EFFECTS OF NUCLEON-NUCLEON CORRELATIONS IN THE MULTIPLICITY DISTRIBUTIONS OF PARTICLES PRODUCED IN HADRON-NUCLEUS COLLISIONS

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(Received March 16, 1977)

It is shown that if one takes into account the nucleon-nucleon short range correlations in the target nucleus and the impact parameter distribution of the elementary production processes, the average multiplicities and multiplicity dispersions decrease by $\sim 10\%$ for light and $\sim 5\%$ for heavy nuclei.

1. Introduction

In this note we apply some of the ideas discussed in the preceding paper [1] and evaluate probability distributions of the number of inelastic collisions in hadron-nucleus interactions. These distributions can be used for analysing the processes of multiparticle production on nuclear targets [2–9]. In some models e. g. the average number of inelastic collisions $\bar{\nu}$ directly determines the average multiplicity [2, 5, 6, 8, 9] and the dispersion of ν gives the dispersion of multiplicities of produced particles [6][1]. The probability distribution of ν is usually calculated [4, 6] from hadron-nucleon cross-sections and the single particle densities of the nuclear targets using standard methods of probability calculus. In view of rapid increase of experimental data and their accuracy and because the commonly accepted methods of analysis of experimental results employ ν distributions [10, 11, 12] it becomes important to know quite accurately ν, its dispersion etc. Therefore, in this note we report the results of the calculations of probability distributions of ν in hadron-nucleus collisions which employ the general form of the target ground state density as an expansion in terms of the nucleon-nucleon correlation function and which keep the impact parameter distribution of the elementary production processes in the incident hadron-target nucleon collision.

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[1] There also exists another parameter which can be equally well used in descriptions of hadron-nucleus interactions: the number of "wounded" nucleons $w = \nu+1$ [9]. However, as was stressed in Ref. [9] these two parameters are *not* equivalent in the case of nucleus-nucleus collisions, where there is no relation between w and ν.

We find that nucleon-nucleon correlations and the finite spatial distribution of the inelastic cross sections decrease the values of the average number of collisions and their dispersion by 5–10%. In view of the present accuracy of the measured average multiplicities and dispersions such corrections should be indeed taken into account in an accurate analysis of the data on multiparticle production in hadron-nucleus collisions.

In our model, as we argued in Ref. [9], the *nondiffractive* production on nuclei is described as an incoherent composition of individual collisions computed by the standard techniques of the probability calculus. One may wonder therefore how does this model reproduce e. g. total cross-sections which have recently been measured up to ~300 GeV [13]. This also raises the problem of relations between the Glauber Model and the classical non-quantal probability calculus. Such relations have already been pointed out [14, 15] and are discussed in more detail in the accompanying paper [1].

In this note, in Section 2, we estimate, using probability calculus, the role of the two particle correlations in the target in the multiparticle production processes off nuclei. Section 3 contains conclusions.

2. *Probability distributions of the number of inelastic collisions for correlated ground state wave functions*

Our basic formula for the hadron-nucleus reaction cross-section is Eq. (1) of the preceding paper [1]:

$$\sigma_R^{(A)} = \int d^2b \int d^3r_1 \dots d^3r_A |\Psi_0(\vec{r}_1, \dots, \vec{r}_A)|^2 \{1 - \prod_{j=1}^{A} (1-\sigma_R(\vec{b}-\vec{s}_j))\}$$
$$= \int d^2b \{1 - \langle \prod_{j=1}^{A} (1-\sigma_R(\vec{b}-\vec{s}_j))\rangle\} \tag{2.1}$$

with same notation as in Ref. [1]. $\langle\dots\rangle$ denotes the ground state average. However, in this note our ground state wave function contains two particle correlations and cannot be factorized into a product of single particle wave functions as in [1]. The probabilistic interpretation of (2.1) leads to a construction of the probability distribution of the number of inelastic collisions and evaluation of the average number of collisions, dispersion, and higher probability moments.

Let us introduce the generating function $\phi(x)$ [9]

$$\phi(x) = \langle \int d^2b \prod_{j=1}^{A} [(1+(x-1)\sigma_R(\vec{b}-\vec{s}_j))-1]\rangle = \sigma_R^{(A)} \sum_{\nu=0}^{A} x^\nu P_\nu, \tag{2.2}$$

where P_ν is the probability of ν inelastic collisions of the incident hadron with target nuleons. From (2.2) we get

$$\sigma_R^{(A)} = -\phi(0), \tag{2.2a}$$

$$\bar{\nu} = \frac{1}{\sigma_R^{(A)}} \left.\frac{d\phi(x)}{dx}\right|_{x=1} = \sum_{\nu=1}^{A} \nu P_\nu = \frac{1}{\sigma_R^{(A)}} \left\langle \int d^2b \sum_{j=1}^{A} \sigma_R(\vec{b}-\vec{s}_j) \right\rangle, \tag{2.2b}$$

$$\overline{\nu(\nu-1)} = \frac{1}{\sigma_R^{(A)}} \left.\frac{d^2\phi(x)}{dx^2}\right|_{x=1} = \sum_{\nu=1}^{A} \nu(\nu-1)P_\nu$$

$$= \frac{1}{\sigma_R^{(A)}} \left\langle \int d^2b \sum_{j\neq k}^{A} \sigma_R(\vec{b}-\vec{s}_j)\sigma_R(\vec{b}-\vec{s}_k) \right\rangle . \tag{2.2c}$$

To perform averaging over the ground state we use the general expansion of the ground state in terms of the two-particle, three-particle etc. correlations [18] and neglect terms depending on correlations of order higher than two. In this approximation we can write the density in the following form

$$|\Psi_0(\vec{r}_1, ..., \vec{r}_A)|^2 = \varrho(\vec{r}_1, ..., \vec{r}_A) \cong \varrho^{(1)}(\vec{r}_1) \dots \varrho^{(1)}(\vec{r}_A) + \sum_{\substack{\text{all possible}\\ \text{pairs of contractions}}} [\varrho^{(1)}(\vec{r}_1) \dots \varrho^{(1)}(\vec{r}_A)] \tag{2.3}$$

where a contraction is defined as follows

$$\Delta(\vec{r}_j, \vec{r}_k) = \varrho^{(2)}(\vec{r}_j, \vec{r}_k) - \varrho^{(1)}(\vec{r}_j)\varrho^{(1)}(\vec{r}_k)$$
$$= \varrho^{(1)}(\vec{r}_j)\varrho^{(1)}(\vec{r}_k)C(\vec{r}_j, \vec{r}_k) \tag{2.4}$$

with

$$\varrho^{(1)}(\vec{r}_1) = \int d^3r_2 \dots d^3r_A \varrho(\vec{r}_1, ..., \vec{r}_A),$$
$$\varrho^{(2)}(\vec{r}_1, \vec{r}_2) = \int d^3r_3 \dots d^3r_A \varrho(\vec{r}_1, ..., \vec{r}_A). \tag{2.5}$$

The dimensionless function $C(\vec{r}_j, \vec{r}_k)$ is the two-particle correlation function which has the following properties

$$C(\vec{r}_j, \vec{r}_k) \to 0 \quad \text{for } |\vec{r}_j - \vec{r}_k| \to \infty$$
$$\int d^3r_j \varrho^{(1)}(\vec{r}_j)C(\vec{r}_j, \vec{r}_k) = 0. \tag{2.6}$$

The exact form of the correlation function $C(\vec{r}_1, \vec{r}_2) = C\left(\frac{\vec{r}_1+\vec{r}_2}{2}, \vec{r}_1-\vec{r}_2\right)$ is known only for some simple cases e. g. for the noninteracting Fermi gas model of nuclear matter or for the harmonic oscillator shell model of the nucleus. Nevertheless, we know quite enough about $C(\vec{r}_1, \vec{r}_2)$ to be able to estimate the role of nucleon-nucleon correlations in the averages (2.2a), (2.2b), (2.2c).

It is useful to introduce the parameter l_c, called the correlation length, which is defined as follows

$$l_c(\vec{R}) = \int_0^\infty dr C(\vec{R}, r), \quad \vec{R} = \tfrac{1}{2}(\vec{r}_1+\vec{r}_2), \quad \vec{r} = \vec{r}_1 - \vec{r}_2, \tag{2.7}$$

where we assumed that C depends on the distance r (but not on its direction). For infinite isotropic media l_c does not depend on $\vec{R}$, but for finite nuclei its values in the middle and on the surface of the nucleus may differ.

Various authors give different estimates for l_c. For example in [19] one finds $l_c = -0.85$ fm and this (rather large) value is supposed to be an effective value averaged over the $\vec{R}$-dependence. In Ref. [20] the authors give $l_c = -0.6$ fm. The lowest estimate of l_c comes from Ref. [21] as is $l_c = -0.3$ fm. Note that they are all negative. From the definition (2.4) we can see that this means a repulsion at small distances.

In this paper we express the effect of correlations only through the correlation length l_c. In doing so we hope that we do not oversimplify the problem. Our view is supported by the existing calculations for hadron-nucleus scattering in Glauber Model [19] which are rather insensitive to the particular form of the correlation function and depend mainly on l_c.

From (2.3) and (2.1) we get

$$\varphi(x) = \int d^2b \left[\sum_{m=0}^{E\left(\frac{A}{2}\right)} \frac{A!}{2^m m!(A-2m)!} S^{A-2m}(\vec{b}, x) T^m(\vec{b}, x) - 1 \right], \tag{2.8}$$

where

$$\begin{aligned} S(\vec{b}, x) &= 1-(1-x) \int d^3r \varrho^{(1)}(\vec{r}) \sigma_R(\vec{b}-\vec{s}), \\ T(\vec{b}, x) &= \int d^3r_1 d^3r_2 [\varrho^{(2)}(\vec{r}_1, \vec{r}_2) - \varrho^{(1)}(\vec{r}_1)\varrho^{(1)}(\vec{r}_2)] \\ &\quad \times (1-(1-x)\sigma_R(\vec{b}-\vec{s}_1))\,(1-(1-x)\sigma_R(\vec{b}-\vec{s}_2)) \\ &= (1-x)^2 \int d^3r_1 d^3r_2 \varrho^{(1)}(\vec{r}_1)\varrho^{(1)}(\vec{r}_2) C(\vec{r}_1, \vec{r}_2) \sigma_R(\vec{b}-\vec{s}_1)\sigma_R(\vec{b}-\vec{s}_2), \end{aligned}$$

where we used the property (2.6). From (2.2a — c) and (2.8) we get

$$\bar{v} = \frac{A\sigma_R}{\sigma_R^{(A)}}, \tag{2.9}$$

$$\begin{aligned} D_v^2 &= \overline{v^2} - \bar{v}^2 = \overline{v(v-1)} - \bar{v}(\bar{v}-1) \\ &= \frac{A(A-1)}{\sigma_R^{(A)}} \int d^2b \{[S(\vec{b}, 0)-1]^2 + T(\vec{b}, 0)\} - \bar{v}(\bar{v}-1), \end{aligned} \tag{2.10}$$

where

$$\sigma_R^{(A)} = \int d^2b \left[1 - \sum_{m=0}^{E\left(\frac{A}{2}\right)} \frac{A!}{2^m m!(A-2m)!} S^{A-2m}(\vec{b}, 0) T^m(\vec{b}, 0) \right]. \tag{2.11}$$

To perform numerical computations we assume $C(\vec{r}_1, \vec{r}_2) = C(\vec{r}_1 - \vec{r}_2)$, with l_c much smaller than the radius of the target nucleus, and give the following spatial distribution to $\sigma_R(s)$:

$$\frac{1}{\sigma_R} \sigma_R(s) = \frac{1}{2\pi a} e^{-s^2/2a}$$

with $\sqrt{a}$ much smaller than the radius of the target nucleus. With this approximation we obtain

$$S(\vec{b}, x) = 1-(1-x)\sigma_{\rm R}\tilde{D}(\vec{b}),$$

$$T(\vec{b}, x) = (1-x)^2 \xi_{\rm c} \sigma_{\rm R}^2 \int_{-\infty}^{\infty} dz \tilde{\varrho}^2(\vec{b}, z),$$

where

$$\tilde{\varrho}^2(\vec{b}, z) = \frac{1}{\pi a} \int d^2 s [\varrho^{(1)}(\vec{s}, z)]^2 e^{-(\vec{b}-\vec{s})^2/a},$$

$$\tilde{D}(\vec{b}) = \frac{1}{2\pi a} \int d^2 s \int_{-\infty}^{\infty} dz \varrho^{(1)}(\vec{s}, z) e^{-(\vec{b}-\vec{s})^2/2a},$$

$$\xi_{\rm c} = \frac{1}{4\pi a} \int d^2 s \int_{0}^{\infty} dz C(s, z) e^{-s^2/4a}.$$

$\xi_{\rm c}$ is an effective correlation length smeared out with the spatial distribution of the projectile-nucleon interaction. It reduces to the correlation length $l_{\rm c}$ when $l_{\rm c} \gg a$.

The generating function $\phi(x)$ may now be written explicitely as a power series in x

$$\varphi(x) = \int d^2 b \sum_{m=0}^{E\left(\frac{A}{2}\right)} \sum_{j=0}^{A-2m} \sum_{k=0}^{2m} \frac{A!}{2^m m!(A-2m)!}$$

$$x^{k+j} \binom{A-2m}{j} (-1)^k \binom{2m}{k} (1-\omega)^{A-2m-j} \omega^j \beta^m, \tag{2.12}$$

where

$$\omega = \sigma_{\rm R} \tilde{D}(\vec{b}), \quad \beta = \xi_{\rm c} \sigma_{\rm R}^2 \int_{-\infty}^{\infty} dz \tilde{\varrho}^2(\vec{b}, z).$$

In (2.12) a term with x^ν is the probability (multiplied by $\sigma_{\rm R}^{(A)}$) of ν inelastic collisions. We get therefore an expansion in powers of $\beta \approx 10^{-1}$:

$$\sigma_{\rm R}^{(A)} P_\nu = \int d^2 b \left\{ \binom{A}{\nu} \omega^\nu (1-\omega)^{A-\nu} + \beta \frac{A(A-1)}{2} \left[\binom{A-2}{\nu} \omega^\nu (1-\omega)^{A-2-\nu} \right.\right.$$

$$\left. -2 \binom{A-2}{\nu-1} \omega^{\nu-1} (1-\omega)^{A-2-(\nu-1)} + \binom{A-2}{\nu-2} \omega^{\nu-2} (1-\omega)^{A-2-(\nu-2)} \right]$$

$$\left. + \text{higher order terms in } \beta \left(\text{up to } E\left(\frac{A}{2}\right) \text{ order} \right) \right\}.$$

From (2.12) we get more explicit expressions than (2.10), (2.11)

$$D_\nu^2 = \frac{A(A-1)}{\sigma_R^{(A)}}\sigma^2 \int d_R^2 b\left[\tilde{D}^2(\vec{b})+\xi_c \int_{-\infty}^{\infty} dz\tilde{\varrho}^2(\vec{b}, z)\right] - \frac{A\sigma_R}{\sigma_R^{(A)}}\left[\frac{A\sigma_R}{\sigma_R^{(A)}}-1\right], \tag{2.13}$$

$$\sigma_R^{(A)} = \int d^2b\left[1-\sum_{m=0}^{E\left(\frac{A}{2}\right)} \frac{A!}{2^m m!(A-2m)!}(1-\omega)^{A-2m}\beta^m\right]. \tag{2.14}$$

In the optical limit we obtain:

$$\lim_{A\to\infty} \sigma_R^{(A)} = \int d^2b[1-\exp(-A\sigma_R\tilde{D}(\vec{b})+A^2\sigma_R^2\xi_c \int_{-\infty}^{\infty} dz\tilde{\varrho}^2(\vec{b}, z))]. \tag{2.15}$$

3. *Calculations and discussion of the results*

To illustrate the role of correlations and the spatial distribution of the hadron--nucleon production processes we computed $\bar{\nu}$, $D_\nu^2 = \overline{\nu^2}-\bar{\nu}^2$ from (2.9) and (2.10) and the average multiplicities and their dispersions given by the model of Ref. [9]

$$\bar{n}_A = \tfrac{1}{2}(\bar{\nu}+1)\bar{n}_H, \tag{3.1}$$

$$D_A^2 = \tfrac{1}{2}(\bar{\nu}+1)D^2+\tfrac{1}{4}D_\nu^2\bar{n}_H^2. \tag{3.2}$$

TABLE I

Average number of collisions and dispersions for σ_R = 30 mb and nuclear densities taken from Ref. [9]

A	$\bar{\nu}$	D_ν^2	D_A	$\bar{n}_A$	$D_A/\bar{n}_A$	ξ_c (in fm) a(in(GeV/c)$^{-2}$
27	1.76	1.08	6.66	11.75	0.57	
64	2.38	2.31	8.48	14.35	0.59	$\xi_c = -0.6$
80	2.58	2.77	9.06	15.20	0.60	$a = 11.5$
207	3.69	5.90	12.19	19.95	0.61	
238	3.90	6.58	12.75	20.81	0.61	
27	1.83	1.27	6.95	12.05	0.58	
64	2.44	2.68	8.90	14.64	0.61	
80	2.64	3.21	9.52	15.49	0.61	$\xi_c = 0.0$
207	3.75	6.70	12.79	20.19	0.63	$a = 11.5$
236	3.95	7.44	13.37	21.04	0.63	
27	1.95	1.47	7.28	12.55	0.58	
64	2.60	2.97	9.26	15.30	0.61	$\xi_c = 0.0$
80	2.80	3.51	9.87	16.19	0.61	$a = 0.0$
207	3.96	6.95	13.03	21.07	0.62	
238	4.16	4.64	13.57	21.95	0.62	

The numerical results for 300 GeV incident nucleons are collected in Table I and Fig. 1. The numerical values of average multiplicities and dispersions go down (relative to the results with no-correlations and point-like production) ~ 10% for light and ~6% for heavy nuclei. Such corrections should already be considered relevant (though not very

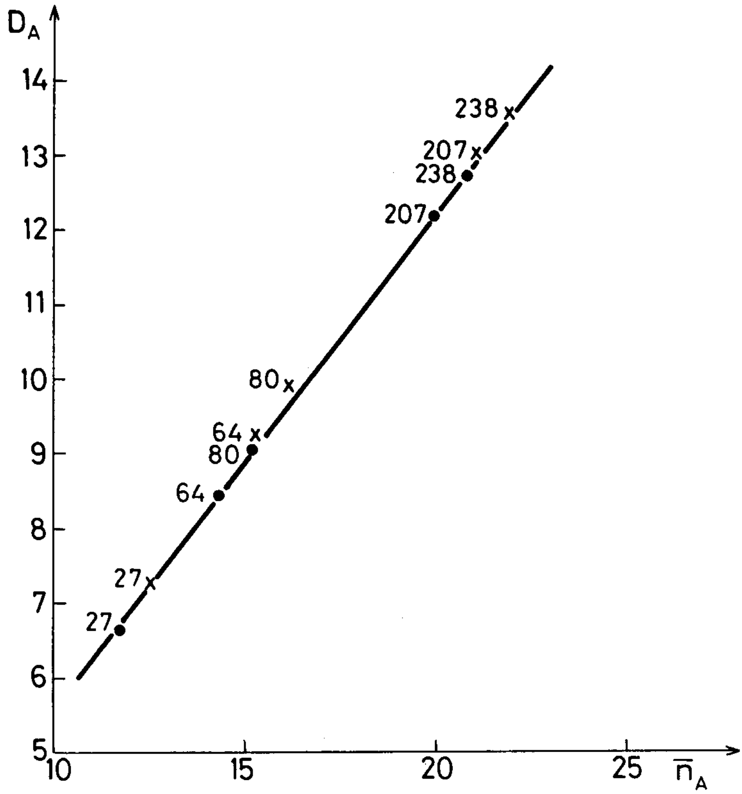

Fig. 1. Dispersion vs average multiplicity for 300 GeV nucleons interacting with five nuclei (whose atomic numbers are shown next to the points). The points were computed from Eqs (3.1) and (3.2) where we used $\bar{n}_H = 8.5$, $D_H = 4.23$, $\sigma_R = 30$ mb. Dots were obtained for $\xi_c = -0.6$ fm, $a = 11.5$ (GeV/c)$^{-2}$, crosses for $\xi_c = 0$, $a = 0$. The straight line is to guide the eye and to visualize the vertical shift of crosses relative to dots

important) in discussions of the recent experimental results. However, the plot of D_A vs $\bar{n}_A$ is hardly influenced by the corrections computed in this note: They shift the points along an approximately "universal" straight line (see Fig. 1). We have also computed the D_A vs $\bar{n}_A$ plot for π-nucleus collisions at 100 and 200 GeV with the same parameters of elementary collisions as in Ref. [6] and the correlated nuclear wave functions with $\xi_c = -0.6$ fm and $a = 7.5$ (GeV/c)$^{-2}$. We obtained a D_A vs $\bar{n}_A$ curve undistinguishable from the one of Ref. [6]. Again, the points corresponding to specific nuclei were merely shifted along the curve D_A vs $\bar{n}_A$ without deviating from it.

Finally, to see how an analysis of an emulsion data is influenced by correlations, we computed P_ν and $\bar{\nu}$ for emulsion with $\sigma_R = 32$ mb from the formulae

$$P_\nu = \frac{\sum_A N_A \sigma_R^{(A)} P_\nu^{(A)}}{\sum_A N_A \sigma_R^{(A)}}, \tag{3.3}$$

$$\bar{\nu}_{Em} = \sigma_R \frac{\sum_A N_A A}{\sum_A N_A \sigma_R^{(A)}}, \tag{3.4}$$

where N_A is the number of nuclei A in cm^3. We found $\bar{\nu}_{Em} = 2.77$ with $\xi_c = 0$, $a = 0$ and $\bar{\nu}_{Em} = 2.52$ for $\xi_c = -0.6$, $a = 11\,(\mathrm{GeV}/c)^{-2}$, hence a 9% negative correction.

The authors are grateful to Dr J. Babecki whose interest in very accurate values of the average number of collisions stimulated these computations.

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