THE UNIVERSAL IMPACT PARAMETER HYPOTHESIS AND THE QUARK MODEL IN THE REACTIONS $0^-\frac{1}{2}^+ \to J^P\,\frac{3}{2}^+$

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Using the universal impact parameter hypothesis and the additive quark model we derive several constraints on the amplitudes and density matrix elements of the process $0^-\frac{1}{2} \to J^P\frac{3}{2}^+$. The comparison of these results with the data implies that the two models should be formulated in the same spin reference system. It is shown that the common frame need not be the s-channel helicity frame.

1. *Introduction*

In a recent paper [1] we have analyzed the additive quark model and the b-universality [2, 3] for the reaction

$$0^-\,\tfrac{1}{2}^+ \to 0^-\,\tfrac{3}{2}^+. \tag{1.1}$$

When used simultaneously, they generate strong constraints on the amplitudes and provide a useful tool to investigate their assumptions. In particular we have checked various hypotheses concerning the additivity frame, i.e. the spin reference frame where the additivity assumption is made.

In this paper we investigate the consequences of the b-universality and the additive quark model for a more general class of reactions

$$0^-\,\tfrac{1}{2}^+ \to J^P\,\tfrac{3}{2}^+, \tag{1.2}$$

where J^P denotes arbitrary particle or system of particles produced peripherally, with a given spin J and parity P. We obtain constraints on the amplitudes and density matrix elements when the models are formulated in different frames. The comparison of these constraints with the data for several reactions shows that the two models should be formulated in the same spin frame. This result contradicts the common belief that the quark

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additivity is valid in the Gottfried-Jackson frame [4] and the b-universality in the s-channel helicity frame [2, 3].

Finally we show that the s-channel helicity frame is not the only one in which the b-universality is valid: the data for reaction $\pi^- p \to \pi^0 n$ agree very well with the b-universality also in the Gottfried-Jackson frame. Thus we conclude that the problem of the common spin system for the two models remains unsolved. The paper is organized as follows: in Section 2 we recall shortly the additive quark model and the b-universality hypothesis. In section 3 we find the consequences of the formulation of the models in two different frames. We compare some of the results with the data in section 4 and perform there a model independent test of the b-universality formulated in the Gottfried-Jackson frame. Our conclusions are listed in Section 5.

2. *The quark model and the b-universality*

The additive quark model states that there is only one quark in the particle which interacts during the collision. The spin quantisation frame in which this assumption is made is called the additivity frame. Several arguments were given that this frame coincides with the Gottfried-Jackson frame [4]. In the following we consider reactions of the type

$$0^- \, \tfrac{1}{2}^+ \to J^P \, \tfrac{3}{2}^+ \tag{2.1}$$

and obtain the quark model predictions applying the additivity assumption only to the baryon vertex [5]. The relations between the transversity amplitudes are then as follows [6]:

$$T^{\mu}_{\frac{3}{2}\frac{1}{2}} = \sqrt{3}\, T^{\mu}_{\frac{1}{2}-\frac{1}{2}}, \quad T^{\mu}_{+\frac{3}{2}\frac{1}{2}} = \sqrt{3}\, T^{\mu}_{-\frac{1}{2}\frac{1}{2}},$$
$$T^{\mu}_{\frac{1}{2}\frac{1}{2}} = T^{\mu}_{-\frac{1}{2}-\frac{1}{2}}, \quad T^{\mu}_{-\frac{3}{2}\frac{1}{2}} = T^{\mu}_{\frac{3}{2}-\frac{1}{2}} = 0 \tag{2.2}$$

with

$$\mu = -J, -J+1, \ldots, J-1, J.$$

(In T^a_{bc} a, b and c are the transversities of the meson state J^P baryon B^* and baryon B respectively.)

The universal impact parameter hypothesis [2, 3] involves the s-channel helicity amplitudes and assumes that

1° all amplitudes $M^{\lambda_d \lambda_b}_{\lambda_c \lambda_a}$ with a given net helicity flip $n = |\lambda_a - \lambda_b - \lambda_c + \lambda_d|$ behave in the same way as functions of the momentum transfer t. (this statement was formulated independently from the b-universality [7]);

2° the amplitudes in the impact parameter representation are expressed through the same function $f(s, b)$

$$\tilde{M}_n(s, b) = a_n(s) b^n f(s, b). \tag{2.3}$$

($a_n(s)$ is an arbitrary function of c. m. energy and all helicities involved and b^n — a kinematical factor). Taking the Fourier transform of Eq. (2.3) we obtain the derivative relations between the helicity amplitudes

$$M_{n'}(s, x) = a_{n'n}(s)x^{n'}\left(\frac{1}{x}\frac{\partial}{\partial x}\right)^{n'-n}\left(\frac{M_n(s, x)}{x^n}\right), \tag{2.4}$$

where $x = \sqrt{-t'}$.

In this paper we call the frame where the b-universality is formulated the BU frame. The arguments that it need not be the s-channel helicity frame are presented in section 5.

3. *The quark model and the b-universality in different frames*

In this section we investigate the two models when formulated in two different frames. Strictly speaking we assume that

$$\vartheta_B - \vartheta_{B*} \neq 0,$$

where ϑ_B and ϑ_{B*} are the rotation angles from the additivity to the BU frame for B and B^*. We stress that this is a typical situation (the b-universality is originally formulated in the helicity frame [2, 3] whereas the quark model mainly in the Gottfried-Jackson frame [4]).

Applying the rotation from the additivity to the BU frame for each particle independently we obtain the quark model relations (2.2) in the transversity BU frame

$$T^{\mu}_{\frac32\frac12} = \sqrt{3}\, T^{\mu}_{\frac12-\frac12}\cdot e^{2i\chi}, \quad T^{\mu}_{-\frac32-\frac12} = \sqrt{3}\, T^{\mu}_{-\frac12\frac12}e^{-2i\chi},$$

$$T^{\mu}_{\frac12\frac12} = T^{\mu}_{-\frac12-\frac12}e^{2i\chi}, \quad T^{\mu}_{-\frac32\frac12} = T^{\mu}_{\frac32-\frac12} = 0,$$

$$\chi = (\vartheta_B - \vartheta_{B_*})/2. \tag{3.1}$$

The relations do not depend on the rotation angle of the meson J^P what is the reflection of the fact that the additivity assumption was applied to the baryon vertex only.

In the frame with the spins projected in the reaction plane the relations look as follows:

$$\sqrt{3}\,(M^{\mu}_{\frac12\frac12} \mp M^{-\mu}_{\frac12\frac12}) = M^{\mu}_{-\frac32\frac12} \mp M^{-\mu}_{-\frac32\frac12}, \tag{3.2}$$

$$\sqrt{3}\,(M^{\mu}_{-\frac12\frac12} \mp M^{-\mu}_{-\frac12\frac12}) = M^{\mu}_{\frac32\frac12} \mp M^{-\mu}_{\frac32\frac12}, \tag{3.3}$$

$$\cos\chi \cdot M^{\mu}_{-\frac32\frac12} = \pm\sin\chi M^{-\mu}_{\frac32\frac12}, \tag{3.4}$$

$$\cos\chi[\sqrt{3}\, M^{\mu}_{-\frac12\frac12} \pm M^{-\mu}_{\frac32\frac12}] = -\sin\chi[\sqrt{3}\, M^{\mu}_{\frac12\frac12} \pm M^{-\mu}_{-\frac32\frac12}]. \tag{3.5}$$

The upper sign holds when J^P is natural and μ odd or J^P unnatural and μ even, and the lower in the remaining cases.

The first immediate observation follows from 1° (in the previous section) and (3.2) and (3.3).

I. *All amplitudes in the* BU *frame with even (odd)* $n = |\lambda_B - \lambda_{B*} + \lambda_J|$ *behave in the same way as functions of* t *(except for the process* $0^-\frac12^+ \to 0^+\frac32^+$*).*

The proof consists of writing the explicit form of Eqs (3.2) and (3.3) starting from $\mu = 0$. In the case of $J^P = 0^+$ there are no relations of this kind and $M_2(s, t)$ need not be proportional to $M_0(s, t)$. We stress that the above theorem is not valid in the additivity frame $\vartheta_B = \vartheta_{B*} = 0$ where all relations (3.2)–(3.5) separate into ones containing the amplitudes with the same net spin flip.

The next important remark is that

II. *the t-dependence of all amplitudes in the* BU *frame is explicitly known when there exist at least two amplitudes with different even (odd) spin flip.*

The construction is demonstrated below.

Supposing there are two amplitudes with even net spin flip $M_{\bar{n}'}$ and $M_{\bar{n}}$ (let $\bar{n}' > \bar{n}$) which have now the same t-dependence.

We write the equation for $M_{\bar{n}}$

$$M_{\bar{n}'}(s, x) = a_{\bar{n}'\bar{n}}(s)x^{\bar{n}'}\left(\frac{1}{x}\frac{\partial}{\partial x}\right)^{\bar{n}'-\bar{n}}\left(\frac{M_{\bar{n}}(s, x)}{x^{\bar{n}}}\right) = \alpha M_{\bar{n}}(s, x), \tag{3.7}$$

where α is a complex constant independent on t. The solution of Eq. (3.7) is

$$M_{\bar{n}}(s, x) = (xy)^{\bar{k}'+\bar{k}}C_{\bar{k}'-\bar{k}}(xy),$$
$$\bar{n}' = 2\bar{k}', \quad \bar{n} = 2\bar{k}. \tag{3.8}$$

$C_l(x)$ is a linear combination of the Bessel functions $J_l(x)$ and $Y_l(x)$ or a linear combination of $I_l(x)$ and $K_l(x)$.

$$y^{\bar{n}'-\bar{n}} = \begin{cases} \dfrac{\alpha}{a_{\bar{n}'\bar{n}}}(-)^{\bar{k}'-\bar{k}} & \text{when } J(x) \text{ and } Y(x) \text{ are used,} \\[2ex] \dfrac{\alpha}{a_{\bar{n}'\bar{n}}} & \text{when } I(x) \text{ and } K(x) \text{ are used.} \end{cases}$$

In the same way we obtain for odd $n' = 2k'+1$ and $n = 2k+1$

$$M_n(s, x) = (xy)^{k+k'+1}C_{k'-k}(xy) \tag{3.9}$$

with the same convention for y.

Next question is how many amplitudes with different spin flip $n = |\lambda_a-\lambda_b-\lambda_c+\lambda_d|$ can exist in a given process. We can prove that

III. *it is impossible to have more than two amplitudes with even (but different)* spin *flips or more than two amplitudes with odd (and different) spin flips, in the* BU *frame.*

Suppose there are more than two such amplitudes (e. g. three: $M_{\bar{n}}$, $M_{\bar{n}'}$ and $M_{\bar{n}''}$). The expression for $M_{\bar{n}}$ is then

$$M_{\bar{n}}(s, x) = (yx)^{\bar{n}''+\bar{n}}C_{\bar{n}''-\bar{n}}(xy) = (xy)^{\bar{n}'+\bar{n}}\bar{C}_{\bar{n}'-\bar{n}}(xy). \tag{3.10}$$

($C_{\bar{n}''-\bar{n}}$ and $\bar{C}_{\bar{n}'-\bar{n}}$ denote two different non-zero linear combinations of the Bessel functions). Eq. (3.10) implies that the ratio of two linear combinations of the Bessel functions is proportional to an integer power of x. This can never happen and it can be checked by taking the series expansion of the Bessel functions [8]. This proves our thesis.

From the above statement it follows that the maximum number of nonvanishing spin flip amplitudes is four. In such case we obtain additionally the following constraint:

IV. *If there exist exactly four non-zero amplitudes with different spin flips in the* BU *frame (i. e. two even* $M_{\bar{n}}$ *and* $M_{\bar{n}'}$ *and two odd* M_n *and* $M_{n'}$ *then their spin flip indices are related by*

$$\bar{n}+\bar{n}' = n+n'. \tag{3.11}$$

To prove it let us consider the shape of the odd spin flip amplitude M_n. It is given by Eq. (3.9) from the relation between the odd amplitudes. On the other hand it is also related to the even spin flip amplitudes $M_{\bar{n}}$ and $M_{\bar{n}'}$ by Eq. (2.4) and its shape following from this relation reads

$$M_n(s, x) = a_{n\bar{n}}\gamma^{2k-2\bar{k}+1}(x\gamma)^{\bar{k}'+\bar{k}}C_{\bar{k}'+\bar{k}-2k-1}. \tag{3.12}$$

Requiring compatibility of both formulas (3.9) and (3.12) we get the constraint (3.11).

The above statement implies for instance vanishing of one of the amplitudes in the process $0^-\frac{1}{2}^+ \to 1^-\frac{3}{2}^+$.

Let us now consider the constraints on the density matrix elements of J^P and B^*. In addition to the known class A relations [9] we obtain in the case of

$$\vartheta_B - \vartheta_{B^*} \neq k\cdot\pi, \qquad k = 0, \pm 1, \pm 2, \ldots$$

our theorem V which states that

$$\mathrm{Re}\,\varrho_{10} = \mathrm{Re}\,\varrho_{\frac{3}{2}\frac{1}{2}} = 0, \tag{3.13}$$

and alternatively

$$\varrho_{11}\pm\varrho_{1-1} = \frac{1}{\sqrt{3}}\,\varrho_{\frac{3}{2}\frac{1}{2}}\pm\varrho_{\frac{3}{2}-\frac{1}{2}} = 0 \tag{3.14}$$

when T_0, T_1 and T_2 exist; +for J^P natural, −for J^P unnatural,

$$\varrho_{1-1} = \varrho_{\frac{3}{2}-\frac{1}{2}} = 0 \tag{3.15}$$

otherwise.

The proof goes by writing the density matrix elements in terms of the amplitudes and using Eqs (3.2)–(3.5). We use also the fact that in the case when there exist at least three different spin flip amplitudes the odd and even spin flip amplitudes are different functions of x (this follows from Eqs (3.8) and (3.12)).

The constraints (3.13) and (3.15) are imposed on the density matrix elements which depend strongly on the angle ϑ_J between the additivity and BU frames. Although ϑ_J seems to be arbitrary (the quark model relations (3.2)–(3.5) do not depend on it), it is actually fixed by the conditions forcing several spin flip amplitudes to vanish in the BU frame (these conditions are not invariant under the rotation of the spin of J^P).

The obtained formulas for the amplitudes and density matrix elements are in fact very strong. In Ref. [1] we have compared the predictions for the amplitudes of the process $0^-\frac{1}{2}^+ \to 0^-\frac{3}{2}^+$ with experimental data. A comparison of our formulas with the measured density matrix elements of ϱ^0 and ω is presented in the following section.

5. Comparison with the data

In this section we check if our contraints on the density matrix elements of the meson J^P are compatible with experimental data. We use the high statistics data [10] of the processes

$$\pi^+ p \to \varrho^\circ \Delta^{++},$$

$$\pi^+ p \to \omega \Delta^{++}$$

at 7 GeV/c.

The only candidate for the BU frame is the Donohue-Högaasen frame [11] where

$$\text{Re } \varrho_{10} = 0$$

(cf. Eq. (3.13)). It can be immediately seen from the data that this frame is significantly different from both the s-channel helicity and the Gottfried-Jackson frames.

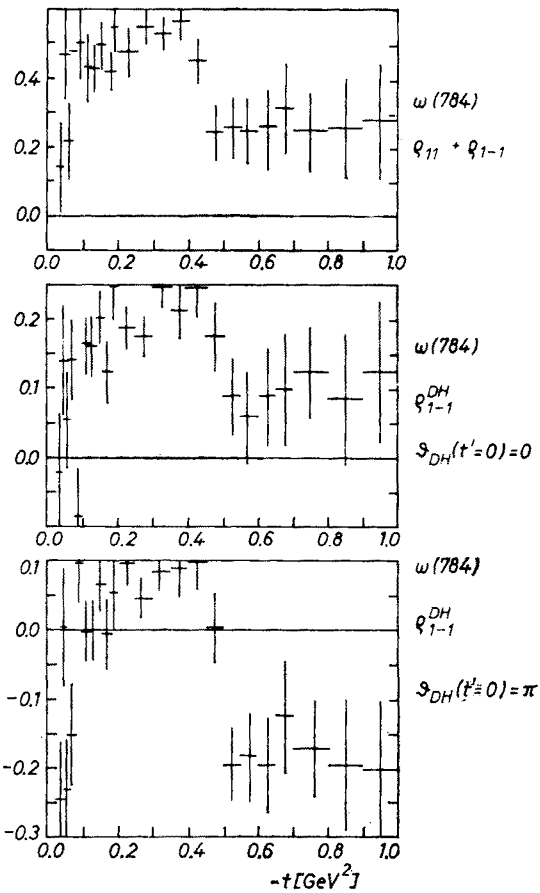

Fig. 1a–c. The density matrix elements of ω (784) in the process $\pi^+ p \to \omega \Delta^{++}$ at 7 GeV/c [10]: a) $\varrho_{11} + \varrho_{1-1}$. b) ϱ_{1-1} in the Donohue-Högaasen frame when $\vartheta_{DH}(t' = 0) = 0$. c) ϱ_{1-1} in the Donohue-Högaasen frame when $\vartheta_{DH}(t' = 0) = \pi$

The rotation to the Donohue-Högaasen frame from the Gottfried-Jackson frame is given in terms of the measurable density matrix elements in the initial frame

$$\tan 2\vartheta_{DH} = -2\sqrt{2}\,\mathrm{Re}\,\varrho_{10}/(\varrho_{00}-\varrho_{11}+\varrho_{1-1}).$$

Having thus determined the angle ϑ_{DH} we are able to compute the remaining matrix elements in the Donohue-Högaasen frame $\varrho_{11}^{DH}+\varrho_{1-1}^{DH} = \varrho_{11}+\varrho_{1-1}$, $\varrho_{1-1}^{DH} = \frac{1}{8}+\frac{3}{4}\varrho_{1-1}-\frac{3}{8}\varrho_{00} \pm\sqrt{8(\mathrm{Re}\,\varrho_{10})^2+(\varrho_{00}+\varrho_{1-1}-\varrho_{11})^2}$; take + sign when $\vartheta_{DH}(t'=0)=0$ or − sign when $\vartheta_{DH}(t'=0)=\pi$.

In Fig. 1. We have plotted the above quantities. It is seen that it is impossible to satisfy simultaneously Eqs (3.13) and (3.14) or (3.13) and (3.15) in a reasonable t-range. This eliminates the Donohue-Högaasen frame as the BU system. Thus are left with the

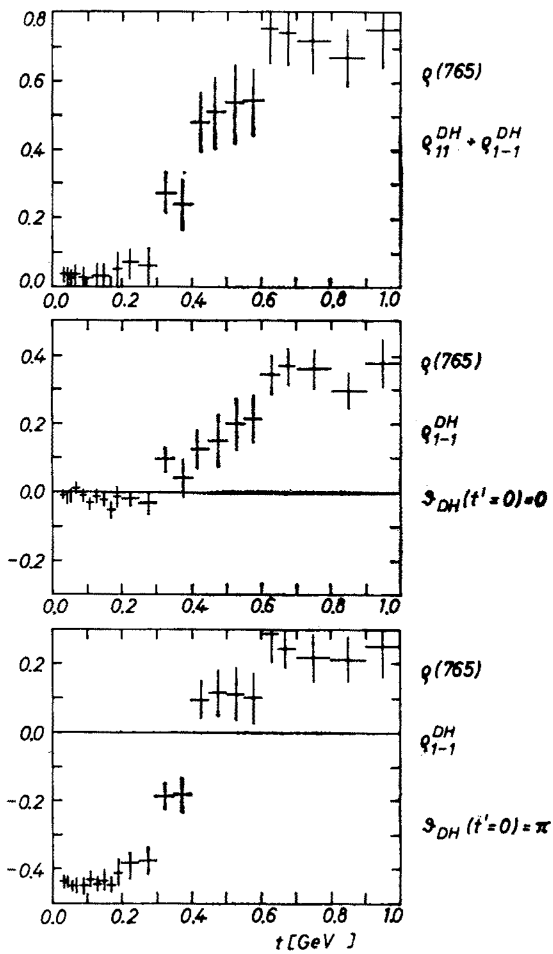

Fig. 1d–f. The density matrix elements of ϱ° (765) in the process $\pi^{+}p \rightarrow \varrho^{\circ}\Delta^{++}$ at 7 GeV/c [10]: d) $\varrho_{11}+\varrho_{1-1}$. e) ϱ_{1-1} in the Donohue-Högaasen frame when $\vartheta_{DH}(t'=0)=0$. f) ϱ_{1-1} in the Donohue-Högaasen frame when $\vartheta_{DH}(t'=0)+\pi$

remaining possibility where

$$\vartheta_B - \vartheta_{B*} = k\pi, \quad k = 0, \pm 1, \pm 2, \ldots .$$

To eliminate odd k in the above formula we use the results of the analysis of the reaction $0^- \frac{1}{2}^+ \to 0^- \frac{3}{2}^+$. The only possible way to fit the differential cross-section was the case where $\vartheta_B - \vartheta_{B*} = 2k\pi$. Combining this fact with the results of this section we state that the additive quark model and the b-universality should be foimulated in two frames which fulfill the constraint

$$\vartheta_B - \vartheta_{B*} = 2k\pi, \quad k = 0, \pm 1, \pm 2, \ldots .$$

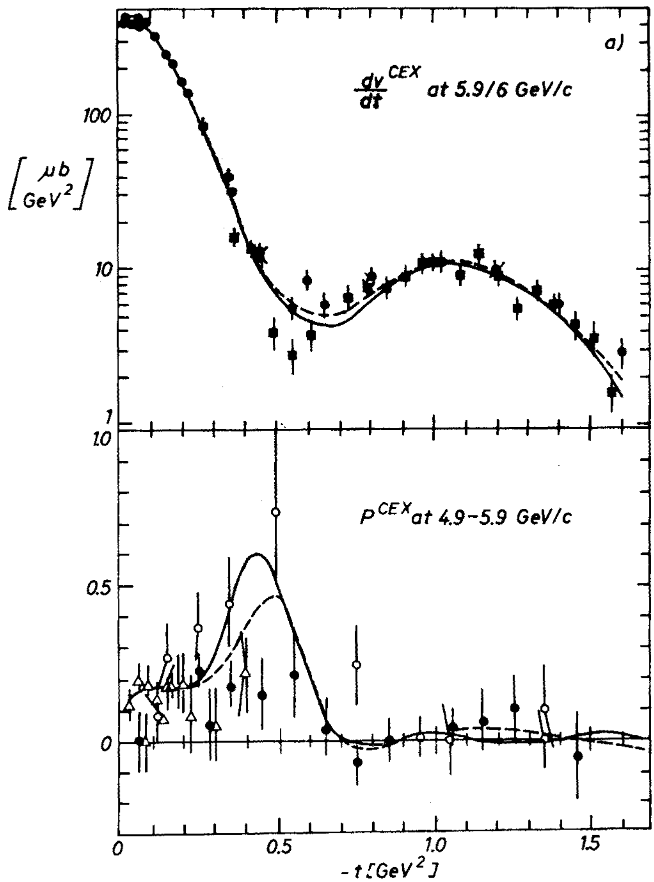

Fig. 2a

It should be stressed that the above analysis eliminates an important and frequently used possibility, namely that the additive quark model is formulated in the Gottfried-Jackson [4] and the b-universality in the helicity frame [2, 3].

Supposing that the both frames coincide, we are facing the following problem: is it possible to decide which common frame of reference should be chosen. As we know there exists an experimentally unsolved ambiguity concerning the additivity frame [4] whereas the b-universality was always formulated in the helicity frame. This situation stimulated us to check if the last system is indeed preferred. In other words, we attempt to find any other frame in which the b-universality holds. Thus we assumed the validity of this hypothesis

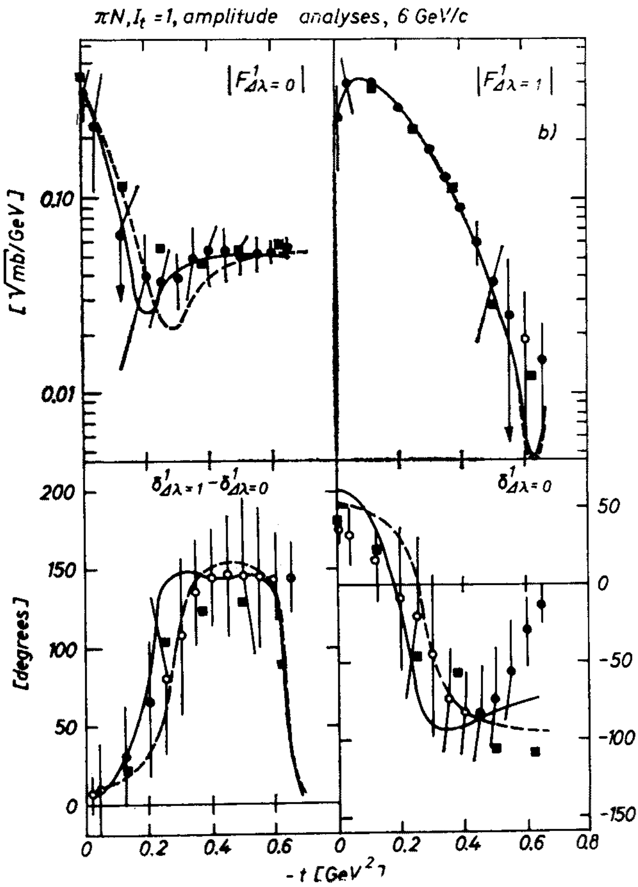

Fig. 2a, b. The results of the model independent test of the b-universality [3]: dashed line — the hypothesis formulated in the helicity frame, solid line — the hypothesis formulated in the Gottfried-Jackson frame. The points taken after Ref. [3] from [12]

in the Jackson frame and performed a model independent test [3] fitting all experimental information ($d\sigma/dt$, polarization and amplitudes [12]) in the charge exchange process

$$\pi^-p \rightarrow \pi^0 n.$$

The results (Fig. 2) show an excellent agreement with the data. Moreover, the Gottfried-Jackson amplitudes (Fig. 3) possess all features suggested by the geometrical picture and expected from the Regge behaviour. This allows us to state that the helicity frame is not the only candidate for the BU frame. Thus the question of a proper spin system for the two considered models remains open.

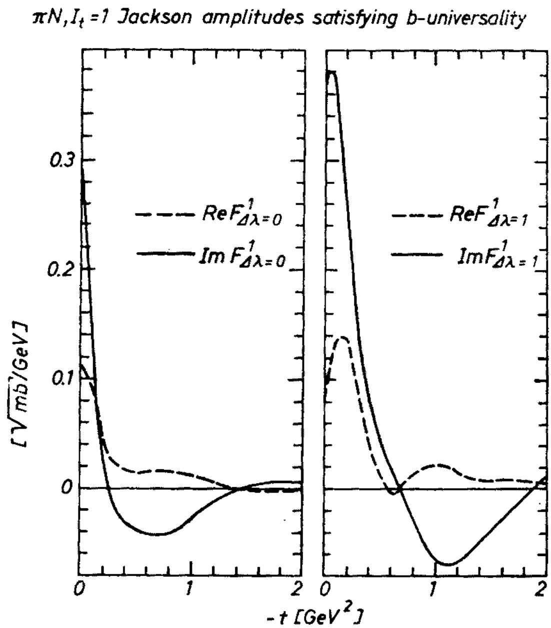

Fig. 3. The amplitudes $M_0^{I=1}(s,t)$ and $M_1^{I=1}(s,t)$ in the Gottfried-Jackson frame satisfying the derivative relations

5. *Conclusions*

In the paper we investigated the additive quark model and the b-universality applied to the processes

$$0^- \tfrac{1}{2}^+ \to J^P \tfrac{3}{2}^+.$$

It turned out that the crucial point is the relative position of spin reference systems in which the models are formulated. In the case when the frames do not coincide ($\vartheta_B - \vartheta_{B^*} \neq 0$) the following constraints were obtained

a — all amplitudes with even (odd) spin flip show the same t-dependence;

b — the form of all amplitudes is uniquely determined (see Eqs (3.8), (3.9));

c — only 4 different spin flip amplitudes are allowed;

d — when in addition $\vartheta_B - \vartheta_{B^*} \neq \pi$ we obtained constraints on the density matrix elements of the baryon B^* and meson state J^P (see Eqs (3.13)–(3.15)).

The comparison of the above predictions (b — for the process $0^- \tfrac{1}{2}^+ \to 0^- \tfrac{3}{2}^+$ and d — for $0^- \tfrac{1}{2}^+ \to 1^- \tfrac{3}{2}^+$) shows discrepancy with the data and implies our main conclusion *the b-universality and the additive quark model are consistent only when formulated in the frames satisfying the constraint*

$$\vartheta_B - \vartheta_{B*} = 2k\pi, \quad k = 0, \pm 1, \pm 2, \dots . \tag{5.1}$$

This constrait is satisfied in particular by two coinciding frames.

Although the *b*-universality was always assumed and tested in the *s*-channel helicity basis this frame is not the only candidate for the common spin system of the two models. We have checked the *b*-universality formulated in the Gottfried-Jackson frame for the process $\pi^- p \rightarrow \pi^\circ n$. Good agreement with the data is observed. Let us finally remark that the choice of the common spin system of the two models is still open.

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