ASYMPTOTIC PROPERTIES OF SCATTERING AMPLITUDES WHERE THE EFFECTIVE NUMBER OF CONTRIBUTING PARTIAL-WAVES IS $L = CS^{1/2}(\mathrm{Ln}\, S)^M$*

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Upper and lower bounds are obtained on the asymptotic behavior of a class of scattering amplitudes. This class of amplitudes is defined by the following two assumptions: (A) the effective number of partial waves contributing to the scattering is $L = CS^{1/2}(\ln S)^M$; (B) as $S \to \infty$, the total cross section has the behavior $\sigma_T = \sigma_0(\ln S)^N$, where $N \leqslant 2M$. The results obtained are relevant to elastic neutrino scattering at asymptotic energies.

1. Introduction

In recent years much interest has been centered on obtaining asymptotic bounds on weak interaction cross sections [1–5]. Unlike the situation for the strong interactions, where only massive particles are exchanged and thus the Froissart bound may be obtained [6], i. e.,

$$\sigma_T < (\pi/\mu^2)(\ln S)^2, \tag{1}$$

for the weak interactions zero mass particles (neutrino-antineutrino pairs) may be exchanged resulting in a singularity of the amplitude at $t = 0$. However, in spite of this difficulty, bounds have been obtained on weak scattering cross sections. The major result of these studies is that at asymptotic energies the weak interaction cross section is bounded by an arbitrarily small power of the energy [1–5],

$$\sigma_T < S^\varepsilon, \quad \varepsilon > 0. \tag{2}$$

Since very little is known about the details of weak scattering amplitudes at asymptotic energies, the study of a class of amplitudes that have total cross sections which are bounded as in equation (2) may give insight into the properties of these important processes.

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The purpose of this paper is to investigate the properties of a certain class of scattering amplitudes at asymptotic energies. This class is defined by the following assumptions: (A) The effective number of partial waves contributing to the scattering is,

$$L = CS^{1/2}(\ln S)^M, \tag{3}$$

where C and M are positive constants. This means that we are assuming that the partial wave series for the scattering amplitude can be truncated after L terms with negligible error as $S \to \infty$. (The concept of "the effective number of partial waves contributing to a scattering process" is very useful and has been used by several authors [7–10] to investigate the properties of amplitudes at asymptotic energies.) (B) As $S \to \infty$, the total cross section has the following energy dependence,

$$\sigma_T = \sigma_0(\ln S)^N, \tag{4}$$

where σ_0 and N are positive constants and $N \leqslant 2M$.

These assumptions are sufficiently general to allow us to discuss scattering processes whose cross section are bounded by an arbitrarily small power of the energy.

However, before we list the results which follow from the above assumptions, we will give a particular class of amplitudes that exhibit explicitly these properties. Let $A(S, t)$ be the absorptive part of an elastic scattering amplitude $F(S, t)$. Consider the following functional form for $A(S, t)$.

$$A(S, t) = \sigma_0 S^{1+\gamma}(\ln S)^N f_1(t) f_2[\alpha(t)(\ln S)^K], \tag{5}$$

where σ_0, γ, N and K are positive constants; and, the functions f_1, f_2 and α are normalized such that $f_1(0) = 1$, $f_2(0) = 1$ and $\alpha(0) = 0$. In addition, assume that $A(S, t)$ has a first derivative. Under these conditions, the class of amplitudes defined by equation (5) has the following upper bound at asymptotic energies,

$$\sigma_T < C_2(\ln S)^K, \tag{6}$$

where C_2 is a constant. To obtain this result, we made use of the fact that if $A(S, t)$ has a first derivative at $t = 0$, then the total cross section is bounded by, [7]

$$\sigma_T < 32\pi d \ln A(S, 0)/dt.$$

Consequently, the class of amplitudes, which have absorptive parts given by equation (5), satisfy the bound of equation (2).

2. *Results*

We now list the most important results obtained under the above assumptions. To obtain these results, we have followed the procedures of Ref. [7–10].

(I) The phase of the forward scattering amplitudes satisfies the upper bond,

$$\left| \frac{\operatorname{Re} F(S, 0)}{\operatorname{Im} F(S, 0)} \right| < \left[\frac{C^2}{\sigma_0} \right]^{1/2} (\ln S)^{M - \frac{N}{2}}. \tag{7}$$

(II) The elastic cross section is bounded above and below by,

$$\left[\frac{\sigma_0^2}{16\pi C_1^2}\right](\ln S)^{2(N-M)} \leqslant \sigma_{el}(S) \leqslant \sigma_0(\ln S)^N. \tag{8}$$

(III) The forward differential cross section is bounded above and below by,

$$\left[\frac{\sigma_0^2}{16\pi}\right](\ln S)^{2N} \leqslant \frac{d\sigma(S,0)}{dt} \leqslant \left[\frac{C\sigma_0^{3/2}}{16\pi}\right](\ln S)^{\frac{3N}{2}+M}. \tag{9}$$

(IV) The non-forward scattering amplitude satisfies the following upper bound,

$$|F(S,\cos\theta)| < C_3 S^{3/4}(\ln S)^{\frac{3M}{2}}/(\sin\theta)^{1/2}. \tag{10}$$

for $0 < \theta < \pi$. A similar bound holds for $A(S,t)$.

(V) The derivative of the absorptive part of the amplitude for non-forward scattering has the upper bound,

$$\left|\frac{dA(S,\cos\theta)}{d\cos\theta}\right| \leqslant C_4 S^{5/4}(\ln S)^{\frac{5M}{2}}/\sin^2\theta, \tag{11}$$

for $0 < \theta < \pi$.

(VI) The width of the diffraction peak has the following upper and lower bounds,

$$\sigma_0/C^4(\ln S)^{4M-N} < \Delta(S) \leqslant 16\pi/\sigma_0(\ln S)^N. \tag{12}$$

We define the diffraction width, $\Delta(S)$, as follows, [11]

$$\sigma_{el}(S) \equiv \Delta(S)\frac{d\sigma(S,0)}{dt}.$$

3. *Summary*

We have obtained properties of scattering processes which have total cross sections that are bounded by an arbitrary small power of the energy in the asymptotic region. These results apply directly to neutrino (weak) scattering amplitudes since these processes obey the bound given by equation (2).

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