DISCRETE "FIREBALL" MASSES IN VERY HIGH ENERGY COLLISIONS

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We show how a discrete mass spectrum for fireballs is consistent with and also a natural explanation for the empirical properties of the energy distributions for pions and gamma rays observed in the experiments of multiple production of hadrons at accelerator and cosmic ray energies. We suggest also that the available data do not support the existence of a limiting temperature of 160 MeV in high energy collisions.

1. Introduction

Two points are discussed in this paper. The first is the case for a discrete mass spectrum of fireballs as being consistent with and also a natural explanation for the empirical properties of the energy distributions for pions and gamma rays observed in the experiments of multiple production of hadrons at accelerator and cosmic ray energies. The second point is the case against the existence of a limiting temperature of 160 MeV for high energy collisions.

Let us recall here how the ideas of fireball and a limiting temperature have emerged in order to appreciate properly what we shall do.

In 1958 Męsowicz and collaborators [1] and also Niu [2] proposed that some characteristics of multiple production of particles observed in cosmic rays experiments could, be described by a model in which, in very high energy hadronic collisions, the main contribution to pion yield would be the isotropic decay of intermediate states (in their own reference frames) trailing the colliding hadrons after the interaction. The Polish group pointed out that the highly anisotropic states of "pionization"[1] could be understood in

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[1] The term pionization is due to B. Peters, *Proc. of Eighth Int. Conf. on Cosmic Rays*, Jaipur, **5**, 423 (1963). Different meanings are attached to his word. Accelerator people think of pionization as a kind of vacuum excitation.

terms of the kinematics of the isotropic decay of two fast moving centers. In these works the masses of the intermediate states were supposed to be a function of the total energy of the collision in the center of momentum system in order to account for the experimental fact that the multiplicity of produced pions increases with incident energy (and they insisted on two intermediate states). We remind here that Cocconi [3] has also arrived at the above conclusions by analysing data of the Polish group and he called the intermediate states fireballs.

More recently Hagedorn [4] has developed a thermodynamical model of strong interaction based on the so called "statistical bootstrap hypothesis" whereby a quantitative treatment of the fireball problem is made possible. The model has been extensively used in the study of particle yields, especially in the energy range covered by the large accelerators.

Now, Hagedorn's thermodynamics of strong interactions contains two very interesting and vulnerable predictions

(*i*) The number of different kinds of hadrons having mass between m and $m+dm$ is given asymptotically by the mass spectrum

$$\varrho(m)dm \xrightarrow[m\to\infty]{} cm^{-a} \exp(m/T_0)dm, \tag{1}$$

where a is a constant not completely determined.

(*ii*) The constant T_0 in the exponential function of the mass spectrum is universal highest temperature above which no matter can be heated. Hagedorn concludes that the data on multiple particle production implies that this temperature is about 160 MeV[2].

It is also the feeling [5] of people working with the thermodynamical model that the data of CERN-ISR with respect to large transverse momenta do not contradict the statistical bootstrap. They think that it is possible to account for the data by assuming that some fraction of the fireballs is produced with nonvanishing transverse momenta.

As we shall see the thermodynamical model is not supported by the data on very high energy collisions ($\langle E_{\text{lab}}\rangle \sim$ 100 TeV) obtained by the Brasil-Japan Emulsion Chamber Collaboration (CBJ) [6].

2. *The case for a discrete mass spectrum of "fireballs"* [7]

During the last 14 years the Brasil-Japan Emulsion Chamber Collaboration [8] has accumulated a great amount of data on nuclear interactions induced by extremely high energy hadrons of cosmic radiation.

In this section we show how the data on energy distributions for pions and gamma rays observed in the experiments of multiple production by CBJ are consistent with a discrete mass spectrum of the intermediate states which trail the colliding hadrons after the interaction, and that this is also the natural explanation for the empirical propreties observed in the spectra. As it is well known [9] the variable $x = E_c/E$ where E_c is the energy of a secondary c (produced in the collision process) and E is the total energy of the collision,

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[2] In fact, the two predictions are only one, because the second is a consequence of the first, see Ref. [4].

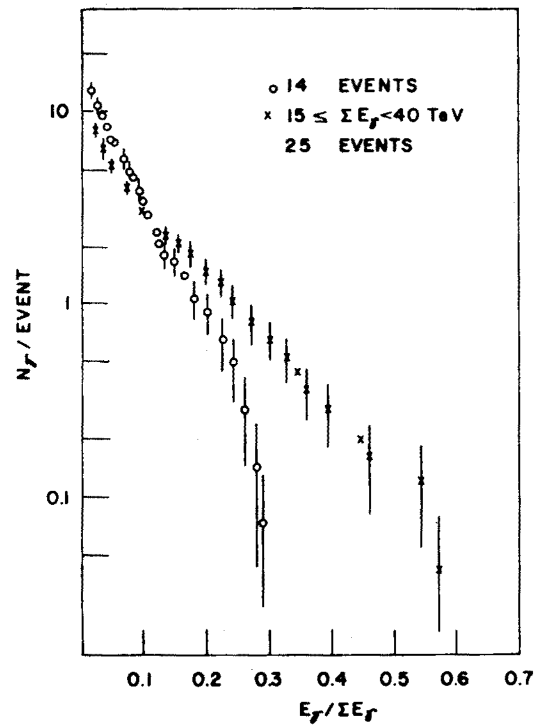

Fig. 1. Integral Energy Distribution of Gamma Rays. The figure is taken from Ref. [7]

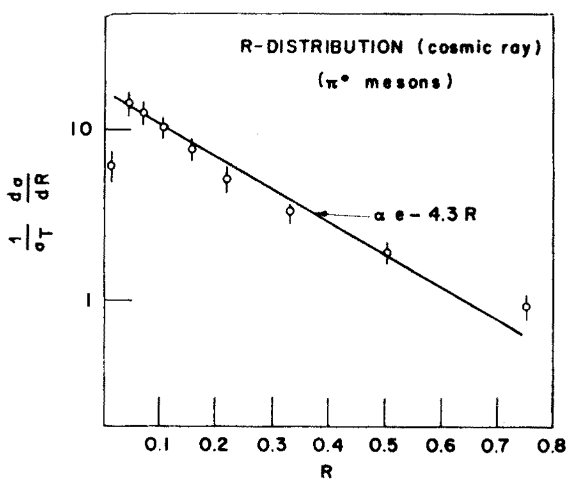

Fig. 2. Fractional energy distribution $\sigma_T^{-1} d\sigma(\pi^0)/dR$ The variable R spans the beam fragmentation region

measured in the laboratory system, spans in the limit $E \to \infty$ the so called beam fragmentation region.

Now, a variable related to x is $R = \varepsilon_\gamma / \sum \varepsilon_\gamma$, the fractional energy, used in the works of CBJ. where $\sum \varepsilon_\gamma$ represents the energy relased in a hadronic collision in the form of gamma rays (resulting from the decay $\pi^0 \to 2\gamma$). These gamma rays are observed in X-ray films and nuclear emulsions, in the emulsion-lead chambers of CBJ at Chacaltaya[3].

The integral spectrum of the energy distribution of gamma rays (Fig. 1) has two branches and each one satisfies a similarity property, i. e., the distributions are only a function of the variable R and no not depend on the collision energy E. We have also verified (see Fig. 2) [10] that the inclusive distribution for π^0's, $\sigma_T^{-1} d\sigma(\pi^0)/dR(R = E\pi^0/\sum E\pi^0)$ is also independent of the collision energy up to $E \simeq 240$ TeV. Such a property of the spectrum is equivalent to the scaling in the beam fragmentation region if the mean inelasticity of the collisions remains constant.

In what follows we assume then that in the process of multiple production of pions one or more intermediate states (fireballs or clusters are how these intermediate states are called) are produced and then decay into pions.

For a given intermediate state we assume that the energy distribution of pions resulting from the decay is such that

$$F(E_\pi^*, \Theta_\pi^*)dE_\pi^* d\Theta_\pi^* = f(E_\pi^*)dE_\pi^* \frac{d\Omega^*}{4\pi}. \tag{2}$$

The aterisks denote variables measured in the rest system of the intermediate state (which we suppose to beanentity independent of the colliding hadrons [11]).

We do not specify $f(E_\pi^*)$ but assume that

$$\int_{E_\pi^{*(\min)}}^{E_\pi^{*(\max)}} f(E_\pi^*)dE_\pi^* = 1, \tag{3}$$

where $E_\pi^{*(\min)} = m_\pi$ and $E_\pi^{*(\max)} = M/2$, m_π being the pion mass and M the mass of the intermediate state, which can in principle be a function of the collision energy[4], that is, $M = M(E)$.

We transform our variables to the laboratory frame and find for the differential energy distribution of pions

$$g(E_\pi, \Gamma) = \tfrac{1}{2} \int_{\bar{E}_\pi^{*(\min)}}^{E_\pi^{*(\max)}} f(E_\pi^*) \frac{dE_\pi^*}{\Gamma\beta p_\pi^*}. \tag{4}$$

In Eq. (4) Γ is the Lorentz factor of the intermediate state (measured in the laboratory frame), p_π^* is the pion momentum and

$$\bar{E}_\pi^{*(\min)} = \Gamma(E_\pi - \beta p_\pi), \quad E_\pi^{*(\max)} = M/2. \tag{5}$$

If we assume now the mass of the intermediate state to be independent of the collision energy and that the mean inelasticity K of the collision is constant when $E \to \infty$, we can write

$$M\Gamma = KE.$$

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[3] For a detailed account of these matters see Ref. [6].

[4] In the thermodynamical model M is a function of the energy, see Ref. [4] for details.

In these conditions (the mass of the intermediate state and the mean inelasticity of the collision being constants) and using approximations valid when $E \gg m_\pi$ and $\Gamma \gg 1$ we obtain

$$g(E_\pi, \Gamma)dE_\pi = \frac{dE_\pi}{2\Gamma} \int_{\frac{E_\pi}{2\Gamma}+\frac{\Gamma m^2_\pi}{2E_\pi}}^{M/2} \frac{f(E_\pi^*)}{P_\pi^*} + O(\Gamma^{-2}), \tag{7}$$

where we have used

$$\overline{E}_\pi^{*(\min)} = \frac{E_\pi^*}{2\Gamma} + \frac{\Gamma m_\pi^2}{2E_\pi}. \tag{8}$$

With the aid of Eq. (6) we can now write the differential distribution for pions in terms of the variable $x = E_\pi/E$ obtaining

$$g(E_\pi, E)dE_\pi = adx \int_{ax+\frac{b}{x}}^{M/2} dE_\pi^* \frac{f(E_\pi^*)}{p_\pi^*} \equiv g(x)dx, \tag{9}$$

where

$$a = M/2K, \quad b = m_\pi^2 K/2M. \tag{10}$$

Thus, if in the collision only one intermediate state is produced

$$\frac{1}{\sigma_T}\frac{d\sigma}{dx} = \frac{d\bar{n}(x, E)}{dx} = \frac{1}{\langle n_\pi \rangle} g(x). \tag{11}$$

As the variable x spans the beam fragmentation region, we see that Eq. (11) is equivalent to the scaling of the inclusive distribution in the beam fragmentation region. If the mean inelasticity of the collision remains independent of the energy, we see that the hypothesis that the mass of the intermediate state is independent of the collision energy is a natural explanation for the distribution of Fig. 2.

If in the collision several intermediate states are produced, all with the same mass, then $\sigma_T^{-1} d\sigma/dx \propto g(x)$ if the Lorentz factors of the intermediate states form a geometrical series, i. e., $\gamma_i/\gamma_{i\pm1}$ = constant. This corresponds to several fireballs being produced along a multiperipheral chain with constant momentum transfer between fireballs[5].

In deriving Eq. (9) we have used the hypothesis that the mass of the intermediate state is independent of the energy of the collision, and so if there are intermediate states with a mass different from M, let us say, M' then the distribution $d\bar{n}^1/dx$ obtained for the object of mass M' also will not be a function of E, although it will be different from $d\bar{n}^1/dx$. Thus studying experimentally the energy (or momentum) distribution we can establish the

[5] See Ref. [7] for this point and also E. L. Feinberg, *Phys. Reports* **5**, (1972).

existence of distinct distributions, if the energy interval under consideration contains contributions from intermediate states of different masses[6].

So the existence of two branches in Fig. 1 for the gamma ray energy distribution is compatible with intermediate states of distinct masses, and with the fact that these masses do not depend on the collision energy. Fig. 1 tells us also that both distributions satisfy a similarity law and now show that this is indeed the case under the hypothesis outlined above.

As is well known [12] the energy distribution of gamma rays in the intermediate state rest system is

$$G(\varepsilon_\gamma^*)d\varepsilon_\gamma^* = d\varepsilon_\gamma^* \int_{\bar{E}_\pi}^{E_\pi^{*(\max)}} \frac{1}{p_\pi^*} f(E_\pi^*)dE_\pi^*, \tag{12}$$

where ε_γ^* is the gamma ray energy, $f(E_\pi^*)$ is again the energy distribution of pions in the intermediate state rest system, and

$$\bar{E}_\pi = \varepsilon_\gamma + \frac{m_\pi^2}{4\varepsilon_\gamma} \tag{13}$$

is the minimum energy a pion must have in order to produce a gamma ray of energy ε_γ^*. Also we have $E_\pi^{*(\max)} = M/2$ and Eq. (12) is valid only in the interval

$$\frac{m_\pi^2}{2M} \leqslant \varepsilon_\gamma^* \leqslant \frac{M}{2}\,. \tag{14}$$

We define

$$\Phi\left[M/2, \varepsilon_\gamma^* + \frac{m_\pi^2}{4\varepsilon_\gamma}\right] = \int_{\varepsilon_\gamma^* + \frac{m^2_\pi}{4\varepsilon_\gamma^*}}^{M/2} dE_\pi^* \frac{f(E_\pi^*)}{p_\pi^*} \tag{15}$$

and assume, as we have already done, isotropic decay of the intermediate state into pions, which implies, due to angular momentum conservation, isotropy of the gamma ray distribution. Consequently,

$$G(\varepsilon_\gamma^*, \cos\Theta_\gamma^*)d\varepsilon_\gamma^* d(\cos\Theta_\gamma^*) = \tfrac{1}{2}\Phi\left[M/2, \varepsilon_\gamma^* + \frac{m_\pi^2}{4\varepsilon_\gamma^*}\right] d\varepsilon_\gamma^* d(\cos\Theta_\gamma^*). \tag{16}$$

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[6] We want to call the attention of the reader to the fact that the distribution of Fig. 2 was constructed without the separation of fireballs into small and large ones. Also due to the dectetion threshold we can see only the more energetic fireballs. See Ref. [10] for details and also. C. M. G. Lattes, W. 'A. Rodrigues, A. Turtelli Jr., E. H. Shibuya, N. Amato, N. Arata, T. Shibata, K. Yokoi, Y. Fujimoto, S. Hasegawa, T. Miyashita, K. Sawayanagi, *Proceed. of International Cosmic Ray Symposium on High Energy Phenomena*, 1, University of Tokyo, Japan 1974.

Using the exact kinematic limits of the appropriate variables [12] and transforming to the laboratory system we get

$$G(\varepsilon_\gamma, \cos\Theta_\gamma)d\varepsilon_\gamma d(\cos\Theta_\gamma)$$

$$= \tfrac{1}{2}\Phi\left[M/2, \Gamma\varepsilon_\gamma(1-\beta\cos\Theta_\gamma) + \frac{m_\pi^2}{4\Gamma\varepsilon_\gamma(1-\beta\cos\Theta_\gamma)}\right]\frac{d\varepsilon_\gamma d(\cos\Theta_\gamma)}{\Gamma(1-\beta\cos\Theta_\gamma)}. \tag{17}$$

In Eq. (17) Γ is the Lorentz factor of the intermediate state as mesured in the laboratory system. From the experimental point of view of CBJ[7] the observable energy region with threshold energy of 0,2 TeV corresponds to the energy interval in the laboratory system

$$\frac{m_\pi^2}{2M\Gamma(1-\beta)} \leqslant \varepsilon_\gamma \leqslant \frac{M}{2\Gamma(1-\beta)}. \tag{18}$$

As here we are only interested in the proof (from our basic assumptions) of the observed similarity property of the integral energy distribution, we calculate $G(\varepsilon_\gamma, \Gamma)$ only for energies inside this interval. The gamma ray energy distribution in the laboratory system is given by

$$G(\varepsilon_\gamma, \Gamma) = \frac{2}{\Gamma\beta}\int\limits_{\Gamma(1-\beta)}^{M/2}\Phi\left[M/2, \varepsilon_\gamma u + \frac{m_\pi^2}{4\varepsilon_\gamma u}\right]\frac{du}{u}, \tag{19}$$

where $u = \Gamma(1-\beta\cos\Theta_\gamma)$. Now, putting $y = Eu$, defining $\eta = \varepsilon_\gamma/E$ and using Eq. (6) we get in the limit $E \gg m_\pi$

$$G(\varepsilon_\gamma, E)d\varepsilon_\gamma = \tfrac{1}{2}\frac{M}{K}d\eta\int\limits_{M/2K}^{M/2\eta}\Phi\left[M/2, y\eta + \frac{m_\pi^2}{4y\eta}\right]\frac{dy}{y}$$

$$= \tfrac{1}{2}\frac{M}{K}h(\eta)d\eta. \tag{20}$$

Then, since

$$G(\varepsilon_\gamma, E) = \frac{1}{\langle N_\gamma\rangle}\frac{d\bar{n}^{(\gamma)}}{d\varepsilon_\gamma} \tag{21}$$

we have

$$\frac{1}{\langle N_\gamma\rangle}\frac{d\bar{n}^{(\gamma)}}{d\eta} = \tfrac{1}{2}\frac{M}{K}h(\eta) \tag{22}$$

for

$$\frac{m_\pi^2 K}{M^2} \leqslant \eta \leqslant K. \tag{23}$$

Eq. (22) implies that the gamma ray-spectrum satisfies up to order $O(E^{-2})$ a similarity law.

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[7] See Ref. [12] for details.

For the integral spectrum $F(\geqslant W, E)$ with $m_\pi^2 \Gamma/M \leqslant W \leqslant M\Gamma$ we immediately get

$$F(\geqslant W, E) = \int_W^{M\Gamma} G(\varepsilon_\gamma, E) d\varepsilon_\gamma \equiv H(R),$$

$$H(R) = \tfrac{1}{2}\frac{M}{K}\int_R^K d\eta \int_{M/2K}^{M/2\eta} \Phi\left[M/2, y\eta + \frac{m_\pi^2}{4y\eta}\right]\frac{dy}{y}. \tag{24}$$

Eq. (24) thus agrees with what can be seen in Fig. 1, that is, with the existence a similarity law for the energy integral spectrum of gamma rays. The emergence of two distinct distributions is then compatible with the existence of two intermediate states with distinct masses.

Now, what about the masses of these intermediate states? Correlation studies by CBJ have shown that the mass of the intermediate state associated with the first branch in Fig. 1 is ~ 3 GeV/c^2 while the mass associated with the intermediate state which determines the second branch is $\gtrsim 20$ GeV/c^2. Also there is strong evidence for the existence of a super-heavy intermediate state with mass $\gtrsim 200$ GeV/c^2, see Refs [6], and [8].

If these results survive after a more detailed experimental investigation then they will become of fundamental importance. In paricular with relation of the thermodynamical model we see that instead of a continuous mass spectrum for fireballs we have a discrete one.

We do not know what these intermediate states are[8] but besides the mass they are characterized by other parameters also, such as the mean multiplicity, the branching ratios for different decays and the "temperature". This point we discuss in next section.

3. The temperature of the "fireball"

In Fig. 3 taken from Ref. [6], we see the integral distribution of transverse momentum for π^0's. We see that for A-jets[9] with 30 TeV $\leqslant \sum \varepsilon_\gamma \simeq$ 400 TeV, $\sum \varepsilon_\gamma$ being the part of the total energy of the collision released to π^0 mesons, the integral distribution, of P_T is twice the one obtained for the C-jets[10] with 3 TeV $\lesssim \sum \varepsilon_\gamma \lesssim$ 20 TeV. The constraint $\sum p_T \geqslant 2.5$ GeV/c imposed on the A-jets implies that the π^0 mesons result from an intermediate state of large mass. This is due to the relation based on isotropic emission

$$M = \frac{4}{\pi}\sum p_T \tag{25}$$

between the mass of the intermediate sate and $\sum p_T$.

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[8] For some speculations see T. Tati, *Progr. Theor. Phys.* **43**, 1956 (1970), and *Suppl. Progr. Theor. Phys.* **54**, 31 (1973).

[9] A-jet is an atmospheric interaction.

[10] C-jets mean local nuclear interactions, C standing for the hydrocarbon target.

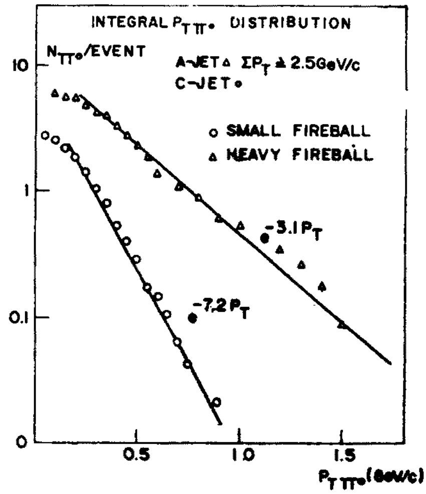

Fig. 3. Integral distribution of transverse momentum of π^0 mesons normalized per event. Open circles are for C-jets and triangles for A-jets. See the text for details

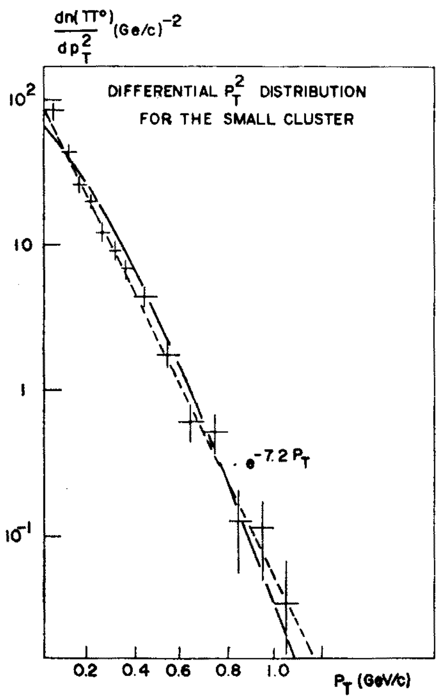

Fig. 4a

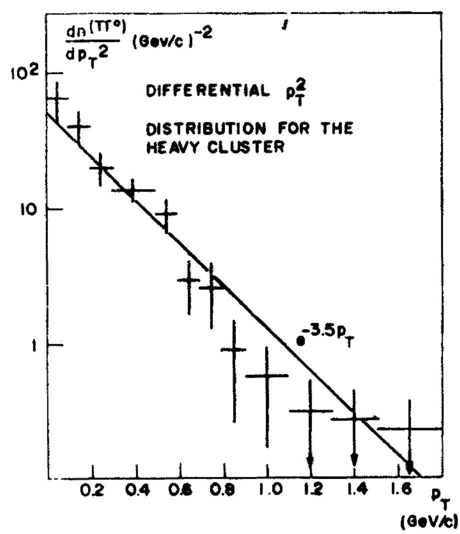

Fig. 4b

Fig. 4a. Distribution of p_T^2 for π^0 mesons produced through the decay of the small mass fireball. The heavy line corresponds to a fit with a Bose-like distribution with equilibrium temperature of 110 MeV

Fig. 4b. Distribution of p_T^2 for π^0 mesons produced through the decay of the large mass fireball

In the C-jets the π^0's detected are mainly produced through the small mass intermediate state, see Ref. [6].

In Figs 4a and 4b we present the data [13] for the $dn^{(\pi^0)}/dp_T^2$ distributions of pions coming through the decays of the small and large mass intermediate states.

Supposing the small mass intermediate state to be a cluster of pions in statistical equilibrium we have determined [10] from the transverse momenta distribution of pions the "equilibrium temperature" as being $T_0 \sim 110$ MeV.

Now, according to Hagedorn's thermodynamical model, there is a maximum temperature for high energy collisions. Consequently we must expect that the decay of the heavy intermediate state of mass $\gtrsim 20$ GeV/c^2 into pions can also be described by an appropriate distribution with an equilibrium temperature not exceeding T_0. Recently Japanese physicists [14] have shown that this cannot be the case. A direct decay of the heavy intermediate state into pions with equilibrium temperature T_0 cannot account for the observed distribution. Supposing that the heavy intermediate states decay first into the small intermediate states (as suggested in Ref. [6]), which later decay into pions, the above mentioned authors have shown that the observed spectra can be reproduced if the heavy intermediate state is a "cluster" of small intermediate states in statistical equilibrium at a "temperature" $T_0' \sim 0.4$ GeV.

Now, if the analysis of the decay of the superheavy "cluster" implies the introduction of a new equilibrium temperature T_0'', we see that if there is a maximum temperature in the universe it must be much greater than the value 160 MeV suggested by Hagedorn.

4. Conclusions

From the analysis outlined above it becomes clear that the data on very high energy collisions agree with general assumptions of a production through intermediate states.

The intermediate states have a discrete mass spectrum and there are different "equilibrium temperatures" characterizing different intermediate states. Obviously the issue with respect to a maximum temperature is not solved, since we do not know where the mass spectrum of intermediate states stops. In fact we know nothing about the real nature of these intermediate states.

We are grateful to Professor F. O. Castro from CBPF for useful discussions.

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