EXTENSION OF THE TOTAL-ABSORPTION MODEL TO INCLUDE PHASE AND SPIN*

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Using crossing symmetry and derivative helicity relations, we show how the total-absorption model may be generalized to include phase and spin effects.

In a recent paper [1] we proposed a model of high-energy hadron-hadron elastic scattering where the first L partial-waves are totally absorbed. We choose L such that the total cross-section had the same energy dependence as the Froissart bound [2], i. e., $L = CS^{1/2} \log S$ and $\sigma_T = 8\pi C^2 (\log S)^2$: C is a constant. We found that the elastic cross-section, the total inelastic cross-section, and the diffraction slope increased in the asymptotic energy region as the square of the logarithm of the energy. In addition, we determined the energy dependence of the differential cross-section at 90° and 180°, and showed, in the near-forward direction, that the amplitude scales.

In this total-absorption model of high-energy scattering, the amplitude is pure imaginary [1]. We did not take intó consideration either phase (real part of the amplitude) or spin effects. The purpose of this paper is to extend this model to include phase and spin effects. We consider, in particular, pion-nucleon elastic scattering.

For pion-nucleon scattering we have two helicity amplitudes [3], $H(S, t)$ and $G(S, t)$; $H(S, t)$ will denote the helicity non-flip amplitude and $G(S, t)$ the helicity-flip amplitude.

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S and t are the usual Mandelstam variables. Our amplitudes are normalized such that the total cross-section and polarization are given by the following expressions,

$$\sigma_T = 4\pi^{1/2}\,\text{Im}\, H(S, t), \tag{1}$$

$$P(S, t) = 2\,\text{Im}\, HG^*/(HH^* + GG^*). \tag{2}$$

Our model may be stated as follows: (*i*) The imaginary part of $H(S, t)$ is obtained by using the result given in reference [1], equation (4). (Note that the relation between the amplitude f of reference [1] and the amplitude H is,

$$\text{Im}\, H = 2\pi^{1/2} f/S^{1/2}.) \tag{3}$$

(*ii*) We calculate the full amplitude, $H(S, t)$, by demanding that $H(S, t)$ be even under crossing, i. e., $H(S, t)$ is crossing symmetric. We may do this by replacing S by $(-i, S)$ in the amplitude f in Eq. (3), [3]. Since, we are in the asymptotic energy region, we will only calculate real-part effects of terms of order $(\pi/\log S)$. (*iii*) To calculate the helicity-flip amplitude, $G(S, t)$, we use the following derivative relation [4,5]

$$G(S, t) = [\lambda/\log(-iS)]\,[dH/d(-t)^{1/2}], \tag{4}$$

where λ is a constant [6].

We now give the properties of our model which follows from the above three assumptions.

First of all, we find that the scattering takes place mainly in the near-forward direction with a backward peak that decreases with energy [1]. In this paper, we will concentrate our consideration on scattering in the near-forward direction. Secondly, we find that in the near-forward direction, i. e., inside the diffraction region, the amplitudes $H(S, t)$ and $G(S, t)$ scale. This means that $H(S, t)$ and $G(S, t)$, except for factors of $\log S$, are functions only of the variable $X = 2C(-t)^{1/2}\log(-iS)$. These amplitudes have the following forms [7],

$$H(S, t) = 2\pi^{1/2} C^2 i[\log(-iS)]^2\,[2J_1(X)/X], \tag{5}$$

$$G(S, t) = -[4\pi^{1/2}\lambda C^3 i]\,[\log(-iS)]^2\,[2J_2(X)/X]. \tag{6}$$

Note that, in the limit as t goes to zero, the helicity-flip amplitude is proportional to $(-t)^{1/2}$. Thus, it has the correct threshold singularity structure in t [3].

We define the phase, ϱ, of an amplitude as the ratio of the real to the imaginary parts of the amplitude. The phases of the helicity non-flip and -flip amplitudes are given, respectively, by the following relations,

$$\varrho(H, \overline{X}) = [\pi/\log S]\left\{1 - \left[\frac{\overline{X} J_2(\overline{X})}{2J_1(\overline{X})}\right]\right\}, \tag{7}$$

$$\varrho(G, \overline{X}) = [\pi/\log S]\left\{1 + (\overline{X}/8)\left[\frac{J_1(\overline{X}) - 3J_3(\overline{X})}{J_2(\overline{X})}\right]\right\}, \tag{8}$$

where $\overline{X} = 2C(-t)^{1/2} \log S$. Note that $\varrho(H, \overline{X})$ has a zero at approximately $\overline{X} = 2.4$; this corresponds to the real part of the helicity non-flip being zero at this point. The imaginary part of H has its first zero at $\overline{X} = 3.83$; see reference [1]. Note also that the real parts of the amplitudes are down by a factor of $(\pi/\log S)$ with respect to the imaginary parts.

We may also calculate and compare the slopes of the real and imaginary parts of the helicity non-flip amplitude. Given an amplitude M, we define the slope, Δ, in the following manner,

$$\Delta = d \log M(S, t)/dt|_{t=0}. \tag{9}$$

Using Eq. (9) and Eq. (5), we find that the slope of the real part of the helicity non-flip amplitude is $(C \log S)^2$ and the slope of the imaginary part is $(C \log S)^2/2$. Thus, the real part has twice the slope of the imaginary part. This is reflected in the fact that, as shown above, the real part has a zero at approximately $\overline{X} = 2.4$ and the imaginary part has its first zero at $\overline{X} = 3.83$.

Finally, we may calculate the polarization for near-forward scattering. Using Eqs (2), (5) and (6), and keeping terms only of order $(\pi/\log S)$, we find,

$$P = -[\pi C\lambda/\log S]\,(J_1^2+J_2^2-J_1J_3-J_0J_2)/(J_1^2+4\lambda^2C^2J_2^2), \tag{10}$$

where $\overline{X} = 2C(-t)^{1/2} \log S$ is the argument of the various Bessel functions.

In summary, we find that *all physical quantities scale*, up to factors of log S.

We conclude with the following comments. First-of-all, the exact nature of the derivative relation, given in Eq. (4), needs to be looked into [5]. Secondly, the application of an analysis, similar to what has been done in this paper, should be applied to scattering in the near-backward direction. Preliminary calculations indicate that for near-backward scattering all physical quantities scale, up to factors of log S [8].

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[8] We have also calculated $G(S, t)$, in our model, using the following derivative relation, $G(S, t) = \lambda\, dH/d(-t)^{1/2}$, where λ is a constant; however, we found that the contribution of the helicity-flip amplitude to the elastic cross-section grows like $(\log S)^4$, which violates the Froissart bound.
[9] J_1 and J_2 are Bessel functions of first and second order, respectively. See, e. g., W. W. Bell, *Special Functions for Scientists and Engineers*, London 1968.
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