ON KINEMATICS OF THREE BODY DECAY: APPLICATION TO MUON CAPTURE WITH NEUTRON EMISSION

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The angular distributions in the three body decay followed by nuclear gamma radiation are analysed in the model independent way in terms of the corresponding helicity decay amplitudes. This analysis is applied to muon capture processes with neutron emission. Comparison of the giant dipole resonance model with the direct process is given. Many conclusions about separation of the nuclear and weak interactions can be easily found from our results.

1. *Introduction*

Problems related to muon-proton primary interaction and to the nuclear structure which are involved in the muon capture process by complex nuclei should be separated as much as possible. Fortunately we may expect that the interference between weak interaction and nuclear physics can be in some cases negligible for the angular distribution in the normal muon capture [1, 2].

For the first time [3] the gamma-neutrino angular distribution measurements for the normal muon capture were performed in ^{28}Si. The phenomenological analysis of these data is given in our Dubna preprint [1]. However, the more exact data are needed for a reliable confirmation of the theoretical assumptions concerned with a fundamental nature of the muon-nuclei weak interaction. At the same time the muon capture process with neutron emission, which is a dominant (approximately 70% of the total muon capture rate, see e. g. [3]) should be a wide source of the important information about mechanism of the weak excitation of the nuclear states as well as about the pure nuclear problems.

Essentially the richest information of such a kind is involved in the angular and polarization distributions. For instance calculated recently the nuclear gamma-neutron angular distributions [4] and polarization of the neutrons as well as the neutron asymmetry in

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polarized muon capture by nuclei [5] clearly show the significance of such phenomena. A lot of theoretical papers (see e. g. the references in [4,5]) and many experiments were devoted to study the nuetron asymmetry in the muon capture [3, 6].

Here we consider the kinematics of the muon capture process with neutron emission. This simple topic of kinematics has been considered frequently but, in our opinion, not as generally and elegantly as in the present paper. Namely we treat process of the muon capture with neutron emission as a slow (weak) three body decay of the mu-mesic atom. This work can be considered partially as an extension of the previous one [2] applied to the normal muon capture. Our treating gives an advantage in the considerable simplification of the formulas. This separation of kinematics and dynamics allows us to give many constructive conclusions e. g. about the interference of the nuclear physics with the weak interaction.

2. *Density matrix*

The density matrix ϱ^F describes a mu-mesic atom in one of the hyperfine (hf) states with the spin F formed from the nuclear (ground) state with spin j_0 and mu-meson on the K-shell. Here $F = j_0 + \frac{1}{2}$. We assume that the mu-mesic atom was formed in a state of orientation with cylindrical symmetry. Obviously this situation occurs always when we consider, as in this paper, the muon capture by unpolarized targets. Let us choose the z-axis along the direction of the cylindrical symmetry; the density matrix is then diagonal.We have (see e. g. [7])

$$\langle M|\varrho^F|M\rangle = \frac{1}{\hat{F}} \sum (-)^{F-M} q^r C^{r0}_{FM\,F-M}. \tag{1}$$

($\hat{F} := \sqrt{2F+1}$). Here q^r are the statistical tensors of the mu-mesic atom normalized as $q^0 = 1$. The statistical tensors for $r \geq 2$ may contribute to density matrix only in the case of the muon capture by oriented or polarized nuclei. For the muon capture by unpolarized targets the density matrix is fully described by only one dynamical parameter q^1. From the other hand the polarization of muons in various hf states of the mu-mesic atom structure $F_+ = j_0 + \frac{1}{2}$ and $F_- = j_0 - \frac{1}{2}$ is characterized by the quantities λ_+ and λ_- respectively. They are defined by the mean value of muon spin in these states

$$\langle \boldsymbol{S}_\pm \rangle = \tfrac{1}{2} \lambda_\pm \boldsymbol{\sigma}, \tag{2}$$

where $\boldsymbol{\sigma}$ is the unit polarization pseudovector of the muon on the K orbit. It is easy to find the relations between the mu-mesic atom statistical tensor q^1 and the quantities λ_+ and λ_-. They look as follows:

$$q^1 = \begin{cases} +\lambda_+ \sqrt{\dfrac{3F}{F+1}} & \text{for } F = j_0 + \frac{1}{2}, \\[2ex] -\lambda_- \sqrt{\dfrac{3(F+1)}{F}} & \text{for } F = j_0 - \frac{1}{2}. \end{cases} \tag{3}$$

Generally for an arbitrary rank r the statistical tensors q^r, which describe the mu-mesic atom state with spin F coupled from two interacting polarized systems with spins j_0 and S, can be expressed as

$$q^r = \frac{(\hat{F})^3}{\hat{j}_0\hat{S}} \sum \hat{p}\hat{g}C^{r0}_{p0g0} \begin{Bmatrix} j_0 & S & F \\ j_0 & S & F \\ p & g & r \end{Bmatrix} Q^F_{pgr}(j_0S). \tag{4}$$

Formula (4) is valid only under the assumption of the cylindrical symmetry of both coupled systems with the z-axis along the common direction of symmetry.

We should note that the quantities A_{pgr} introduced by Bukhvostov and Popov [8] are essentially our Q_{pgr} because

$$A_{pgr} = (-)^g \hat{r}\hat{g}C^{r0}_{p0g0}Q^F_{pgr}(j_0S). \tag{5}$$

For two non-interacting systems we easily find the following factorization

$$Q^F_{pgr}(j_0S) = q^p(j_0)q^g(S). \tag{6}$$

For muon capture by unpolarized targets ($r \leqslant 1$) the Bukhvostov and Popov formula [8] may be expressed shortly as follows (C–G coeficient should be expressed explicitly):

$$Q^F_{pgr}(j_0, \tfrac{1}{2}) = \hat{p}(\lambda_F)^r C^{j_0m}_{j_0mp0},$$

where

$$m = \begin{cases} j_0 & \text{for } F = F_+, \\ -j_0-1 & \text{for } F = F_-. \end{cases} \tag{7}$$

This shorthand but rather formal expression (7) after substitution to formula (4), gives us immediately relations (3).

3. The angular distributions in terms of the helicity amplitudes

The muon capture process with neutron emission is a slow (weak) three body decay of corresponding mu-mesic atom. In the rest frame of the mu-mesic atom with spin F let $\boldsymbol{n}$, $\boldsymbol{\nu}$ and $\boldsymbol{p}$ be the momenta of the neutron, neutrino and final nucleus state (in general, excited state with spin j_1), respectively. Then, the amplitude describing the decay of a system with spin F and with the z-component M into neutron, neutrino and some nuclear state with the helicities λ_n, $h = -\frac{1}{2}$, and λ_1 respectively, can be written [7]

$$\langle [j_1]\lambda_1\boldsymbol{p}; [\tfrac{1}{2}]\lambda_n\boldsymbol{n}; [h]\boldsymbol{\nu}|T|\,[F]M\rangle = \sum_k T_{k\lambda_1\lambda_n}(E_nE_\nu)\overset{*}{D}{}^F_{Mk}(\alpha, \beta, \gamma). \tag{8}$$

Here E_n and E_ν are the energies of the neutron and neutrtino. The angles $\alpha\beta\gamma$ determine uniquely the plane of the decay (for details and notations we refer to the book by Werle [7] § 30). In [8] the complex functions $T_{k\lambda_1\lambda_n}$ are the helicity mu-mesic atom decay amplitudes.

We wish to describe the angular distributions for two steps process in which the muon capture is followed by the emission of the nuclear deexcitation gamma ray $j_1 \xrightarrow{\gamma} j_2$. Therefore the density matrix of the final states should be

$$\varrho_{\mathrm{FINAL}} = HT\varrho^F T^+ H^+, \tag{9}$$

where H describes the nuclear radiation process. For the matrix elements we find[1]

$$\langle \lambda_1 \lambda_n | \varrho_{\mathrm{FINAL}} | \lambda_1' \lambda_n' \rangle = \frac{1}{2F+1} \sum T_{k\lambda_1\lambda_n} \overset{*}{T}_{k'\lambda_1'\lambda_n'}$$
$$\left(\sum \hat{F}(-)^{F-k} C^r_{Fk'F-k} q^r D^r_{0k'-k}(\alpha\beta\gamma)\right)$$
$$\left((-)^{\lambda_1-\lambda_1'} \sum \hat{S} B_S C^{j_1\lambda_1}_{j_1\lambda_1'S\lambda_1-\lambda_1'} D^S_{\lambda_1-\lambda_1'0}(\varphi, \Theta, 0)\right). \tag{10}$$

Due to supposed cylindrical symmetry of decaying system we may choose the Euler angle $\alpha = 0$ in (10). The last term (in parentheses) determines the nuclear radiation process. The angles φ, Θ describe the direction of gamma ray momenta in the coordinate frame where the z-axis is parallel to the initial polarization of muons and angle $\alpha = 0$ fixes the remainder axes.

If the circular polarization of the nuclear deexcitation gamma-ray is denoted by $\eta = \pm 1$ (for right and left-polarized radiation, respectively) then the term $B_S \equiv B_S(j_1 \xrightarrow{\gamma} j_2)$ looks like [9]

$$(1+\delta^2)B_S = R_S(LL) + 2\eta\delta R_S(LL+1) + \delta^2 R_S(L+1L+1),$$

where

$$R_S(LL'):= (-)^{L-\eta} \hat{L}\hat{L}'\hat{j}_1 C^{S0}_{L'\eta L-\eta} W(j_2 L j_1 S j_1 L'), \qquad B_0 = 1. \tag{11}$$

The ratio of reduced matrix elements δ (the mixing ratio) is defined by Rose and Brink [9].

The multipolarity L of the gamma radiation is $L = |j_1 - j_2|$ if $j_1 \neq j_2$ and $L = 1$ for $j_1 = j_2 \neq 0$. From (10) we easily find e. g. the general formula for gamma-neutron angular distribution for muon capture by completely depolarized muons ($r = 0$)

$$W(E_n, E_\nu, \hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{k}}) = \frac{1}{2F+1} \sum_{\lambda_1} a_{\lambda_1}(E_n, E_\nu) \sum_S \hat{S} B_S C^{j_1\lambda_1}_{j_1\lambda_1 S0} P_S(\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{k}}),$$

where

$$a_{\lambda_1}(E_n, E_\nu) = \sum |T_{k\lambda_1\lambda_n}(E_n, E_\nu)|^2. \tag{12}$$

It is easy to see the relation between our correlation coefficients a_{λ_1} and a_S calculated explicitly in the standard theory in Ref. [4] (formula (23) in [4]). In formula (12) and below we denote by $\hat{\boldsymbol{n}}$ the unit vector in the direction of $\boldsymbol{n}$, etc.

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[1] Arbitrary normalization.

If the neutron emission is the two step process i. e. we are dealing with the compound nucleus mechanism then the E_n and E_ν in formula (8) may take a number of discrete values only, contrary to the direct process mechanism in which the energies change continuously in the region of the Dalitz plot.

Schematically for giant dipole resonance mechanism the muon capture process with neutron emission may be described as

$$\begin{array}{l}\text{Mu-mesic atom } [F] \to [j]+\nu \\ \qquad\qquad\qquad\quad \hookrightarrow [j_1]+n \\ \qquad\qquad\qquad\qquad\quad \hookrightarrow [j_2]+\gamma.\end{array} \tag{13}$$

For this particular mechanism the matrix element of the first weak two-body decay is described by complex numbers $T_\lambda(j)$ — the helicity mu-mesic atom decay amplitudes which are related to the normal muon capture [2]. Consequently the second step: the nuclear two-body decay is described by the complex numbers $N^j_{\lambda\lambda_n}$ — the helicity two body decay amplitudes. The definitions are as follows [7]

$$\langle \boldsymbol{\nu}[h]; -\boldsymbol{\nu}[j]\lambda|T|\,[F]M\rangle = \frac{\hat{F}}{\sqrt{4\pi}}\, T_\lambda(j)\overset{*}{D}{}^F_{M\lambda-h}(\hat{\boldsymbol{\nu}}),$$

$$\langle \boldsymbol{n}[\tfrac{1}{2}]\lambda_n; -\boldsymbol{n}[j_1]\lambda_1|N|\,[j]\lambda\rangle = \frac{\hat{j}}{\sqrt{4\pi}}\, N^j_{\lambda_n\lambda_1}\overset{*}{D}{}^j_{\lambda\lambda_n-\lambda_1}(\hat{\boldsymbol{n}}). \tag{14}$$

Therefore the formula (10) in this case looks like

$$\begin{aligned}\langle\lambda_1\lambda_n|\varrho_{\mathrm{FINAL}}|\lambda_1'\lambda_n'\rangle = (4\pi)^{-2}\sum\{T_\lambda(j)\overset{*}{T}_{\lambda'}(j')\hat{j}\hat{j}'(-)^{\lambda-\lambda_n+\lambda_1}N^j_{\lambda_n\lambda_1}\overset{*}{N}{}^{j'}_{\lambda_n'\lambda_1'}\} \\ \{\hat{F}\sum q^r(-)^{F-h+\lambda}C^r_{Fh-\lambda'\,F-h+\lambda}D^r_{0\lambda-\lambda'}(\hat{\boldsymbol{\nu}})\} \\ \{\sum C^{p\lambda'-\lambda}_{j'\lambda'j-\lambda}C^p_{j'\lambda_n'-\lambda_1'j\lambda_1-\lambda_n}D^p_{\lambda'-\lambda\;\lambda_1-\lambda_1'+\lambda_n'-\lambda_n}(\hat{\boldsymbol{n}})\} \\ \{(-)^{\lambda_1-\lambda_1'}\sum\hat{S}B_SC^{j_1}_{j_1\lambda_1'S\lambda_1-\lambda_1'}D^S_{\lambda_1-\lambda_1'0}(\hat{\boldsymbol{k}})\}.\end{aligned} \tag{15}$$

Particularly the following factorization holds

$$\frac{1}{2F+1}\sum_k T_{k\lambda_1\lambda_n}\overset{*}{T}_{k\lambda_1'\lambda_n'} = (4\pi)^{-2}\left(\sum_\lambda |T_\lambda|^2\right)N_{\lambda_n\lambda_1}\overset{*}{N}_{\lambda_n'\lambda_1'}. \tag{16}$$

If we denote by l and J the orbital and total angular momentum of the neutron, we may introduce usual multipole couplings instead of the helicity coupling. The multipole reduced amplitudes which describe the neutron emission are connected with the helicity amplitudes

$$a_{lJ} = \sqrt{\pi}\sum_{\lambda_n\lambda_1} i^{-l}C^J_{l0\frac{1}{2}\lambda_n}(-)^J C^j_{J-\lambda_n j_1\lambda_1}\frac{\hat{l}}{\hat{j}}N_{\lambda_n\lambda_1}. \tag{17}$$

The transition normal capture rate is given by

$$\Lambda^c = E_\nu^2\sum_\lambda |T_\lambda|^2. \tag{18}$$

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