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ON THE ENERGY LEVEL STRUCTURE OF THE ^{10}B NUCLEUS

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The clustering nature of each energy level of the ^{10}B nucleus has been investigated considering admixed wave functions involving higher radial quanta and in an overlap integral formalism. By evaluating the ground state nuclear quadrupole moment a suggested parameter of deformability of the nucleus in that state is estimated from the experimental value.

1. *Introduction*

Cohen and Kurath [1] shell model calculations of the ^{10}B nucleus are often quoted in literature for comparison with experimentally observed levels. Two types of studies to improve on the shell model wave functions have been recently reported, following the work of Cohen and Kurath. The first by Varma and Goldhammer [2] which considers the influence of effective three body interactions on the normal shell model levels. Second type of work involves consideration of the iso-spin mixing effect of shell model wave-functions and a number of investigations are directed towards the study of this effect [3–6].

Varma and Goldhammer adopted the wave functions obtained by the Goldhammer, Hill and Nachamkin [7] method, to estimate static magnetic moments, M1 transition rates, two nucleon and single nucleon spectroscopic factors and Gamow–Teller transitions, but they caution that their adopted wave functions are best suited only to calculate the energies.

They point out the following important features for $A = 10$ nuclei.

1. A definite trend away from LS coupling seems to occur.
2. The ground state magnetic moment agrees with the experimental value but the Gamow–Teller transition

$$^{10}\mathrm{Be}(JT = 01) \rightarrow {}^{10}\mathrm{B}(JT = 10)$$

is strongly inhibited.

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3. The spectroscopic factor from the ground state ^{9}Be to the $J = 0$, $T = 1$ level of ^{10}B is about 0.98 while the experimental value is 1.67. It may be noted that Cohen and Kurath estimate this to be about 2.35.

Relative spectroscopic factors S_{re} between the companion (d, n) and (^{3}He, d) reactions leading to the same final states having different iso-spin in odd-odd light mass nuclei exhibited a number of discrepancies. The disparity has been claimed to be most distinctive between the (d, n) and (^{3}He, d) of the ^{10}B nucleus. The S_{re} value of the $T = 1$, 1.74 MeV level from (d, n) was found to be smaller than that for (^{3}He, d) by a factor of 3 when both of the reactions were normalized to unity. The (d, n) reaction at 7 MeV was studied by Buccino and Smith [8] while the (^{3}He, d) reaction at 17 MeV by Siemssen et al. [9].

At 5 MeV the reaction (d, n) was studied by Fife et al. [10] and at 10 MeV, the reaction (^{3}He, d) by Crosby et al. [11]. The two different (d, n) works, however, show significant differences in S_{re} value for the $T = 0$, 0.72-MeV level as well as for the $T = 1$, 1.74-MeV level, as found recently by Bingham et al. [12].

But the spectroscopic investigation made by Park et al. [13] on ^{10}B levels from the ^{9}Be (d, n) ^{10}B reaction, shows that the extent of discrepancy in the spectroscopic factors between (d, n) and (^{3}He, d) leading to the states with different isospin within the same final nucleus ^{10}B, is not as large as it had been previously reported, a factor of < 2 instead of ~ 3.

Moreover, the (^{3}He, d) reaction systematics shows dependence on deuteron energy, hence it does not confirm the discrepancy established, unless this is studied at much higher energies where interference from the compound-nucleus mechanism is small. Inclusion of such effects, as the two-step process, the recoil effect, the D-component of the deuteron wavefunction etc. may explain the discrepancy to an extent.

Again a non-resonant contribution to the reaction of γ-ray yield from the reaction ^{9}Be (p, γ)^{10}B has been cited, by Renan et al. [14] as a factor which leads to substantially reduced values for the iso-spin mixing coefficient. Large non resonant contributions are found especially for transitions to the low-lying excited states, with the process involved being direct radiative capture.

Tamura [15] attempted to resolve the discrepancy by including a charge-exchange coupling process between (d, p) and (d, n) channels but he found it insufficient to give the desired results. Robson and Catonch [16] more recently attempted, with the idea of the charge exchange process represented, by the iso-spin dependence of distorted wave and form factors, to resolve this discrepancy. Their preliminary findings show that the discrepancy in S_{re} values virtually disappears.

2. *α-cluster effects on ^{10}B levels*

Shell model calculations for $A = 6-14$ with a realistic interaction have been reported by Hague and Maripuu [17]. Their conclusions are

(1) It is necessary to assume the single particle energy splitting to vary for each mass number.

(2) The three types (2p, 3plh, 4p2h) of second order perturbation corrections are found to be important.

(3) The harmonic oscillator size parameter b is found to be relatively constant this is perhaps the reason why many-parameter fitting procedures of Cohen and Kurath [18], Norton et al. [19], Goldhammer et al. [7], Varma et al. [2] are so highly successful.

(4) $b = 1.4$ fm yields the best results for $A = 8$ spectrum indicating that the Sussex and Hamada–Johnston potentials are of a similar nature.

But there are important failures of these calculations (1). They fail to account for the lowest three "α-cluster" states of ^{8}Be (2); the predicted values of the 7.35 and 10.3 MeV of ^{12}C, "α-cluster" states are 3 MeV too high, as in other shell-model calculations. (Halbert et al. [20], Inglis [3], Barker [5], Norton et al. [19]).

3. *Spin assignments and experimental levels*

Spin and parity assignments of the ^{10}B nucleus determined experimentally prior to 1966 have been sumarized by Lauritsen and Ajzenberg–Selove [21]. During the last decade, however, new wealth of information has been accumulated about the level structure

TABLE I

The experimental energy level structure of the ^{10}B nucleus according to Stehle et al. The paranthesized values are the ambiguous assignments

E	J^π	T
(MeV ± KeV)		
4.774 ± 3	(2^+)	0
5.114 ± 4	$(2)^-$	0
5.166 ± 4	2^+	1
5.183 ± 8	1^+	0
5.923 ± 4	2^+	0
6.133 ± 4		
6.566 ± 6		
6.884	1^-	0
7.00		
7.431 ± 10	2^-	0
(7.468 ± 10)	(2^+)	
7.479 ± 2	(2^-)	(1)
7.561 ± 1	0^+	1
7.62 ± 50	(1^+)	(0)
7.77 ± 30	2^-	1
8.07 ± 100	(2^-)	(0)
8.892 ± 6	$3(^-)$	(1)
8.896 ± 2	2^+	1
9.7		1
10.83 ± 30		1
11.4	(+)	1
14.0	(+)	

of this nucleus. For example, a report in 1974 of low-lying levels of ^{10}B, by Karadeniz [22] who adopted a new emulsion method to examine neutrons from ^{9}Be (d, n) ^{10}B reaction, mentions that the 2.9 MeV level may be actually composed of two levels, one at 2.75 MeV and the other at 3.17 MeV. Again the experimental studies of the break up reaction ^{10}B (α, 2α) ^{6}Li at $E = 24$ MeV performed by Stehle et al. [23] showed that sequential decay via excited levels of ^{10}B predominates, with negligibly small contributions of both the $\alpha-\alpha$ quasifree scattering and sequential decay via ^{8}Be levels. The study reveals 17 new levels of ^{10}Be with excitation energies between 8.665 and 16.0 MeV. The list of levels given by Stehle et al. is given in Table I.

4. *High energy states of* ^{10}B

The long lived states at high excitation energies in ^{8}Be and ^{10}B have also been studied by Ajzenberg-Selove et al. [21] by means of the reactions ^{9}Be (^{3}He, α) ^{8}Be and ^{11}B (^{3}He, α) ^{10}B conducted at E (^{3}He) = 49.3 MeV. Three new states at 22.05, 22.63 and 22.98 MeV (± 0.1 MeV) have been observed for the ^{8}Be nucleus. A new state ^{10}B (13.49) and two broader states of ^{10}B are also reported by them. Confirmation of the previously observed levels of ^{10}B (10.85, 11.51 and 12.55) with slightly higher widths is another outcome of these studies.

Purvis et al. [24] found only the states at 11.53 ± 0.04 and 12.57 ± 0.03 MeV to be definitely quite sharp. But Fisher [25] gave an account of ^{9}Be (p, γ_0) and (p, γ) experiments which indicate several more sharp reasonances in the range 10.8 – 19.7 MeV. The results of ^{9}Be (p, p_0) by Votava et al. [26] ^{10}B (e, e′) by Kossanyi-Demay et al. [27] and ^{11}B (p, d) by Bachelier et al. [28] are not conclusive.

By ^{9}Be (p, d) ^{10}Be reaction, Anderson et al. [29] have reported states at 9.27, 9.4, 10.57 ± 0.03, 11.76 ± 0.02 MeV while by the study of ^{9}Be (p, π^{+}) ^{10}Be the analogue states of ^{10}B at 11.0, 11.1, 12.3 and 13.5 MeV have been stated to occur by Dahlgren et al. [30].

5. *Radiative capture on* ^{10}B

Baer et al. [31] have recently studied the (π, γ) reaction on ^{10}B and ^{14}N targets for the photon spectra in the capture of stopped pions (π^{-}). They confirm the 1.74, 5.11 and 7.477 MeV levels of ^{10}B. The findings of these experiments agree with the calculations of Mukopadhyay [32] for μ capture which predict the existence of an additional 2^{+} state at 8.89 MeV and 4^{+} state at about 10 MeV. The ^{10}B (π, γ) data show little resolved structure in the giant dipole resonance region and no clear separation between quasi-free and resonance capture could be ascertained.

The states with strong 'M1' transitions to the (3^{+}, $T = 0$), (0^{+}, $T = 1$), (1^{+}, $T = 1$) and similar states of ^{10}B are those which could give rise to strong (π, γ) transitions. A few of these states studied by Baer et al. [31] are (2^{+}, $T = 1$), (3^{+}, $T = 1$) and (4^{+}, $T = 1$). The *jj* coupling wavefunctions, constructed within an SU(3) scheme for these levels and for the higher excited 2^{+} states are given by them. The eigen-states, described in this manner are, however, found not to possess a simple structure in the Wigner supermultiplet scheme. The reason being that the use of an additional quantum number, as in the Wigner supermultiplet theory, requires one to distinguish the states of a degeneracy under that quantum number and the states thus generated do not have a physical meaning.

6. The model and the method of calculations

The ^{10}B nucleus has been chosen for such a study since it may be regarded as a composite of α-clusters plus an extra two nucleons. Because of the arguments cited above it is possible to assume that the α-clusters are separated by a distance $R = 2.8$ fm. The degree of freedom in the Berggren's formalism allows one to adopt the following simple and resonable set of wave functions to evaluate the energy level scheme of the ^{10}B nucleus.

We assume that an energy level of the ^{10}B nucleus is characterized by the orbital angular momentum L, the total angular momentum J, iso-spin T and the set of values (p, q) which arise in[1]

$$\Psi_{JLT} = \Psi_{JL}\psi_T \tag{1}$$

with

$$\psi_{JL} = \sum_i C_i(\xi_1^p\xi_2^q \pm \xi_1^q\xi_2^p)e^{-\alpha(\xi_1{}^2+\xi_2{}^2)}\chi_s, \tag{2}$$

where ξ_1 and ξ_2 represent the two nucleon position co-ordinates in a central co-ordinate system of the ^{10}B nucleus and χ_s the spin part of it. The C_i are the coefficients of linear combination which are to be determined. The ± sign in the wave function ψ_{JL} is determined by the appropriate symmetry character of how the "L" couples with "S", the spin angular momentum giving rise to the total angular momentum "J". In turn for each set of JT values, a number of possible wave functions may be generated with the differing sets of lowest possible values of (p, q) which by *an additive law* give the L value.

We concur with the argument of Berggren that a number of nuclear quantities are expected to be reproduced with a chosen set of wave functions, a fact which largely makes up for the lack of simple orthogonality and completeness relation of the same. The ψ_{JLT} set is analogous with the symbolic nuclear overlap integral $\langle\varphi(\xi_1)\varphi(\xi_2)\varphi_A^\alpha\varphi_B^\alpha|\Phi\rangle$ of two--particles and two clusters of a nucleus. Here $\varphi(\xi_1)$ and $\varphi(\xi_2)$ represent the two nucleon part of wave functions, the φ_A^α, φ_B^α, stand for the two α-clusters of the ^{10}B nucleus, and Φ the total nuclear wave function of the nucleus.

Eq. (9) of Berggren's formalism [33] shows clearly how a nuclear overlap integral may be utilised to evaluate the energy levels of a nucleus in terms of those of its clusters. The analogy implies however the following important features.

1. Higher radial quanta[2] of nucleon motion are involved in ψ_{LJT}.

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[1] (a) The above wave function equation (2) is analogous with the wave function given by Dreizler et al. [54] in equation (3.2) of their paper on two centre Hartree-Fock problems. (b) Lin et al. [55] in their paper on spontaneous fission half-life for ^{8}Be state that there is distance of 1.8 fm units only between α-particles, each of radius 1.7 fm with little overlap of the two α-clusters.

[2] Berggren in his article (Ref. [33]) states by theoretical considerations that the overlap integral may involve admixtures of many different radial quanta. Practical calculations on ^{18}O nucleus involving higher orbital quantum states have been earlier reported by Macfarlane and French, *Rev. Mod. Phys.* **32**, 567 (1960).

2. The relative distance between the centres-of-mass of the clusters is considered to be a constant.

3. Explicitly in ψ_{LJT} the α-particle wave functions do not occur since we presume that the two nucleons move essentially in an independent shell model average potential field characterized by two centres[3].

Again the question of whether the wave functions given above are really the nuclear overlap integrals or at least play a role similar to them and/or the argument for the lack of simple orthogonality properties (which is the case also in the earlier study on ^{6}Li nucleus) becomes evidently clarified only when such studies are performed on a number of other nuclei and comparative study is made. To that end the present study is just a beginning.

The method of the present work essentially involves solving the secular equation

$$|H_{ij} - E\Delta_{ij}| = 0, \tag{3}$$

for each set of JT values, assuming that the isospin is a reasonably good quantum number. The Hamiltonian H_{ij} is written down taking into account:

(*i*) δ interaction potential between the two extra nucleons as,

$$\frac{V}{\xi_1 \xi_2}\,\delta(\xi_1 - \xi_2)\delta(\eta_1 - \eta_2)\delta(\varphi_1 - \varphi_2), \tag{4}$$

where $\eta = \cos\theta$ and φ are the angle variables and V strength of the local interaction;

(*ii*) The kinetic and potential energies relative to the centres-of-mass of the two α-clusters;

(*iii*) The nature of the exchange potential as a Rosenfeld mixture given by

$$-0.13P_W + 0.93P_M + 0.46P_B - 0.26P_H \tag{5}$$

where P_W, P_M, P_B and P_H are the Wigner, Majorana, Bartlett and Hisenberg projection operators;

(*iv*) A parameter U is also considered to take into account the combined effects of the respective binding energies of the clusters, a possible interaction between the clusters separated by a fixed distance R as mentioned above and the absolute energy of a ^{10}B nuclear level.

In Table II are listed the set of $(pq)LSJT$ values adopted to evaluate the energy levels of the ^{10}B nucleus.

A computer programme has been developed to evaluate energy levels treating U, V, α as variable parameters typical for each specified J, T set of energy levels. For each (J, T) value these three parameters denoted by U, V, α are determined so as to obtain the best agreement with experimental data, the object being that whenever we find agreement with experiment, we can express all experimental nuclear energies in a certain region in

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[3] From nuclear Hartree-Fock calculations it has been shown by Muthukrishna and Baranger, *Phys. Lett.* **18**, 160 (1965), that harmonic oscillator functions are good as approximations to the single particle wave functions. For a two centre Hartree-Fock problem refer the work by Dreizler, Galbraith and Lin, Proc. of Int. Conf. on Nuclear Structure and Spectroscopy, part. I, Amsterdam 1974, p. 19, [54], [55].

TABLE II

The list of (p, q) LSJT values adopted in the computer programme and the values of exponent parameter for each JT set which give the best fit to the experimental energy levels as in Fig. 1. S.: symmetric, A.S.: antisymmetric

p	q	L	S	J	T	2α
3	1	$3^{+\text{A.S.}}$	$0^{\text{A.S.}}$	$3^{+\text{S.}}$	$0^{\text{A.S.}}$	0.550
2	0	$2^{+\text{S.}}$	$1^{\text{S.}}$			
1	1	$2^{+\text{S.}}$	$1^{\text{S.}}$			
0	0	$0^{+\text{S.}}$	$0^{\text{A.S.}}$	$0^{+\text{A.S.}}$	$1^{\text{S.}}$	1.50
1	1	$0^{+\text{S.}}$	$0^{\text{A.S.}}$			
0	0	$0^{+\text{S.}}$	$1^{\text{S.}}$	$1^{+\text{S.}}$	$0^{\text{A.S.}}$	0.75
2	0	$2^{+\text{S.}}$	$1^{\text{S.}}$			
1	1	$2^{+\text{S.}}$	$1^{\text{S.}}$			
2	0	$2^{+\text{S.}}$	$0^{\text{A.S.}}$	$2^{+\text{A.S.}}$	$1^{\text{S.}}$	3.20
1	1	$2^{+\text{S.}}$	$0^{\text{A.S.}}$			
2	0	$2^{+\text{A.S.}}$	$0^{\text{A.S.}}$	$2^{+\text{S.}}$	$0^{\text{A.S.}}$	4.25
3	1	$2^{+\text{A.S.}}$	$0^{\text{A.S.}}$			
1	0	$1^{-\text{A.S.}}$	$0^{\text{A.S.}}$	$1^{-\text{S.}}$	$0^{\text{A.S.}}$	0.750
1	0	$1^{-\text{S.}}$	$1^{\text{S.}}$			
3	0	$3^{-\text{A.S.}}$	$1^{\text{S.}}$	$2^{-\text{A.S.}}$	$1^{\text{S.}}$	4.290
1	0	$1^{-\text{S.}}$	$1^{\text{S.}}$	$2^{-\text{S.}}$	$0^{\text{A.S.}}$	2.95
2	1	$1^{-\text{S.}}$	$1^{\text{S.}}$			

terms of a few parameters. In an ultimate theoretical formulation these parameters should be calculable from realistic nuclear cluster models, and the estimated parameters as mentioned above from the analysis of experimental data may be useful to this end.

However, these parameters can not be treated entirely as free parameters, since there are a number of criteria which fix their range of probable variation. The parameter U can not be expected to change by more than 8 MeV, which is the value of the separation energy of a single nucleon. One may contend against this criteria by the argument that U involves other types of energy contributions also. But we note that the saturation property of nuclear forces does certainly restrict this parameter U from changing too strongly in going from one set of (J, T) to another one. An examination of the data given in Fig. 1 shows indeed that the change in the value of U is not beyond 6 MeV. The two nucleon interaction parameter V may also be expected to change appreciably from one set of (J, T)

to another, since the exchange contributions differ much among these different levels. Also the expectation values of kinetic and potential energies relative to centres-of-mass of two clusters are different and characteristic of each set of (J, T) levels. Again from the data and Fig. 1 we note the change in V is about 50 MeV, a reasonable value in comparison with "free" two nucleon interaction energy changes in different quantum orbital states.

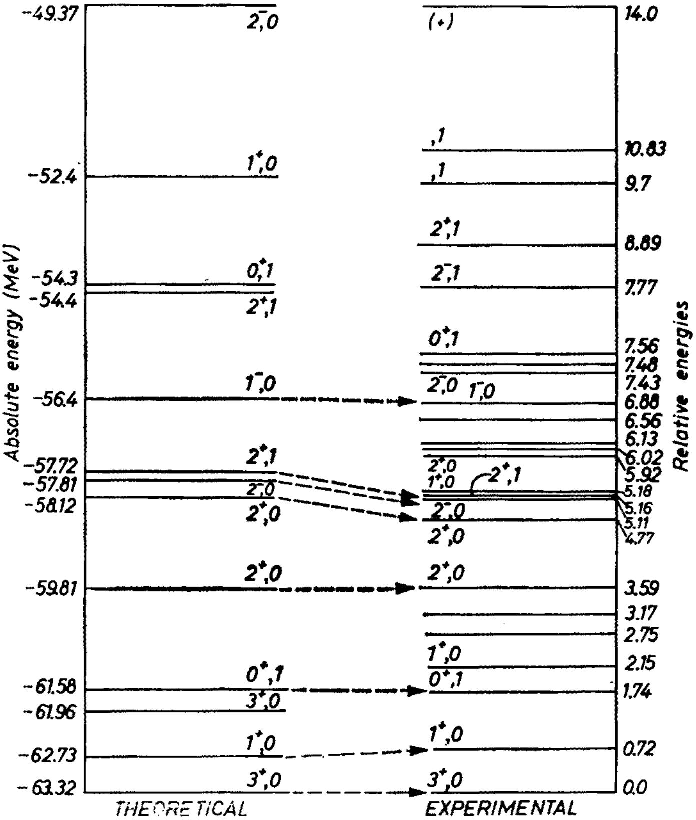

Fig. 1. Comparison of the theoretical and experimental energy level schemes

The parameter α characteristic of a given J, T set of levels, represents the totality of the exponents of the set of adopted wave functions.

The initial choice of the value of the α parameter for the computer programme has been made utilising the fact that it is an average, over the two individual nucleon wave function exponent values with different (L, S) values constituting each chosen set of (J, T) values. The two individual nucleon wave functions are characterized by the set of (pq) values. For the sake of clarity, one may regard the set of (pq) values analogous to the shell model orbital quantum numbers. In contrast we note that our model which adopts a unique α parameter value for each (J, T) set, treats the nucleus characterized by that set of (J, T) values as exhibiting a collective behaviour of individual nucleons. The i and j of Eq. (3) refer to two different types of possible wave functions within a set of (J, T) values. Numerical calculations have been performed on a CDC 3600 computer to evaluate the Eq. (1) for the following set of eight (J, T) values.

$$[3^+, 0], [0^+, 1]\ [1^+, 0], [2^+, 1], [2^+, 0],$$

$$[1^-, 0], [2^-, 1], [2^-, 0].$$

The values of the exponent parameter 2α giving the best fit with experimental nuclear energy level data of the ^{10}B nucleus are listed in Table II, corresponding to each of the (J, T) set of values.

7. *Comparison of the theoretical and experimental energy levels*

The comparison of the calculated and experimental energy levels is made in Fig. 1. The dotted lines in the Fig.1 relate the corresponding predicted levels of the present work with those experimentally observed.

Purvis et al. [24] and Ajzenberg-Selove et al. [34] have compiled the data for the energy levels of the ^{10}B nucleus and we note from these studies that the levels below 9 MeV have been well established experimentally.

0.717 MeV level

Historically there was a controversy whether this level is to be assigned a 3^+ or 1^+ spin-parity. Recent shell model calculations of Cohen and Kurath confirm that only one 3^+ occurs below 4 MeV. Both the Park et al. [13] and Cohen and Kurath investigations agree that the second $(3^+, 0)$ exists at about 4.77 MeV.

Model calculation of the present work reproduced the 0.717 MeV level to be $(1^+, 0)$ with an absolute energy of -62.73 MeV, in accordance with currently accepted spin-parity assignment for this level.

1.74 MeV level

From the time of flight method studies of $p + {}^{13}C$ reaction products, Oberg et al. [35] find that the angular distributions indicate that the 1.74 MeV level of ^{10}B has $J^\pi = 0^+$. But at certain proton energies they find interesting dips in the 1.74 MeV state cross-section, a feature which is interpreted as due to interference involving an unidentified $T = 1$ state. A similar dip is also observed in the 5.17 MeV ^{10}B level cross-section for the reaction ^{13}C (p, α) ^{10}B, at nearly the same proton energy. Whether the features are related is unclear.

Model calculations of the present work generates this level to be at an absolute energy -61.58 MeV.

Levels around 2.9 MeV

First Bonner and Brubaker [36], then Stub and Stephens [37], Dyer and Bird [38], Reid [39], Genine [40], Hjalmar and Statis [41], Galloway and Sillitto [42], Combe and Walker [43], Srivastava and Saha [44] claimed the presence of an energy level around 2.9 MeV. Combe and Walker [43] were the first to propose that probably the 2.9 MeV is a composite level, the constituents of this level being one at 2.7 MeV and the other at 3.1 MeV.

But the time of flight experiments have not given any evidence of such levels except for the one by Good [45].

Karadeniz, using a new emulsion method and examining the neutrons from the ^{9}Be (d, n) ^{10}B reaction at deuteron energy of 600 KeV on a ^{9}Be target of 4 mm thickness, could confirm that the controversial 2.9 MeV level is composed of two levels, one at 2.75 MeV and the other at 3.17 MeV. The spin and parity assignments of these two separate levels are not available.

The model of the present work implies (a possibility) that one of these can be the second (3^+, 0) level, unlike the Cohen and Kurath prediction that this level occurs at 4.72 MeV. (Othei possibilities are also given in what follows below). Exact reproduction of these levels by suitable adjustment of parameters based on the model of the present work need only to be made, when the level assignments of the 2.75 and 3.17 MeV are known experimentally.

3.59 MeV level

The 2^+ assignment of spin-parity has been preferred by Park et al. [13] instead of (3^+) since, angular distributions of (d, n) fit well with calculations involving an admixture of $p_{1/2}, p_{3/2}$ and $f_{7/2}$ orbitals. The existence of this level has been confirmed by most of the studies made to-date on the ^{10}B nucleus.

The calculations of the present work shown that (2^+, 0) assignment for this level is reasonable.

4.77 MeV level

Crosby and Legg [11], Meyer-Schützmeister et al. [46] and Warburton et al. [47] assigned the spin-parity of this level as 2^+. But Stehle et al. [23] exclude this possibility and assign the spin 3. Park et al. [13] confirm, following the shell model prediction of a (3^+, 0) level at 4.72 MeV, that the 4.77 level has the 3^+ spin-parity assignment.

The Park et al. [13] studies of ^{9}Be (d, n) ^{10}B reaction indicates this level to be weakly populated and not well resolved from the ground state of ^{13}N.

Our calculations indicate the possibility that 4.77 MeV has the spin-parity, isospin assignment (2^+, 0).

The second lowest (3^+, 0) level of our model occurs at around 1.09 MeV, but it may be shifted up by a parameter adjustment. But if we adopt the 4.77 MeV level to have a (2^+, 0) assignment, then the second lowest (3^+, 0) level of our calculations can by a parameter adjustment be identified with one of the two levels, viz. the 2.75 MeV level or the 3.15 MeV level. But it is reasonable to attempt this only after an experimental confirmation and spin-parity assignments of these two latter levels are available.

5.11, 5.16, 5.18 MeV levels

5.18 MeV level could not be resolved in the ^{9}Be (d, n) ^{10}B work of Park et al. [13] which made it difficult to confirm its assignment (1^+, 0) as the previous (d, n) works indicated. Dearnaley et al. [48], and Meyerhof et al. [49] assigned 5.11 MeV to be a 2^-, the 5.16 MeV (and 5.92 MeV) level to have 2^+, and the 6.02 MeV level to be a 4^+. Forsyth et al. [50] from the ^{9}Be (^{3}He, d) ^{10}B reaction studies find these assignments to be correct.

5.16 MeV level has the assignment (2^+, 1) according to the calculations in the present work. The 5.18 MeV (1^+, 0) level is not reproduced readily by the model predictions in the present work.

5.92 MeV level

A remarkably good fit of the calculations has been obtained by Park et al. to assign this level to be a 2^+ state. On the other hand the Cohen and Kurath shell model calculations give rise to two 2^+ levels, one at 5.53 MeV with $T = 0$ and the other level at 5.58 MeV with $T = 1$. Park et al. could not ascertain which of these isospin assignments would correspond to the observed 5.92 MeV level.

This level, our model implies, in the case the predicted (2^+, 0) level identification as the one occuring at the experimentaly observed level 4.77 MeV is valid, may be a centre--of-mass excitation level of the α-clusters to the 2^+ angular momentum state and coupled to the zero angular momenta of the two extra nucleons.

6.024 MeV and 6.03 MeV levels

Stehle et al. [23] and Forsyth et al. [50] favour the assignment 4^+ for the 6.024 MeV level while the 6.03 MeV level suggested by the (d, n) work of Fife, Neilson and Dawson [10] could not be confirmed by Park et al.

5.58 MeV and 6.40 MeV levels

According to Forsyth et al. [50], the existence of these levels could not be confirmed by the (^{3}He, d) reaction studies.

6.13 MeV level

The spin-parity of this level is most controversial. The shell model calculations of Cohen and Kurath predict a 1^+ level at 6.19 MeV. Park et al. and Young, Lindgren and Reichart [51] favour a negative parity assignment, with a spin assignment of 3 by the latter workers, while the (^{3}He, d) works by Forsyth, Knudson and Young [50] and by Crosby and Legg [11] propose this as a positive parity state.

6.57 MeV level

The parity assignment for the level is controversial. A positive parity assignment favoured by the work on (^{3}He, d) while ^{6}Li$+\alpha$ scattering experiment indicates a negative parity assignment.

6.88 MeV, 744 MeV levels

Cooper et al. [52] and Roush et al. [53] suggested that 7.44 MeV level in ^{10}B has $J^\pi = 1^-$ and $T = 1$. The ^{6}Li$+\alpha$ experiments lead to the assignment of 6.88 MeV level as $J^\pi = 1^-$ and an isospin $T = 0$. Although transitions to the 0.72 and 2.15 MeV levels from the 6.88 MeV level are forbidden by $\Delta T = 0$, selection rule, they are observed, hence it may be an admixed state of about 25% isospin $T = 1$. Renan et al. [14] estimate the wave function of 6.88 MeV level to contain $\alpha = 0.92$, $T = 0$ component, and $\beta = 0.39$, $T = 1$ component. 7.44 MeV level is the other possible isospin mixed pair.

7.497 MeV, 7.561 MeV and 7.77 MeV levels

Previous investigations [8, 10–12, 21, 50] assign $(J^\pi, T) = (2^-, 1), (0^+, 1)$ and $(2^-, 1)$ respectively for these levels. Stehle et al. find these assignments to hold good from the studies they made on the sequential decay at $E_\alpha = 24$ MeV of the break up reaction $^{10}\mathrm{B}\,(\alpha, \alpha)\,^{10}\mathrm{B}^*\,(\alpha)\,^{6}\mathrm{Li}$.

8.896 MeV level

This has the experimental assignment $J^\pi = 2^+$, $T = 1$ (Stehle et al. [23]).

Other levels

The spin-parity assignments are not certain for levels higher than 9 MeV in Fig. 1, although levels up to 16 MeV are listed.

8. *Ground state quadrupole moment*

Since the method used here essentially involves a nonorthogonal and incomplete set of wave functions, a recourse has to be made to the Schmidt orthogonalisation procedure to obtain the exact wave functions for estimating static properties such as nuclear quadrupole moments [56].

The coefficients of linear combinations of the wave function fot the ground state are calculated by Schmidt's orthogonalisation procedure to be as follows:

$$C_a = +0.4123 \times 10^{-2},$$

$$C_d = -0.5186 \times 10^{-2},$$

$$C_c = -1.008 \times 10^{-2}.$$

for $\Psi = C_a\Psi_a + C_d\Psi_d + C_c\Psi_c$, where Ψ_a, Ψ_d, Ψ_c, respectively are the three wave function with (p, q) $LSJT$ values listed in Table II, for $J = 3$, $T = 0$.

The nuclear overlap integral formalism approach implies the definition of quadrupole moment as,

$$Q = Q_{\mathrm{sub}} + Q_\psi, \tag{6}$$

where Q_{sub} is the contribution from the subsystems evaluated relative to an origin is situated at the centre of ^{10}B nucleus, and Q_ψ is the quadrupole contribution arising from the linear combination of the wave functions defined in Eq. (2). We get

$$Q_{\mathrm{sub}} = +4e\left(\frac{R}{2}\right)^2, \tag{7}$$

where e is the proton charge. The quadrupole moment[4] operator Q_{op} for an extra nucleon at the position (ξ_1, η_1) is defined to be

$$Q_{\text{op}} = -\left\langle \frac{e}{2}\,[3(\cos\theta_1+\varepsilon)^2-1]\xi_1^2 \right\rangle$$

$$= -\left\langle \frac{e}{2}\,[3(\eta_1+\varepsilon)^2-1]\xi_1^2 \right\rangle ,$$

where ε is expected to indicate the contribution from higher order $(p\, q)$ set of values neglected and of the angular excitation contributions. Utilising the experimentally [57] observed value of the quadrupole moment 0.074×10^{-24} cm^2 the value of ε is calculated to be 1.71 for the ground state. The value thus obtained for ε seems to be a reasonably[4] low value.

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