$$\times e^{k[\sqrt[4]{h^2+\nu^2} + \sqrt[4]{p^2+\nu^2} - \sqrt[4]{(h+p)^2+\nu^2}]}. \tag{4.14}$$

We can simplify this still further if we try a fit involving only one non-zero value a_n. Then the cross-section constraint (2.5) can easily be incorporated. First we use (2.6) and (4.9) to write

$$A e^{-k\sqrt[4]{p^2+\nu^2}} = \int_0^\infty \sigma(\lambda) a(\lambda) e^{-\lambda p} d\lambda \tag{4.15}$$

$$= \int_0^\infty \sigma(\lambda) a_n \lambda^n e^{-\lambda p} d\lambda \tag{4.16}$$

with our assumption about $a(\lambda)$. We multiply this by p^{n-1} and integrate over p from zero to infinity, thereby obtaining

$$A \int_0^\infty p^{n-1} e^{-k\sqrt[4]{p^2+v^2}} dp = a_n (n-1)! \int_0^\infty \sigma(\lambda) d\lambda, \tag{4.17}$$

or

$$\frac{a_n \sigma^{\text{inel}}}{A} = \frac{1}{(n-1)!} \int_0^\infty p^{n-1} e^{-k\sqrt[4]{p^2+v^2}} dp. \tag{4.18}$$

This, inserted into (4.14) gives our final expression for $Z_h(p)$. It has one free parameter, n. We have taken this to be an integer although the same expressions could probably be obtained without this restriction. The integral in (4.18) is evaluated numerically unless n is an even integer.

As an example we consider the case where n equals 2; i.e. we have a constant average multiplicity (see (2.4)).

Then,

$$Z_h(p) = \left[\frac{3}{k^2} + \frac{3\sqrt{v}}{k} + \frac{3v}{2} + \frac{kv^{3/2}}{2}\right] \frac{1}{\sqrt[4]{(h^2+v^2)((h+p)^2+v^2)}}$$
$$\times e^{k[\sqrt[4]{h^2+v^2} + \sqrt[4]{p^2+v^2} - \sqrt[4]{(h+p)^2+v^2} - \sqrt[4]{v^2}]}. \tag{4.19}$$

This is plotted in the figure. We see that it gives a reasonable fit to the data — assuming that, since we have no recoil effect in our model, we should roughly "average" the same/opposite side correlation.

To see the difference made by changing the form of $a(\lambda)$ we have also calculated (4.14) with other values of n. The results are shown in the figure. We anticipate that a suitable form for $a(\lambda)$ could be found such that the predicted $Z_h(p)$ lies in the required range for all p. However more detailed fits are not significant until we have studied recoil effects. The very small p region could cause problems but the data suggests there are experimental uncertainties.

5. *Conclusions*

Any attempt to describe in detail the available correlation data must take account of recoil and also discuss the longitudinal distribution. Since our model does not include these effects we must be content with a very crude comparison with data. With this proviso it is clear from the figure that the multitemperature model with an approximately constant multiplicity as a function of temperature can describe both the single-particle and the two-particle inclusive distributions as functions of p_T. On the other hand the two-particle distribution does not appear to be compatible with the existence of a linear relation between inverse temperature and impact parameter as in the model of Fröyland [4].

Since the model effectively predicts the conditional inclusive spectrum, for p_T in the range 0.5 to 3.0 GeV/c, from the fully inclusive spectrum for $p_T > 3.0$ GeV/c (this is particularly clear in Eq. (4.6)), it suggests that there is a common mechanism for the whole p_T range. The success of our prediction would seem to be accidental in parton-type models.

Finally we note that models of the type considered here automatically yield the universal dependence on mass and p_T of produced particles, through the combination $(p_T^2+m^2)$, which has been proposed by Michael [8] and appears to be supported by the data.

Note added in proof: This point is further discussed by Safari and Squires, *J. Phys.* **G 3**, L45 (1977).

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