A NOTE ON THE SOLUTION OF THE EQUATION

$$g_{\underset{+-}{\mu\nu;\lambda}} - \frac{2}{3}(\Gamma_\nu g_{\mu\lambda} + \Gamma_\lambda g_{\mu\nu}) = 0$$

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The affine connection of Einstein's weak and strong unified field theories can be expressed in terms of the fundamental tensor by solving a system of sixty-four algebraic equations. In the Einstein-Kaufman theory, there is again a system of sixty-four algebraic equations but it is shown that the rank of this system is only sixty.

1. *Introduction*

Mme Tonnelat obtained (Ref. [1]) a general solution of the equation

$$g_{\underset{+-}{\mu\nu;\lambda}} \equiv g_{\mu\nu,\lambda} - \Gamma^{\sigma}_{\mu\lambda} g_{\sigma\nu} - \Gamma^{\sigma}_{\lambda\nu} g_{\mu\sigma} = 0, \tag{1}$$

of Einstein's non-symmetric unified field theory, expressing the components $\Gamma^{\lambda}_{\mu\nu}$ of the affine connection in terms of the components $g_{\mu\nu}$ of the fundamental tensor and their first derivatives. Further work (Ref. [2]) on the above theory revealed that it may be more appropriate to consider first the equation

$$g_{\underset{+-}{\mu\nu;\lambda}} - \frac{2}{3}(\Gamma_\nu g_{\mu\lambda} + \Gamma_\lambda g_{\mu\nu}) = 0. \tag{2}$$

Since these may be written in the form

$$g_{\underset{+-}{\mu\nu'\lambda}}(\Delta^{\alpha}_{\beta\gamma}) = 0, \tag{3}$$

where

$$\Delta^{\alpha}_{\beta\gamma} = \Gamma^{\alpha}_{\beta\gamma} + \frac{2}{3}(\delta^{\alpha}_{\beta}\Gamma_\gamma), \tag{4}$$

for which

$$\Delta_\beta \equiv \Delta^{\alpha}_{\underset{\vee}{\beta\alpha}} \equiv 0, \tag{5}$$

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(in Einstein's and Tonnelat's notation adopted throughout, a hook denoting the skew symmetric part), it appears that equation (2) cannot determine Γ_λ. However, an explicit proof of this, namely that the equation (2) considered as an algebraic equation for $\Gamma^{\lambda}_{\mu\nu}$ is of rank 60, can be obtained only by following through the Tonnelat solution for the new equations. The purpose of this note is to sketch such a proof while emphasising the generality of the Tonnelat method.

2. *The Tonnelat method*

The general method depends on splitting the equation (2) into forty equations for the symmetric part $\Gamma^{\lambda}_{\underline{\mu\nu}}$ of the affine connection and twenty four equations for its skew symmetric part (a tensor) $\Gamma^{\lambda}_{\check{\mu\nu}}$, and solving the latter by purely tensorial techniques. If we write

$$g_{\underline{\mu\nu}} = h_{\mu\nu} = h_{\nu\mu}, \qquad g_{\check{\mu\nu}} = k_{\mu\nu} = -k_{\nu\mu}, \tag{6}$$

$h^{\mu\nu}$, $k^{\mu\nu}$ respectively, for their tensorial inverses and h, k for their determinants (so that $h < 0$ and $k \geqslant 0$) we readily find that

$$\Gamma^{\lambda}_{\underline{\mu\nu}} = \begin{Bmatrix} \lambda \\ \mu\nu \end{Bmatrix} + u^{\lambda}_{\mu\nu}, \tag{7}$$

where

$$u_{\mu\nu,\varrho} = h_{\varrho\lambda} u^{\lambda}_{\mu\nu} = \Gamma^{\sigma}_{\check{\mu\varrho}} k_{\sigma\nu} + \Gamma^{\sigma}_{\check{\nu\varrho}} k_{\sigma\mu} - \tfrac{1}{3}\,\Gamma_{\mu} g_{\varrho\nu} - \tfrac{1}{3}\,\Gamma_{\nu} g_{\varrho\mu}, \tag{8}$$

and $\begin{Bmatrix} \lambda \\ \mu\nu \end{Bmatrix}$ are the Christoffel brackets formed from $h_{\mu\nu}$. Also

$$\Gamma_{\check{\mu\nu},\varrho} \equiv h_{\varrho\lambda}\Gamma^{\lambda}_{\check{\mu\nu}} = -\tfrac{1}{2}\,k_{\mu\nu\varrho} + \nabla_{\varrho} k_{\mu\nu} - (u^{\sigma}_{\mu\varrho} k_{\sigma\nu} - u^{\sigma}_{\nu\varrho} k_{\sigma\mu}) + \tfrac{1}{3}\,\Gamma_{\mu} g_{\nu\varrho} - \tfrac{1}{3}\,\Gamma_{\nu} g_{\mu\varrho} - \tfrac{2}{3}\,\Gamma_{\varrho} k_{\mu\nu}, \tag{9}$$

where $k_{\mu\nu\varrho} = k_{\underset{\cdot}{\mu}\underset{\cdot}{\nu},\underset{\cdot}{\varrho}}$ (the cycle sum) (we shall use a similar notation for other three index quantities, $\Gamma_{\mu\nu\varrho}$ and $u_{\mu\nu\varrho}$), and Δ_{ϱ} is the covariant differentiation operation with respect to $\begin{Bmatrix} \lambda \\ \mu\nu \end{Bmatrix}$. Then

$$u_{\mu\nu\varrho} = -\tfrac{2}{3}\,\Gamma_{\underset{\cdot}{\mu}} h_{\underset{\cdot}{\nu}\underset{\cdot}{\varrho}} \qquad \text{and} \qquad \Gamma_{\mu\nu\varrho} = -\tfrac{1}{2}\,k_{\mu\nu\varrho}. \tag{10}$$

Algebraic equations for $\Gamma_{\check{\mu\nu},\varrho}$ are obtained from the equations (8) and (9) in a self evident way but their solutions relies crucially on the introduction of two linearly independent operations. The first is a repeated raising of a covariant index with $h^{\mu\nu}$ and lowering of it with $k_{\mu\nu}$, which is called the "barring" of that index. Thus

$$A^{\cdots}_{\cdots\bar{\lambda}\cdots} = k_{\lambda\alpha} h^{\alpha\beta} A^{\cdots}_{\cdots\beta\cdots};$$

$$A^{\cdots}_{\cdots\bar{\bar{\lambda}}\cdots} = k_{\lambda\alpha} h^{\alpha\beta} k_{\beta\gamma} h^{\gamma\varepsilon} A^{\cdots}_{\cdots\varepsilon\cdots}, \tag{11}$$

etc. This, in a way, avoids raising indices with $k^{\mu\nu}$ (which may not exist, although we can always write

$$k^{\mu\nu} = \frac{1}{2\sqrt{k}}\, \varepsilon^{\mu\nu\alpha\beta} k_{\alpha\beta}, \qquad k_{\mu\nu} = \tfrac{1}{2} \sqrt{k}\, \varepsilon_{\mu\nu\alpha\beta} k^{\alpha\beta}. \tag{12}$$

Since

$$\sqrt{k} = \tfrac{1}{8}\, \varepsilon^{\mu\nu\varrho\sigma} k_{\mu\nu} k_{\varrho\sigma},$$

and

$$g = h + k + \tfrac{1}{2}\, h^{\mu\varrho} h^{\nu\sigma} k_{\mu\nu} k_{\varrho\sigma},$$

it is not difficult to show that

$$A_{\bar{\bar{\bar{\varrho}}}} = -\frac{k}{h} A_{\varrho} - \frac{1}{h} (g - h - k) A_{\bar{\bar{\varrho}}}, \tag{13}$$

$$\equiv \omega^2 A_{\bar{\bar{\varrho}}} + (1 - j^2) A_{\bar{\bar{\bar{\varrho}}}}, \quad \text{say, where} \quad \omega = \sqrt{-\frac{k}{h}} \quad \text{and} \quad j^2 = \frac{g-k}{h}.$$

The second operation is the well known duality one:

$$\overset{*}{A}_{\cdots\mu\nu\cdots} = \frac{\sqrt{-h}}{2}\, \varepsilon_{\mu\nu\alpha\beta} h^{\alpha\varrho} h^{\beta\sigma} A_{\cdots\varrho\sigma\cdots} \tag{14}$$

We have a useful relation between these, valid for any vector A_λ:

$$2h^{\sigma\lambda} k_{\lambda\varrho} k_{\sigma[\overset{*}{\mu}} A_{\nu]} = -\sqrt{-h}\, h^{\sigma\lambda} \varepsilon_{\mu\nu\bar{\bar{\varrho}}\sigma} A_\lambda, \tag{15}$$

the square bracket this time denoting the skew symmetric part and the star referring (as throughout the sequel) to the pair $(\mu\nu)$ only. The barring and starring operations can be performed on both sides of a tensor equation (the latter, of course, only with respect to a skew symmetric pair of indices), leading in general to additional equations. There are some other identities which it is worthwhile to recall in the reduction of the equations of interest to us. These are

$$h \varepsilon_{\mu\nu\alpha\beta} h^{\alpha\lambda} h^{\beta\sigma} = \varepsilon^{\lambda\sigma\alpha\beta} h_{\alpha\mu} h_{\beta\nu}, \tag{16}$$

and

$$\varepsilon_{\alpha\lambda\mu\nu} \varepsilon^{\alpha\beta\gamma\eta} = (\delta^\beta_{\underset{\cdot}{\lambda}} \delta^\gamma_{\underset{\cdot}{\mu}} \delta^\eta_{\underset{\cdot}{\nu}} - \delta^\beta_{\underset{\cdot}{\mu}} \delta^\gamma_{\underset{\cdot}{\lambda}} \delta^\eta_{\underset{\cdot}{\nu}}),$$

$$\varepsilon_{\alpha\beta\mu\nu} \varepsilon^{\alpha\beta\gamma\eta} = 2!(\delta^\gamma_\mu \delta^\eta_\nu - \delta^\gamma_\nu \delta^\eta_\mu), \qquad \varepsilon_{\alpha\beta\gamma\nu} \varepsilon^{\alpha\beta\gamma\eta} = 3! \delta^\eta_\nu. \tag{17}$$

It follows now from the equations (8) and (9) that

$$\Gamma_{\underset{\smile}{\mu\nu},\varrho} - h^{\sigma\lambda} [\bar{\bar{\Gamma}}_{\mu\lambda\varrho} k_{\sigma\nu} - \bar{\bar{\Gamma}}_{\nu\lambda\varrho} k_{\sigma\mu}] - 2h^{\sigma\lambda} [\Gamma_{\underset{\smile}{\varrho\lambda},\bar{\mu}} k_{\sigma\nu} - \Gamma_{\underset{\smile}{\varrho\lambda}\bar{\nu}} k_{\sigma\mu}] - \bar{\bar{\bar{\Gamma}}}_{\mu\nu\varrho} + \Gamma_{\mu\nu,\bar{\bar{\bar{\varrho}}}}$$

$$= -\tfrac{1}{2}\, k_{\mu\nu\varrho} + \nabla_\varrho k_{\mu\nu} + \tfrac{1}{3} [\Gamma_\mu (h_{\nu\varrho} + h^{\sigma\Lambda} k_{\lambda\varrho} k_{\sigma\nu}) - \Gamma_\nu (h_{\mu\varrho} + h^{\sigma\lambda} k_{\lambda\varrho} k_{\sigma\mu})], \tag{18}$$

where $\bar{\Gamma}_{\mu\nu\varrho} = \Gamma_{\underset{\smile}{\mu\lambda},\bar{\varrho}}$, $\bar{\bar{\Gamma}}_{\mu\lambda\varrho} = \Gamma_{\mu\lambda,\bar{\bar{\varrho}}}$, the bars not shifting, of course, in the cyclic summation.

The quantities in the square brackets on the left hand side of the equation (18) and $\overline{\overline{\Gamma}}$ have been calculated in Tonnelat's paper directly from definitions to give the equation

$$\Gamma_{\underset{\vee}{\mu\nu},\varrho}-2\omega\Gamma_{\underset{\vee}{\mu\nu},\varrho}+\Gamma_{\underset{\vee}{\mu\nu},\varrho} = -\tfrac{1}{2}\,k_{\mu\nu\varrho}+\nabla_\varrho k_{\mu\nu}-\tfrac{1}{2}\,(1-j^2)k_{\mu\nu\varrho}$$
$$+\tfrac{1}{2}\,h^{\sigma\lambda}h^{\tau\pi}(\sqrt{k}\,\varepsilon_{\mu\nu\sigma\tau}-k_{\sigma\tau}k_{\mu\nu})k_{\varrho\lambda\pi}-\frac{\sqrt{k}}{4}\,\varepsilon_{\mu\nu\varrho\lambda}h^{\alpha\sigma}h^{\lambda\tau}k_{\sigma\tau}k^{\beta\gamma}k_{\alpha\beta\gamma}$$
$$+\tfrac{1}{3}\,[\Gamma_\mu(h_{\nu\varrho}+h^{\sigma\lambda}k_{\lambda\varrho}k_{\sigma\nu})-\Gamma_\nu(h_{\mu\varrho}+h^{\sigma\lambda}k_{\lambda\varrho}k_{\sigma\mu})]$$
$$-\sqrt{k}\,\varepsilon_{\mu\nu\lambda\pi}h^{\lambda\sigma}h^{\pi\tau}k_{\sigma\varrho}B_\tau+\sqrt{k}\,\varepsilon_{\mu\nu\varrho\sigma}k^{\lambda\sigma}A_\lambda-2k_{\mu\nu}A_\varrho, \tag{19}$$

where

$$A_\varrho = \tfrac{1}{2}\,h^{\mu\lambda}h^{\nu\tau}k_{\mu\nu}\Gamma_{\underset{\vee}{\lambda\tau},\varrho} \quad \text{and} \quad B_\varrho = \tfrac{1}{2}\,k^{\mu\nu}\Gamma_{\underset{\vee}{\mu\nu},\varrho}. \tag{20}$$

3. Calculation of A_ϱ & B_ϱ and the final equation

Contracting equations (2) with $g^{\mu\nu}$ gives

$$\Gamma^{\sigma}_{\underline{\varrho\sigma}} = \partial_\varrho \ln\sqrt{-g}\,-\tfrac{5}{3}\,\Gamma_\varrho. \tag{21}$$

Using equations (21) and (9) we obtain directly from the definition of B_ϱ

$$B_\varrho = -\tfrac{1}{4}\,k^{\mu\nu}k_{\mu\nu\varrho}+\partial_\varrho \ln\sqrt{-\frac{k}{g}}+\tfrac{1}{6}\,k^{\mu\nu}(\Gamma_\mu g_{\nu\varrho}-\Gamma_\nu g_{\mu\varrho})+\tfrac{1}{3}\,\Gamma_\varrho. \tag{22}$$

The terms proportional to Γ_λ are absent from the corresponding formula of Mme Tonnelat.

Similarly, contracting equation (18) with $k^{\mu\nu}$ and substituting from (22), gives

$$A_\varrho = -\tfrac{1}{4}\,h^{\alpha\mu}h^{\beta\nu}k_{\alpha\beta}k_{\mu\nu\varrho}+\partial_\varrho \ln\sqrt{\frac{g}{h}}+\tfrac{1}{3}\,h^{\sigma\lambda}\Gamma_\sigma k_{\lambda\varrho}. \tag{23}$$

We may note that since B_ϱ appears in the equation (19) multiplied by $\sqrt{k}$, the result which we are about to write down is still valid when the determinant of $k_{\mu\nu}$ vanishes.

We now substitute for A_ϱ and B_ϱ into the equation (19) and obtain

$$\Gamma_{\underset{\vee}{\mu\nu},\varrho}-2\omega\Gamma^{*}_{\underset{\vee}{\mu\nu},\varrho}+\Gamma_{\underset{\vee}{\mu\nu},\overline{\overline{\varrho}}} = J_{\underset{\vee}{\mu\nu},\varrho}+W_{\underset{\vee}{\mu\nu},\varrho}, \tag{24}$$

where

$$J_{\underset{\vee}{\mu\nu},\varrho} = -\tfrac{1}{2}\,k_{\mu\nu\varrho}+\nabla_\varrho k_{\mu\nu}+\tfrac{1}{2}\,\omega k^{*}_{\mu\nu\varrho}+\tfrac{1}{4}\,\omega k^{*}_{\mu\nu}k^{\sigma\tau}k_{\sigma\tau\varrho}$$
$$-(k_{\mu\nu}\delta^\lambda_\varrho-\tfrac{1}{2}\,\sqrt{k}\,\varepsilon_{\mu\nu\varrho\sigma}k^{\lambda\sigma})\partial_\lambda \ln(j^2-\omega^2)-\tfrac{1}{2}\,\omega(\varepsilon^{*}_{\mu\nu\varrho\sigma}k^{\lambda\sigma}-k^{*}_{\mu\nu}\delta^\lambda_\varrho)\partial_\lambda \ln\left(\frac{j^2-\omega^2}{\omega^2}\right), \tag{25}$$

and is the same as the corresponding expression of Mme Tonnelat, and

$$W_{\underset{\vee}{\mu\nu},\varrho} = \tfrac{2}{3}\,h^{\sigma\lambda}[k\varepsilon_{\mu\nu\varrho\sigma}\Gamma_\lambda+k_{\lambda\varrho}k_{\sigma[\mu}\Gamma_{\nu]}]+\tfrac{2}{3}\,\Gamma_{[\mu}h_{\nu]\varrho}. \tag{26}$$

Equation (24) is now in the form in which it can be readily solved by Tonnelat's method. The main difference is that previously Γ_λ was known and now it can only be determined, if at all, from equation (24) itself.

4. *Solution of the equation* (24)

Let us apply to the equation (24) in succession the star operation, the double bar operation and the two operations combined. Remembering that

$$A^{**}_{\cdots\mu\nu\cdots} = -A_{\cdots\mu\nu\cdots}, \tag{27}$$

and recalling the identity (13) we obtain besides (24) the following additional equations for the four unknowns $\Gamma_{\underset{\vee}{\mu\nu},\varrho}$, $\Gamma^*_{\underset{\vee}{\mu\nu},\varrho}$, $\Gamma_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}}$, and $\Gamma^*_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}}$

$$2\omega\Gamma_{\underset{\vee}{\mu\nu},\varrho}+\Gamma^*_{\underset{\vee}{\mu\nu},\varrho}+\Gamma^*_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}} = J^*_{\underset{\vee}{\mu\nu},\varrho}+W^*_{\underset{\vee}{\mu\nu},\varrho},$$

$$\omega^2\Gamma_{\underset{\vee}{\mu\nu},\varrho}+(2-j^2)\Gamma_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}}-2\omega\Gamma^*_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}} = J_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}}+W_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}},$$

$$\omega^2\Gamma^*_{\underset{\vee}{\mu\nu},\varrho}+2\omega\Gamma_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}}+(2-j^2)\Gamma^*_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}} = J^*_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}}+W^*_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}}. \tag{28}$$

From (24) and (28), we get

$$(a^2+b^2)\Gamma_{\underset{\vee}{\mu\nu},\varrho} = U_{\underset{\vee}{\mu\nu},\varrho}+(2\omega b-ac)W_{\underset{\vee}{\mu\nu},\varrho}+(2\omega a+bc)W^*_{\underset{\vee}{\mu\nu},\varrho}+aW_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}}-bW^*_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}}, \tag{29}$$

where

$$a = 5\omega^2+j^2-2, \quad b = 2\omega(3-j^2), \quad c = 2-j^2,$$

and

$$U_{\underset{\vee}{\mu\nu},\varrho} = (2\omega b-ac)J_{\underset{\vee}{\mu\nu},\varrho}+(2\omega a+bc)J^*_{\underset{\vee}{\mu\nu},\varrho}+aJ_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}}-bJ^*_{\underset{\vee}{\mu\nu},\bar{\bar{\varrho}}},$$

is the Tonnelat solution for $\Gamma_{\underset{\vee}{\mu\nu},\varrho}$ in the case when $\Gamma_\lambda = 0$.

Since then $\Gamma^\lambda_{\mu\nu}$ is determined by the equations (3) (with $\Delta^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu}$) for which

$$\sqrt{-g}\,\Gamma_\mu = h_{\mu\nu}\mathfrak{g}^{\underset{\vee}{\nu\sigma}}{}_{,\sigma}, \tag{30}$$

it follows that if

$$g \neq 0 \quad \text{and} \quad \mathfrak{g}^{\underset{\vee}{\mu\nu}}{}_{,\nu} = 0,$$

then

$$U_\mu = h^{\nu\varrho}U_{\underset{\vee}{\mu\nu},\varrho} = 0, \tag{31}$$

Unless $a = 0$ and $b = 0$, the Tonnelat solution is unique.

If we now let

$$A_{\mu\nu,\lambda} = -\tfrac{2}{3}\Gamma_\nu h_{\mu\lambda}, \tag{32}$$

so that

$$A_{\underset{\vee}{\mu\nu},\lambda} = \tfrac{2}{3}\,\Gamma_{[\mu}h_{\nu]\lambda}, \tag{33}$$

then

$$\begin{aligned} A^{*}_{\underset{\vee}{\mu\nu},\varrho} &= \frac{\sqrt{-h}}{2}\,\varepsilon_{\mu\nu\alpha\beta}h^{\alpha\gamma}h^{\beta\delta}A_{\gamma\delta,\varrho} \\ &= \frac{\sqrt{-h}}{6}\,\varepsilon_{\mu\nu\alpha\beta}h^{\alpha\gamma}h^{\beta\delta}(\Gamma_{\gamma}h_{\delta\varrho}-\Gamma_{\delta}h_{\gamma\varrho}) \\ &= -\frac{\sqrt{-h}}{3}\,\varepsilon_{\mu\nu\varrho\sigma}h^{\sigma\lambda}\Gamma_{\lambda}, \end{aligned}$$

or

$$-2\omega A^{*}_{\underset{\vee}{\mu\nu},\varrho} = \tfrac{2}{3}\sqrt{k}\;h^{\sigma\lambda}\varepsilon_{\mu\nu\varrho\sigma}\Gamma_{\lambda}. \tag{34}$$

Also

$$\begin{aligned} A_{\underset{\vee}{\mu\nu},\overset{=}{\varrho}} &= k_{\varrho\sigma}h^{\delta\gamma}k_{\gamma\beta}h^{\beta\alpha}A_{\underset{\vee}{\mu\nu},\alpha} \\ &= \tfrac{1}{3}\,k_{\varrho\delta}h^{\delta\gamma}k_{\gamma\beta}h^{\beta\alpha}(\Gamma_{\mu}h_{\gamma\alpha}-\Gamma_{\nu}h_{\mu\alpha}) \\ &= \tfrac{1}{3}\,k_{\varrho\delta}h^{\delta\gamma}k_{\gamma\nu}\Gamma_{\mu}-\tfrac{1}{3}\,k_{\varrho\delta}h^{\delta\gamma}k_{\gamma\mu}\Gamma_{\nu} \\ &= \tfrac{2}{3}\,h^{\sigma\lambda}k_{\lambda\varrho}k_{\sigma[\mu}\Gamma_{\nu]}. \end{aligned} \tag{35}$$

Hence

$$A_{\underset{\vee}{\mu\nu},\varrho}-2\omega A^{*}_{\underset{\vee}{\mu\nu},\varrho}+A_{\underset{\vee}{\mu\nu},\overset{=}{\varrho}} = W_{\underset{\vee}{\mu\nu},\varrho}.$$

Consequently, equation (29) is equivalent to

$$(a^2+b^2)\Gamma_{\underset{\vee}{\mu\nu},\varrho} = U_{\underset{\vee}{\mu\nu},\varrho}+\tfrac{2}{3}\,(a^2+b^2)\Gamma_{[\mu}h_{\nu]\varrho}$$

from which Γ_{μ} cannot be determined.

In conclusion, we note that the above method of solution applies to any equation with scalar coefficients α, β, γ, δ, of the form

$$\alpha A_{\underset{\vee}{\mu\nu}\varrho}+\beta A^{*}_{\underset{\vee}{\mu\nu}\varrho}+\gamma A_{\underset{\vee}{\mu\nu}\overset{=}{\varrho}}+\delta A^{*}_{\underset{\vee}{\mu\nu}\overset{=}{\varrho}} = W_{\underset{\vee}{\mu\nu}\varrho}.$$

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