SPIN MAY PREVENT FORMATION OF SMALL BLACK HOLES*

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It is shown that in the Einstein-Cartan theory of gravitation, black holes of arbitrary small mass could not be formed. The minimal mass of a primordial black hole is estimated.

It was shown recently by Hawking [1] that a black hole of mass M will radiate particles of all kinds as if it were a hot body with temperature $T \approx 10^{-7} M_{\odot}/M$ K. As a result a black hole will loose mass therefore increasing temperature and this self perpetuating process will lead inevitably to an explosive end. Masses of black holes which form in the process of gravitational collapse are of the order of magnitude of the solar mass and therefore temperature of those black holes will be much lower than the temperature of the black body background radiation. The process of evaporation is important only for black holes of small mass as $\frac{dM}{dt} \approx \hbar \left(\frac{c^2}{GM}\right)^2 = 10^{20} \left(\frac{10^{15}\text{g}}{M}\right)^2$ erg s^{-1} so the black hole in otherwise empty space would radiate away all its mass in a time scale of the order of $\tau = 10^{10} \left(\frac{M}{10^{15}g}\right)$ y. Hence only black holes with mass smaller than 10^{15}g could have evaporated during the lifetime of the Universe which is estimated as some 10^{10} y. However, small balck holes could not be formed in the process of gravitational collapse. In the standard big-bang theory they could have been created at very early stages of the evolution of the Universe when the energy density of matter was very high.

It would like to point out that this might not be possible if one takes into account the influence of spin on the spacetime geometry. The simplest cosmological model in the Einstein-Cartan theory discussed by Trautman [2] is non-singular. In this model the space is uniformly filled with a spinning dust with all spins aligned along one axis. It was shown

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that those assumptions are compatible with the Robertson-Walker line element

$$ds^2 = c^2dt^2 - R^2(t)\,(dx^2+dy^2+dz^2) \tag{1}$$

with the scaling function $R(t)$ satisfying a modified Friedmann equation

$$\tfrac{1}{2}\dot{R}^2 - \frac{GM}{R} + \frac{3G^2S^2}{2c^2R^4} = 0, \tag{2}$$

where $M = \frac{4}{3}\pi\varrho R^3 = \text{const}$ and $S = \frac{4}{3}\pi\sigma R^3 = \text{const}$, σ being the density of spin. This equation has an exact solution

$$R(t) = \left(\frac{3GS^2}{2Mc^4} + \frac{9GM}{2}t^2\right)^{1/3}. \tag{3}$$

The minimal value of the radius of the Universe is therefore given by

$$R(0) = \left(\frac{3GS^2}{2Mc^4}\right)^{1/3}. \tag{4}$$

Equation (3) allows also a static solution with R given by relation (4). Let us look for a condition under which $R(0)$ will be equal to the gravitational radius of a body with mass M; we have

$$\left(\frac{3GS^2}{2Mc^4}\right)^{1/3} = \frac{2GM}{c^2} \tag{5}$$

which leads to

$$M = \frac{3^{1/4}}{2}\left(\frac{Sc}{G}\right)^{1/2}, \tag{6}$$

assuming that $M = m_n N$, $S = \frac{1}{2}\hbar N$, where m_n – mass of the neutron; for number of particles N we obtain $N = \frac{\sqrt{3}}{8}\hbar c/Gm_n^2$ and therefore the mass M is given by $M = \frac{\sqrt{3}}{8} \times \hbar c/Gm_n$. For more massive body the gravitational radius is greater than $R(0)$ and therefore it could form a black hole. The critical mass of the smallest black hole in the Einstein-Cartan theory is therefore $6 \cdot 10^{13}$ g. Black holes of smaller mass could not be formed in the Einstein-Cartan theory of gravitation. Mass spectrum of primordial black holes in cosmological models based on the Einstein-Cartan theory of gravitation should therefore have cut off at masses smaller than $6 \cdot 10^{13}$ g. Primordial black holes of this mass should have evaporated at $z \approx 100$ and the emitted radiation could only contribute to the observed background gamma radiation.

REFERENCES

[1] S. W. Hawking, *Commun. Math. Phys.* **43**, 199 (1975).
[2] A. Trautman, *Nature* **242**, 7 (1973).