CONSERVATION OF ISOSPIN AND THE UNCORRELATED JET MODEL

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In is shown that the requirement of exact conservation of the isospin is in contradiction with the basic ingredients of the uncorrelated jet model. The nature of the strong correlations resulting from an attempt to build an isospin conserving uncorrelated jet model is studied. Modifications that make the correlations less visible are discussed.

The purpose of this paper is to comment on the idea of building an isospin conserving uncorrelated jet model [1–4]. In particular we want to show that exact isospin conservation cannot be satisfied without giving up two essential ingredients of the uncorrelated jet model: the factorization and the absence of strong correlations.

The basic assumption in the uncorrelated jet model is that the scattering amplitude for the process $2 \to n$ factorizes to factors each depending on the state of one final state particle only. Equivalently we can say that the final state wave function for fixed initial state has the form [5]

$$|\psi_n\rangle = \frac{1}{\sqrt{n!}}\left[\int d\boldsymbol{q} f(\boldsymbol{q}) a^\dagger(\boldsymbol{q})\right]^n |0\rangle. \tag{1}$$

Here we have assumed that all particles are identical and neglected the energy momentum conservation constraint, because we shall be interested in internal quantum numbers only. The operator $a^\dagger(\boldsymbol{q})$ creates a particle of momentum $\boldsymbol{q}$. The function $f(\boldsymbol{q})$ controls the shape of the single particle distribution: a suitable energy dependent choice of this function guarantees that the four-momentum is conserved on the average. The differential $d\boldsymbol{q}$ is the invariant phase space element $d\boldsymbol{q} = d^3\boldsymbol{q}/2E$.

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We can generalize the above model to the production of pions of three different charges by writing [1-3]

$$|\psi_n(\vec{\tau})\rangle = \frac{1}{\sqrt{n!}} [\int dq f(q) \vec{\tau}(q) \cdot \vec{a}^\dagger(q)]^n |0\rangle. \tag{2}$$

Now we have an isotriplet of creation operators $\vec{a}^\dagger(q) = (a_1^\dagger(q), a_2^\dagger(q), a_3^\dagger(q))$. The neutral and charged pions are created by operators

$$a_0^\dagger(q) = a_3^\dagger(q),$$

$$a_\pm^\dagger(q) = \frac{1}{\sqrt{2}} (a_1^\dagger(q) \pm i a_2^\dagger(q)). \tag{3}$$

The vector function $\vec{\tau}(q)$ is chosen to be a unit vector. A possible choice is $\vec{\tau} = (\sqrt{2/3}, 0, \sqrt{1/3})$. In order to produce all types of pions with equal probability we must have

$$|\tau_0(q)| = |\tau_+(q)| = |\tau_-(q)|. \tag{4}$$

The second equation follows also from charge conservation and requires that τ_1 and τ_2 are relatively real. Thus they can be chosen to be real without loss of generality. Now the state $|\psi_n(\vec{\tau})\rangle$ is a direct product of n isovectors and contains thus in general components of all isospins $I = 0, 1, \ldots, n$. In all cases at least the isospin $I = n$ is present. Taking into account the isospins of initial particles and leading final state particles the conservation of isospin sets in the case of pp-scattering, e. g., the limit $I \leqslant 2$ [3]. The model described by formula (2) is thus in explicit conflict with the isospin conservation. This is obviously true for any model, where the matrix element for producing n particles of nonzero isospin factorizes in the above mentioned sense.

A state of a definite isospin can, however, be formed by taking a suitable linear combination of the states $|\psi_n(\vec{\tau})\rangle$. In particular the state

$$|\psi_n, I, I_3\rangle = \sqrt{\frac{2I+1}{4\pi}} \int d\Omega Y_{II_3}(\Omega) \, |\psi(\vec{\tau}(\Omega))\rangle \tag{5}$$

has the isospin I with the three-component I_3 [1, 3]. The function $Y_{II_3}(\Omega) = Y_{II_3}(\Theta, \Phi)$ is a spherical harmonic function. The vector $\vec{\tau}(q, \Omega)$ is obtained from $\vec{\tau}(q)$ by the related rotation:

$$\tau_m(q, \Omega) = \sum_{m'} D^{(1)}_{m'm}(\Phi, \Theta, 0) \tau_{m'}(q), \tag{6}$$

where m and m' refer to indices $+$, 0 and $-$, and D is a Wigner rotation matrix. If we sum finally over all values of I_3, the result is not sensitive to the exact value of I. So we choose for simplicity $I = 0$.

It is clear that the requirement for $\vec{\tau}(q, \Omega)$ (4) cannot be satisfied for all Ω. The second equation, related to charge conservation, remains however valid if we choose $\vec{\tau}(q)$ to be real. Then $\vec{\tau}(q, \Omega)$ covers the full solid angle for each q in the integral (5). For simplicity we choose $\vec{\tau}(q)$ to be independent of q. Then all real choices are equivalent. From the choice $\vec{\tau}(q) = (0, 0, 1)$ follows

$$\vec{\tau}(\Omega) = (\sin\Theta\cos\phi, \sin\Theta\sin\phi, \cos\Theta). \tag{7}$$

Now it is easy to see that in the model defined by the equation (2) with $\vec{\tau}$ given by (7) the multiplicity distribution of neutral pions is binomial with an average value

$$\langle n_0 \rangle = n\cos^2\Theta. \tag{8}$$

Thus the ratio $R \equiv n_0/n$ has the expectation value $\langle R \rangle = \cos^2\Theta$ and a variance proportional to $1/n$. In particular we get only neutral pions, when $|\cos\Theta| = 1$ and only charged pions, when $\cos\Theta = 0$.

Expanding $\langle \psi_n, I = 0 | \psi_n, I = 0 \rangle$ in terms of states with definite number of pions of each charge, a straightforward calculation gives the following results: 1) $n_+ = n_-$, 2) n_0 and n must be even, 3) the probability of n_0 neutral pions is given by (n_0 even):

$$p(n_0|n) \propto \frac{1}{n_0![(\frac{1}{2}(n-n_0))!]^2} \int d\Omega d\Omega' (\cos\Theta\cos\Theta')^{n_0} (\sin\Theta\sin\Theta')^{n-n_0}. \tag{9}$$

Using the fact that the integral is the square of a Euler beta function we get [3] for $n_0 \leqslant n$ and even

$$p(n_0|n) \propto \frac{(n_0-1)!!}{n_0!!}. \tag{10}$$

For large n the probability density of the ratio $R = n_0/n$ approaches rapidly the limiting behaviour

$$p(R) = \frac{1}{2\sqrt{R}}. \tag{11}$$

This is exactly the probability distribution of $\cos^2\Theta$ on the unit sphere and it could have been deduced directly from Eq. (8). A striking feature of the formula (11) is that the fluctuations in R are practically independent of the total multiplicity. This is due to the fact that for all n the quantity R is controlled essentially by one parameter: $\cos\Theta$ — not by an increasing number of independent variables as one would expect in a model of uncorrelated emission. The fact that the above procedure in model building leads to strong correlations has been observed previously [2, 3], but the connection between these correlations and the dominance of one parameter has not been discussed.

The name uncorrelated jet model (or independent emission model) seems completely inappropriate for a model with these extreme correlations. In taking the linear combination (5) we loose naturally also the factorization property used generally in defining uncorrelated emission.

There are several modifications that can be made to the model to weaken or hide the correlations, but none of these leads to a strictly independent emission combined with the exact isospin conservation. One possibility is to make the model directly for the absolute square of the matrix element, but then the exact isospin conservation cannot be defined at all. Another change is to make $\vec{\tau}(q)$ to depend on q. It is, however, clear that this does not reduce the amount of correlations — it only makes them more subtle and difficult to observe. One can also introduce more particle types. Biebl, Klein, and Nahnhauer [4] have proposed a model, where π's and ϱ's are produced in the same coherent state. This reduces strongly the observable correlations because neutral pions appear together with neutral ϱ's, which decay always to π^- and π^-. One could also require the total isospin of the pions and the ϱ's to be separately zero, i. e. introduce two independent vectors $\vec{\tau}_\pi$ and $\vec{\tau}_\varrho$ and average separately over both. The resulting correlations would be weaker than in the present model but stronger than in Ref. [4], if described in terms of the correlation parameters f_{cc}, f_{00}, f_{c0} etc. On the amplitude level the correlations are, of course, always weakened by the introduction of more independent parameters $\vec{\tau}$. This observation leads to one further possibility. The range of the correlations may be reduced by averaging over $\vec{\tau}$ separately in different parts of the phase space (e. g. in several rapidity intervals). The result is then that the isospin is conserved separately in each of these intervals. One should, however, avoid making the intervals too small, because the production of one particle only in an interval is of course forbidden, if the particle has a nonzero isospin.

Finally we notice that an attempt to build an uncorrelated jet model for particles with nonzero spin leads to similar problems as discussed above for the isospin case.

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