A NOTE ON THE HADRONIC FORM FACTOR AND ITS POSSIBLE FORM

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The form factor of hadrons in exclusive scattering initiated by hadrons and photons has been investigated on the basis of the dual parton model of hadrons developed by Bandyopadhyay and De. Also, it has been shown that this form factor arises from the electromagnetic form factor of pion in the structure of hadrons. Finally from the scaling behaviour of the differential cross-section of the exclusive scattering of the type $AB \to CD$, predicted by Brodsky and Farrar, in the asymptotic region $s \to \infty$, $t \to \infty$, t/s fixed, an analytical form of the form factor in the hadronic two body exclusive interaction has been suggested.

1. *Introduction*

In search of the fundamental constituents of matter much effort has recently been devoted and in that course many models for the structure of hadrons have been developed. One of the much discussed models is the parton model originally proposed by Feynman. Introducing quarks as the partons in that model, Bjorken and Paschos (1969) did not meet with much success, because it gives a mean square charge per parton much higher than experimentally observed. Again Gardiner and Majumdar (1970) have pointed out that the experimental results indicate that the partons are highly integrally charged rather than fractionally charged and most of the partons in the proton are neutral. Recently the various strong interaction processes have been investigated on the basis of a dual parton model of hadrons as developed by Bandyopadhyay and De (1972). Indeed, taking into account the suggestion by Bloom and Gilman (1970) that a substantial part of the scaling curves for the structure functions, is, in fact, built up from resonances, it has been shown in the previous paper (Bandyopadhyay et al. 1972) that a five parton model for the nucleon can be constructed so that the ep deep inelastic scattering can be supposed to be contributed to by resonances in conformity with the concept of duality and there is no contribution from the meson cloud. In this five parton model, partons are assumed to be spin $\frac{1}{2}$ point-like constituents with integral charges. Here, a

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proton is considered to be composed of five spin $\frac{1}{2}$ partons with one having positive charge $+1$ and all others being neutral; and to get a good fit in the en scattering the neutron is assumed to be a mixture of the following two configurations: (*i*) all the partons are neutral and (*ii*) two of them have charges $+1$ and -1 and the remaining three are neutral. Thus the configuration of a proton, in this picture, is $(\chi^+\bar{\chi}^0\chi^0\bar{\chi}^0\chi^0)$ where χ^+, χ^- and χ^0 are the positively, negatively charged partons and the neutral one respectively, and the bars denote the antiparticles of the partons (antipartons). Similarly a neutron is a combination of the two configurations $\alpha(\chi^0\bar{\chi}^0\chi^0\bar{\chi}^0\chi^0)+\beta(\chi^+\chi^-\chi^0\bar{\chi}^0\chi^0)$, where α and β denote the weightage of the respective configurations. Again, we have shown that duality can be incorporated from the very dynamics of strong interactions in view of this five-parton model provided we assume that any two spin $\frac{1}{2}$ point-like constituents (parton-antiparton pair can form a π-meson cluster in the structure of baryons and the strong interactions) actually involve the interaction of this π-meson with the incident hadron (Bandyopadhyay and De, 1972). Actually, a suitable form factor can be introduced so that the clustering effect will not alter the scaling behaviour in the high energy limit (Okumura, 1971). With this view we have supposed that a nucleon can be represented as $N = (\pi\pi C)$ where π is composed of any two spin $\frac{1}{2}$ constituents and C represents the unbound parton. Specifically we can write the configuration of a proton as $(\pi^+\pi^0 C)$ and that of a neutron as $\alpha(\pi^0\pi^0 C)+\beta(\pi^+\pi^- C)$ with the respective weightages α and β. This model of strong interactions satisfies the requirement of duality in MB scattering such as in the π N interaction in the sense, that both the *s*- and *t*-channel amplitudes are contributed to by the same meson (viz., the ϱ-meson). Indeed, in this scheme it can be assumed that a baryonic resonance occurs when a π-meson cluster in the structure of a baryon is exited to the level of the ϱ-meson. The most interesting aspect of this interpretation of duality is that the inconsistencies which crop up in $B\bar{B}$ scattering in the naive quark model are removed in this scheme because this interaction also involves the $\pi\pi$-interaction, where all the pions are in the structure of the baryons (Bandyopadhyay and De, 1972).

On the basis of this model, we have analysed processes like $\pi N \to \pi N$, $\pi N \to N\varrho$, $\pi N \to N\omega$, $\gamma N \to N\pi$, $\gamma N \to N\varrho(\omega)$, in a series of papers (Bandyopadhyay and De, 1973 and 1975; Bandyopadhyay et al., 1973 and 1973) where it has been assumed that the incident hadron (photon) interacts with the π-meson in the structure of a nucleon. To calculate the amplitudes for the relevent processes, we also take into account a factor corresponding to the structural rearrangement of partons involved in the duality diagrams.

The number of partons involved in the duality diagrams is found to be related to the energy dependence of the respective processes. The differrential cross-sections of the above processes have been calculated on the basis of these ideas and found to give results which are in excellent agreement with the experimental results. Also, it has been shown (Bandyopadhyay and De) that this model can nicely interpret the occurence or nonoccurrence of dips in the differential cross-sections of the various processes in consistency with the experimental results, where it is assumed that a dip is a resonance effect (Imachi et al. 1970). Recently, we have shown that the rearrangement of the constituents (partons) of hadrons in hadronic strong interactions helps us to effectively avoid the difficulties encountered in Lagrangian field theory, *S*-matrix approach and the conventional dual models and as such proper dynamics of strong interactions can be envisaged on the basis of the structure of hadrons (Bandyopadhyay and De, 1975).

The scheme of this interaction for a specific case such as $\pi^- p \to n\pi^0$, can be written as

$$\pi^- p = \pi^- + \begin{Bmatrix} \pi^+ \\ \pi^0 \\ C \end{Bmatrix} \to \begin{Bmatrix} \varrho^0 \\ \pi^0 \\ C \end{Bmatrix} \to \begin{Bmatrix} \pi^+ \\ \pi^- \\ C \end{Bmatrix} + \pi^0 = n\pi^0 \tag{1}$$

or,

$$\pi^- + \begin{Bmatrix} \pi^0 \\ \pi^+ \\ C \end{Bmatrix} \to \begin{Bmatrix} \varrho^- \\ \pi^+ \\ C \end{Bmatrix} \to \begin{Bmatrix} \pi^- \\ \pi^+ \\ C \end{Bmatrix} + \pi^0 = n\pi^0.$$

Thus we write the forward scattering amplitude of the process $\pi^- p \to n\pi^0$ in terms of the amplitude for the pion-pion interaction $\pi\pi \to \varrho \to \pi\pi$ and an amplitude related to the rearrangement of the partons involved in duality diagrams (Bandyopadhayay and De, 1973).

$$A(\pi^- p \to n\pi^0) = \tfrac{1}{3} A(\pi\pi \to \pi\pi) T(s, t). \tag{2}$$

The factor $\frac{1}{3}$ has been introduced in the amplitude because, as revealed in Eq. (1), the configuration for the neutron obtained in this process is always $(\pi^+\pi^- C)$, whereas the full configuration of the neutron is given by the mixture of the configurations $n = \alpha(\pi^0\pi^0 C) + \beta(\pi^+\pi^- C)$ with $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{3}$. Here $T(s, t)$ denotes the factor which arises from the structural rearrangement of partons involved in duality diagrams in the process $\pi^- p \to n\pi^0$. Indeed, this factor may be considered to come from the overlap integrals of the wave functions or the form factor of hadrons as the composite system of partons. In fact, we have found that the number of partons involved in the rearrangement in duality diagrams is related to the energy dependence of the process concerned. The $\pi\pi \to \varrho \to \pi\pi$ amplitude can be derived from the well known Lagrangian

$$L_I = i g_{\varrho\pi\pi} \left(\bar{\pi} \frac{\partial \pi}{\partial x_\mu} - \pi \frac{\partial \bar{\pi}}{\partial x_\mu} \right) \varrho_\mu. \tag{3}$$

With all these, we find that the differential cross-section up to $-t = 0.6\ \text{GeV}^2$, where the dip is found to occur, is given by

$$\frac{d\sigma}{dt} = 13.6 \frac{1}{(m_\varrho^2 - t)^2} \frac{1}{s^{1.5}} F(t)\ \text{mb/GeV}^2 \tag{4}$$

where the form factor $F(t)$ is parametrized as follows:

$$F(t) = \frac{1}{1 + a|t| + bt^2}, \tag{5}$$

so that $F(0) = 1$. Taking $a = 8$ and $b = 200$, a good fit to the experimental data in the region $0 \leqslant |t| \leqslant 0.5$ has been achieved (Bandyopadhyay and De, 1973). The same procedure can be applied to $\pi^+ p \to \pi^+ p$ and $\pi^- p \to \pi^- p$ scattering. In forward elastic scattering the diffraction contribution will dominate in the high energy region. In the backward region, the differential cross sections are found to be given by the relations (Bandyopadhyay and De, 1973).

$$\frac{d\sigma}{dt}(\pi^+ p \to \pi^+ p) = 4.896\times 10^3 \frac{1}{(m_\varrho^2-u)^2}\frac{1}{s^{2.25}} F'(u)\mu b/\text{GeV}^2,$$

$$|u| < 0.2\ \text{GeV}^2, \tag{6}$$

$$\frac{d\sigma}{dt}(\pi^- p \to \pi^- p) = 3.4\times 10^3 \frac{1}{(m_\varrho^2-u)^2}\frac{1}{s^3} F''(u)\mu b/\text{GeV}^2. \tag{7}$$

In the case of $\pi^+p \to \pi^+p$ backward scattering, we have a dip at $|u| \simeq 0.2$ GeV^2 and in the region $0 < |u| < 0.2$ GeV^2, $F'(u)$ is taken to be of the form

$$F'(u) = \frac{1}{1+a|u|+bu^2},$$

so that $F'(0) = 1$. With $a = 50$, $b = 400$, a good fit to the experimental data is obtained. In the case of $\pi^-p \to \pi^-p$ backward scattering, with $a = 8$, $b = 4$ in the relation

$$F''(u) = \frac{1}{1+a|u|+bu^2},$$

a good fit with the experimental data is obtained (Bandyopadhyay and De, 1973).

Now it should be noted that in all the above equations like (4), (6) and (7) the form factors $F(t)$, $F'(u)$ and $F''(u)$ are unknown and seem to be arbitrary at a first glance, although we have fitted them for the respective interactions with different values of the parameters a and b for small values of t and u. In this note we shall try to demonstrate certain physical characteristics of these form factors and to throw some light on the possible form of these functions which should fit, not only for small t (or u) but also for any value of t (or u) from the possible scaling behaviour of the exclusive processes.

2. *Physical characteristics of the form factor*

First, we shall see whether the body form factor of the proton, derived from the wave function of a proton, contribute to the form factors which we are discussing. If we assume that the configuration of a nucleon is given by N = ($\pi\pi$C), the wave function of a nucleon can be represented as

$$\psi_N = \phi(\pi\pi)\phi(C). \tag{8}$$

It is noted that the unbound parton C does not contribute to strong interactions in this scheme. The symmetric wave function $\phi(\pi\pi)$ can be reasonably chosen as

$$\phi_{\text{space}}(\pi\pi) = e^{-\beta^2(r_1^2+r_2^2)}. \tag{9}$$

Then the wave function leads to a body form factor $F(q^2)$ of the form e^{-Z} where $Z \simeq q^2/\beta^2$. It may be remarked here that a wave function of the form (8) avoids the difficulties which crop up in the three fermion model (such as quark model) for nucleons. Indeed, for a three fermion model the simplest wave function is of the form

$$\psi_{\text{space}} = (r_1^2 - r_2^2)(r_2^2 - r_3^2)(r_3^2 - r_1^2)\phi_{\text{sym}}(r_1, r_2, r_3),$$

with

$$\phi_{\text{sym}}(r_1, r_2, r_3) = e^{-\beta^2(r_1{}^2 + r_2{}^2 + r_3{}^2)}.$$

An analysis of this antisymmetric wave function for baryons by Mitra and Majumdar (1966) shows that the form factor should vanish at $Z = 3.8$ where $Z = q^2/12\beta^2$, which is not observed in experiments. Evidently, this difficulty is avoided in our present model of the nucleon. It is to be noted that in the high energy limit, pions in the configuration ($\pi\pi$C) will behave as free pions. Obviously, in that limit the body form factor coming from the wave function (8) will not contribute to the form factors of this scheme.

Now in the case of high energy $\pi N \to \pi N$ scattering, as we have discussed above, the process is contributed to by the interaction between the incident pion and the pion in the configuration of the proton. Let us now regard this high energy pion-pion scattering as due to two absorptive spheres of some given spatial density distribution $\varrho_\pi(\vec{r})$, going through each other as suggested by Chou and Yang (1967). The absorption should then have the form

$$\eta(b) = \exp\left[-K \int d^2\vec{b}' D_\pi(\vec{b}') D_\pi(\vec{b} - \vec{b}')\right], \tag{10}$$

where K is some (assumed constant) absorption coefficient and where

$$D_\pi(\vec{b}) = \int_{-\infty}^{\infty} dz \varrho_\pi(x, y, z) \tag{11}$$

is the two-dimensional density distribution of the particles at an impact parameter $b = (x^2 + y^2)^{1/2}$. For small absorption (η near 1) we can now expand the exponential of (10) to get

$$\eta(b) = 1 - K \int d^2\vec{b}' D_\pi(\vec{b}') D_\pi(\vec{b} - \vec{b}'), \tag{12}$$

which means that the amplitude

$$\begin{aligned} f(t) &= 2 \int d^2\vec{b}[1 - \eta(b)] e^{i\vec{q}\cdot\vec{b}} \\ &= 2K \int d^2\vec{b} e^{i\vec{q}\cdot\vec{b}} \int d^2\vec{b}' D_\pi(\vec{b}') D_\pi(\vec{b} - \vec{b}') \\ &= 2K |D_\pi(t)|^2, \quad [\vec{q}^2 = -t] \end{aligned} \tag{13}$$

where

$$D_\pi(t) = \int d^2\vec{b} e^{i\vec{q}\cdot\vec{b}} D_\pi(\vec{b}). \tag{14}$$

Now, in this droplet model the density distribution (or rather its Fourier transform $D(t)$) is chosen as the electro-magnetic form factor of the hadron concerned. Then we get for elastic πp scattering at high energy,

$$f(t) \propto |F_\pi(t)|^2, \tag{15}$$

where $F(t)$ is the electromagnetic form factor of the pion involved. This we can write

$$\frac{d\sigma/dt}{(d\sigma/dt)_{t=0}}(\pi p \to \pi p) = |F_\pi(t)|^4. \tag{16}$$

The difference in form factors in π^+p and π^-p elastic scattering as depicted in Eqs. (6) and (7), can be regarded as due to the difference in the electromagnetic form factors of π^+ and π^0. For, in π^-p scattering, π^- interacts with both π^+ and π^0 in the structure of a proton (p = ($\pi^+\pi^0$C)) while in π^+p elastic scattering π^+ interacts only with π^0 in the configuration, since duality arguments suggest that the contribution from ϱ and f exchanges will cancel in $\pi^+\pi^+$ scattering as there is no resonance in this channel. It should be clearly understood that in this analysis a proton itself is not considered to be a droplet. Actually, the density distribution of the proton whose composite configuration is of the form ($\pi\pi$C) should be considered as those of the pions occupying some spatial regions in the proton structure. Thus from the above analysis it is clear that, in any hadron-hadron interaction the form factors should arise from the electromagnetic form factors of pions which take part in the interaction as two absorptive spheres going through each other. The expressions for these form factors are evident from equation (16).

3. A possible form

Now let us turn our attention to the analytical form of the form factors, from the possible scaling behaviour of hadronic exclusive scattering, such as $\pi p \to \pi p$ in the limit $t \to \infty$, $s \to \infty$ while t/s remains fixed, demonstrated by Brodsky and Farrar (1973). Theoretically, such a type of scaling phenomena was achieved by Brodsky and Farrar for exclusive scattering and derived relations for processes like $\pi p \to \pi p$, $pp \to pp$, $\gamma p \to \pi p$, $\gamma p \to \gamma p$ etc., in this asymptotic region. In this consideration a hadron would become a collection of free quarks with equal momenta if the strong interactions were turned off. Noting that the dimensions of the associated invariant amplitude $M_{n_i \to n_f}$ with $n_i + n_f$ external lines is (length)$^{n_i+n_f-4}$, they derived the result

$$\left(\frac{d\sigma}{dt}\right)_{AB \to CD} \sim s^{2-N} f(t/s), \tag{17}$$

where N is the total number of leptons, photons and quark components (i.e., elementary fields) of the initial and final states. However, this analysis will not be true in our present dual parton model of hadrons. For in our model, no individual parton can interact strongly. It is only the bound state of two partons (parton-antiparton pair) i.e., a π-meson cluster in the structure of a hadron is the basic unit of all strong interactions. But such scaling behaviour could be achieved in our case from the differential cross-sections for processes like $\pi^- p \to n\pi^0$ (i.e., from Eq. (4)) or from the generalised amplitude for two-body processes of the type $AB \to CD$ without any exchange of hypercharge, obtained in our earlier work (Bandyopadhyay and De, 1975). This amplitude which also satisfies crossing, can be written in the form

$$A(s, t) = \alpha g^2 \frac{s+t-\mu^2}{(s-\mu^2)(t-\mu^2)} \frac{1}{(s+t)^{n\gamma}} \frac{\hat{F}(s)\hat{F}(t)}{\hat{F}(s+t)} + (t \to u) + (u \to s). \tag{18}$$

Here α is a parameter dependent on the number of interacting pions, g is the $\varrho\pi\pi$-coupling constant, μ is the mass of the exchanged meson (viz. the ϱ meson) in the $\pi\pi$ interaction, n is a numerical factor depending in the number of rearranged partons, γ is a suitable parameter (may not be constant) and $\hat{F}(x)$ is a form factor [$|\hat{F}(t)|^2$ is the form factor of Eq. (4)] parametrized as $\hat{F}(0) = 1$. Now we take $\hat{F}(x)$ to be of the general form

$$\hat{F}(x) = \frac{1}{1+a_1|x|+a_2x^2+ \ldots +a_r|x^r|}. \tag{19}$$

Now, in the scaling limit, $s \to \infty$, $t \to \infty$, t/s fixed we have the following behaviour for $A(s, t)$,

$$A(s, t) \sim s^{-(n\gamma+r+1)} f(t/s). \tag{20}$$

If future experiments indicate such scaling phenomena, then from the behaviour of the amplitude indicated in (20) or from that of the differential cross-section calculated from this amplitude for various two-body exclusive processes, the parameter r can be found and thereby a possible form of the unknown form factor can be found. As an example, we consider the process $\gamma p \to n\pi^+$ to get a scaling behaviour in agreement with the result of Brodsky and Farrar for this process. For this we use the differential cross--section based on the present model (Bandyopadhyay et al. 1973),

$$\left(\frac{d\sigma}{dt}\right)_{\gamma p \to n\pi^+} = 300 \frac{1}{m_\varrho^2 - t} \frac{1}{s^2} F(t) \mu\text{b/GeV}^2$$

and assume, for all s and t,

$$F(t) = \frac{1}{1+a|t|+bt^2+c|t^3|+dt^4}. \tag{21}$$

It should be noted that for this $F(t)$ we should have $r = 2$ in Eq. (19) for $F(t)$.

With these, in the asymptotic limit $s \to \infty$, $t \to \infty$ and t/s fixed we have the approximate relation,

$$\left(\frac{d\sigma}{dt}\right)_{\gamma p \to n\pi^+} \sim s^{-7} f(t/s). \tag{22}$$

Similarly for the process $\pi^- p \to n\pi^0$ if we take $r = 2$ in $\hat{F}(t)$ and use the expression (4) for the differential cross-section we obtain

$$\left(\frac{d\sigma}{dt}\right)_{\pi^- p \to n\pi^0} \sim s^{-7.5} f(t/s), \tag{23}$$

which is in fair agreement with the result of Brodsky and Farrar,

$$\left(\frac{d\sigma}{dt}\right)_{\pi N \to \pi N} \sim s^{-8} f(t/s). \tag{24}$$

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