RELATION BETWEEN THE GLAUBER MODEL AND CLASSICAL PROBABILITY CALCULUS

By A. Białas*, **, M. Błeszyński* and W. Czyż*

Institute of Nuclear Physics, Cracow*

Institute of Physics, Jagellonian University, Cracow**

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It is shown that the Glauber model, for factorized ground state densities and purely imaginary elastic amplitudes, can be viewed as a consequence of the classical probability calculus used for computing incoherent cross-sections, supplemented with the optical theorem to obtain the coherent effects.

Many theoretical models of multiple particle production in hadron-nucleus interactions contain as an essential ingredient probability distributions of the number of inelastic collisions of the projectile with the target nucleons [1–8]. These probability distributions are usually calculated from classical (non-quantal) probability calculus. On the other hand, many features of hadron-nucleus interactions are well reproduced by the Glauber model which is based on the principle of additivity of phase-shifts [9]. This raises the problem of relations between these two approaches.

In the present paper we discuss this question and show that there exist some far reaching similarities between these two approaches and, in many cases, they even lead to identical results.

Let us first consider the probability calculus [8]. For the hadron-nucleus reaction cross section we have

$$\sigma_r^A(\boldsymbol{b}) = \int d^3r_1 \ldots d^3r_A |\Psi_0(\boldsymbol{r}_1 \ldots \boldsymbol{r}_A)|^2 \{1 - \prod_{j=1}^{A} [1-\sigma_r(\boldsymbol{b}-\boldsymbol{s}_j)]\}. \tag{1}$$

Here, Ψ_0 is the ground state wave function of the target nucleus, $\boldsymbol{b}$ the impact parameter, $\sigma_r(s)$ is the impact parameter distribution of the hadron-nucleon inelastic cross section normalized as follows (note that $\sigma_r(s)$ is dimensionless)

$$\sigma_r = \int d^2s\sigma_r(s). \tag{2}$$

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* Address: Instytut Fizyki Jądrowej Radzikowskiego 152, 31-342 Kraków, Poland.

** Address: Instytut Fizyki UJ, Reymonta 4, 30-059 Kraków, Poland.

In the following we will consider only the case of factorized ground state densities

$$|\Psi_0(\boldsymbol{r}_1 \ldots \boldsymbol{r}_A)|^2 = \prod_{i=1}^{A} \varrho(\boldsymbol{r}_i). \tag{3}$$

With this assumption, Eq. (1) can be written as

$$\sigma_r^A(\boldsymbol{b}) = 1-[1-\tilde{\sigma}_r(\boldsymbol{b})]^A, \tag{4}$$

where

$$\tilde{\sigma}_r(\boldsymbol{b}) = \int d^2 s D(\boldsymbol{s})\sigma_r(\boldsymbol{b}-\boldsymbol{s}). \tag{5}$$

Here

$$D(\boldsymbol{s}) = \int_{-\infty}^{+\infty} dz \varrho(\boldsymbol{s}, z). \tag{6}$$

To see that Eq. (4) does indeed originate from probability calculus one can rewrite it in the form

$$\sigma_r^A(\boldsymbol{b}) = \sum_{\nu=1}^{A} \binom{A}{\nu} [\tilde{\sigma}_r(\boldsymbol{b})]^\nu [1-\tilde{\sigma}_r(\boldsymbol{b})]^{A-\nu}, \tag{7}$$

where the consecutive terms correspond to exactly one, two, three etc. collisions with the target nucleus.

Similarly, one gets the following expression for the total inelastic cross section

$$\sigma_{in}^A(\boldsymbol{b}) = 1-\{1-[\tilde{\sigma}_t(\boldsymbol{b})-\tilde{\sigma}_{el}^*(\boldsymbol{b})]\}^A, \tag{8}$$

where

$$\tilde{\sigma}_t(\boldsymbol{b}) = \int d^2 s D(\boldsymbol{s})\sigma_t(\boldsymbol{b}-\boldsymbol{s}) \tag{9}$$

is the impact parameter distribution of the total cross section of the projectile with one nucleon in the target nucleus. Here $\int d^2 b\, \sigma_t(\boldsymbol{b}) = \sigma_t$. Note that $\tilde{\sigma}_t(b)$ distribution is already smeared over the target nucleus. The second term, $-\tilde{\sigma}_{el}^*(\boldsymbol{b})$, takes care of the fact that some interactions leave the target nucleus in its ground state, hence do not contribute to inelastic cross section $\sigma_{in}^A(\boldsymbol{b})$. When the elastic amplitude is purely imaginary, which is approximately the case, this term can be explicitely calculated from the optical theorem and we obtain

$$\tilde{\sigma}_{el}^*(\boldsymbol{b}) = |\tfrac{1}{2}\, \tilde{\sigma}_t(\boldsymbol{b})|^2. \tag{10}$$

Assuming again that the elastic scattering is purely imaginary, the elastic amplitude $\Gamma(\boldsymbol{b})$ is calculated from the optical theorem

$$2\Gamma(\boldsymbol{b}) = |\Gamma(\boldsymbol{b})|^2 + \sigma_{in}^A(\boldsymbol{b}), \tag{11}$$

using Eq. (8) as the input. The result is

$$\Gamma(b) = 1-\{1-[\tilde{\sigma}_t(b)-\tfrac{1}{4}\tilde{\sigma}_t^2(b)]\}^{\frac{A}{2}}$$
$$= 1-\{1-\tfrac{1}{2}\tilde{\sigma}_t(b)\}^A. \tag{12}$$

This is the well-known formula of the Glauber model.

To complete the proof that the two approaches considered give indeed identical results, we have to show that reaction cross section in Glauber models is identical with expression (4). To this end, let us recall first the formula for quasielastic cross section in the Glauber model [10]

$$\sigma^A_{\mathrm{q.el,G}}(b) = [1-\tilde{\sigma}_r(b)]^A-[1-\tilde{\sigma}_t(b)+\tfrac{1}{4}\tilde{\sigma}_t^2(b)]^A. \tag{13}$$

Now, the reaction cross section is given by

$$\sigma^A_{\mathrm{r,G}}(b) = \sigma^A_{\mathrm{in,G}}(b)-\sigma^A_{\mathrm{q.el,G}}(b) = 2\Gamma(b)-|\Gamma(b)|^2-\sigma^A_{\mathrm{q.el,G}}(b). \tag{14}$$

Using Eqs (13) and (12) we thus obtain

$$\sigma^A_{\mathrm{r,G}}(b) = 2-2[1-\tfrac{1}{2}\tilde{\sigma}_t(b)]^A-[1-(1-\tfrac{1}{2}\tilde{\sigma}_t(b))^A]^2$$
$$-[1-\tilde{\sigma}_r(b)]^A+[1-\tilde{\sigma}_t(b)+\tfrac{1}{4}\tilde{\sigma}_t^2(b)]^A$$
$$= 1-[(1-\tfrac{1}{2}\tilde{\sigma}_t(b))^A]^2-[1-\tilde{\sigma}_r(b)]^A+[1-\tilde{\sigma}_t(b)+\tfrac{1}{4}\tilde{\sigma}_t^2(b)]^A = 1-[1-\tilde{\sigma}_r(b)]^A, \tag{15}$$

and we recover Eq. (4).

Thus we have shown that the formulae for elastic, total and reaction cross sections in the Glauber model are identical to those derived from the classical probability calculus. Since the quasielastic cross section is given by

$$\sigma_{\mathrm{q.el}} = \sigma_t-\sigma_{\mathrm{el}}-\sigma_r,$$

it follows that also $\sigma_{\mathrm{q.el}}$ is given by the same formula (13) in both approaches. This can be also seen directly by computing the probabilities similarly as in derivation of Eq. (8).

Let us now summarize our conclusions. Our arguments show that the Glauber model can be viewed as a consequence of the classical probability calculus used for computing inelastic cross sections coupled with the optical theorem to obtain the coherent effects. We should stress however, that an essential element in our proof of the equivalence was the probabilistic independence of the interactions with nucleons in the target. At present we do not know how far one can get without this assumption.

We feel that this result gives some new insights into the validity region of the Glauber model by relaxing the very stringent conditions usually required [9]. Indeed, in the probabilistic approach the problem is shifted to calculation of σ_{in} where the only important restriction is the independence of elementary interactions. Consequently, at energies where the wave length of the incident particle is smaller than the average distance between nucleons in the target, the Glauber model should be valid.

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