IMPACT PARAMETER ANALYSIS OF MULTIPLICITY DISTRIBUTION IN HIGH ENERGY PROTON-NUCLEUS COLLISIONS

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Assuming the geometrical model of particle production, the average multiplicity of negative particles produced in high-energy proton-nucleus collisions at fixed impact parameter is determined from experimental multiplicity distribution and elastic scattering data. The results are compared with the similar analysis performed earlier for proton-proton collisions.

In this paper we continue the discussion of the multiplicity distribution in the impact parameter representation. Using the method proposed in Ref. [1] we attempt to determine the average multiplicity of particles produced in hadron-nucleus collision at given impact parameter.

Our starting point is so-called geometrical model [2, 3, 4]. In this model the multiplicity distribution is represented by the formula

$$P(n) = \frac{1}{\sigma}\int d^2b\sigma(b)p(n, b), \tag{1}$$

where $\sigma = \int d^2b\sigma(b)$ is the total inelastic cross-section and $\sigma(b)$ is the total inelastic cross-section at impact parameter b. $p(n, b)$ is the multiplicity distribution of particles produced in the collision at impact parameter b. It is furthermore assumed that at high energy the distribution $p(n, b)$ is very narrow and can be approximated by δ-function

$$p(n, b) \sim \delta(n-\bar{n}(b)), \tag{2}$$

where $\bar{n}$ (b) is the average multiplicity of the collision at impact parameter b. It is this function $\bar{n}(b)$ which we want to determine from experimental data.

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To this end we use the equation

$$\overline{N}P(n) = \frac{2\pi b\sigma(b)}{\sigma} \frac{\overline{N}}{\left|\frac{d\overline{n}(b)}{db}\right|_{b=b_n}} \tag{3}$$

which follows from Eqs (1), (2) of [1, 2]. Here b_n is the solution of the equation

$$\overline{n}(b_n) = n \tag{4}$$

and $\overline{N}$ is the average multiplicity of the collision

$$\overline{N} = \sum_n nP(n) = \frac{1}{\sigma} \int d^2b\sigma(b)\overline{n}(b). \tag{5}$$

As shown in reference [1] equation (3) can be considered as a differential equation for $\overline{n}(b)$

$$\frac{dw}{db} = \pm \frac{2\pi b\sigma(b)}{\sigma} \frac{1}{\psi(w)}, \tag{6}$$

where for convenience we denoted

$$w(b) = \frac{\overline{n}(b)}{\overline{N}}. \tag{7}$$

The sign ambiguity of equation (6) implies that there are two solutions for $\overline{n}(b)$. In this paper we consider only the so-called "intuitive" solution which is obtained by taking a negative sign of the right-hand side of Eq. (6). This corresponds to the geometrical picture of the collision in which the "central" collisions at small impact parameter lead to the production of many particles, whereas in "peripheral" collisions at large impact parameter only a few particles are produced.

The solution $w = w(b)$ of Eq. (6) is obtained by integration

$$\int_0^{w(b)} \psi(w)dw = \frac{1}{\sigma} \int_b^{\infty} d^2b\sigma(b). \tag{8}$$

In reference [1] $w(b)$ was found from Eq. (8) and known KNO function [5] $\psi(z)$ and inelastic cross-section $\sigma(b)$ for p–p collision. In the present paper we solve Eq. (8) for the case of p-nucleus collision.

As seen from equation (8), to determine $w(b)$, it is necessary to know $\sigma(b)$ and $\psi(z)$. For a given nucleus of mass number A the inelastic cross-section $\sigma(b)$ can be very well estimated from multiple-scattering formula

$$\sigma_A(b) = 1-[1-\sigma D(b)]^A, \tag{9}$$

where

$$D(b) = \int_{-\infty}^{+\infty} \varrho(b, z)dz \tag{10}$$

and ϱ is the nuclear density. σ is the total proton-proton inelastic cross-section. The nuclear densities were taken in the form of Saxon-Woods distribution.

$$\varrho(\vec{r}) = \frac{\varrho_0}{1+e^{\frac{(r-R)}{a}}}, \tag{11}$$

where the nuclear parameters R and a were taken from reference [7].

For the KNO function we assume that it is the same for all nuclei, i. e. equal to that of the hydrogen. This assumption is consistent with available experimental data [6]. In the present calculation we used the Møller fit [8]

$$\psi(z) = Az^C \exp(-Bz^{C+1}) \tag{12}$$

with $A = 1.43$, $B = 0.758$, $C = 0.886$.

When equation (12) is introduced into equation (8) we obtain

$$w(b) = \exp\left[\frac{1}{C+1}\ln\left(-\frac{1}{B}\ln\left(\frac{\pi}{\sigma}\int_0^{b^2}\sigma_A(b)db^2\right)\right)\right]. \tag{13}$$

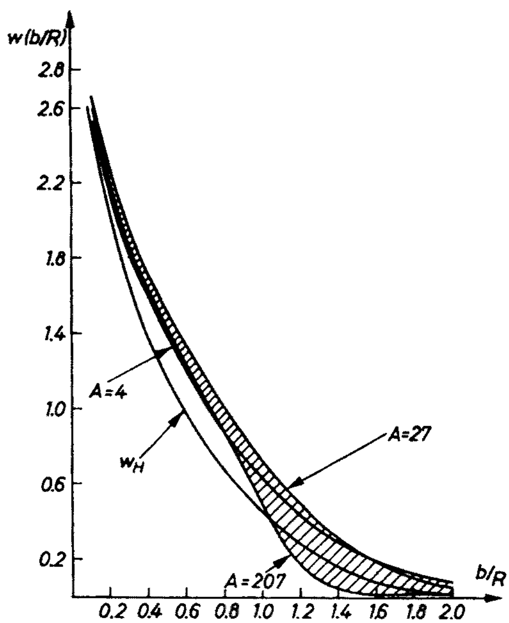

Fig. 1. Average multiplicity vs relative impact parameter. The curves for the nuclei from $A = 27$ till $A = 207$ lie in the shadowed area

The formula (13) was evaluated for several nuclei from $A = 4$ to $A = 238$. The results are shown in Fig. 1 where we have plotted w versus the variable

$$\beta = b/R \tag{14}$$

for $A = 4$, $A = 27$, and $A = 207$. It is seen that the curves coincide at small β but starting from $\beta = 1$ no clear systematic trend is observed.

In figure (2) we show w as function $\nu/\bar{\nu}$, where ν is the average number of collisions at given impact parameter

$$\nu = \frac{AD_A(b)\sigma}{\sigma_A(b)} \tag{15}$$

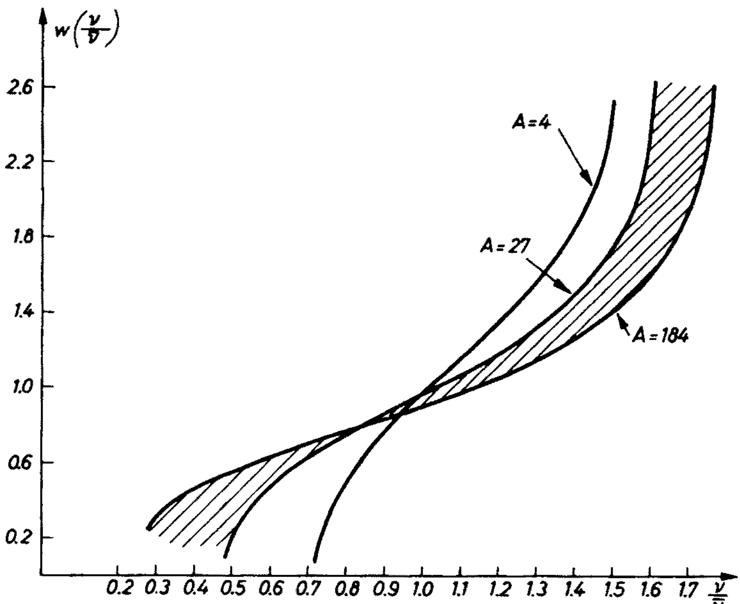

Fig. 2. Average multiplicity vs average number of collisions at fixed impact parameter. The curves for the nuclei from $A = 27$ till $A = 207$ lie in the shadowed area

and $\bar{\nu}$ is the average number of all collisions

$$\bar{\nu} = \frac{A\sigma}{\sigma_A}. \tag{16}$$

There one observes again noticeable differencies between different nuclei, particularly at the end of the curves.

Finally in figure 3 we show w versus the variable $\Omega/\Omega_{\max}$, where Ω is the eikonal defined by the formula

$$\sigma_A(b) = 1 - e^{-2\Omega(b)}. \tag{17}$$

It is seen that, plotted in this variable, w (Ω/Ω_{max}) has a systematic trend to decrease with increasing atomic number. Furthermore, starting from $A = 12$, there is no significant change with A. These observations seem to be in agreement with expectations from eikonal model [9, 10] which suggests that eikonal Ω is the most natural variable for

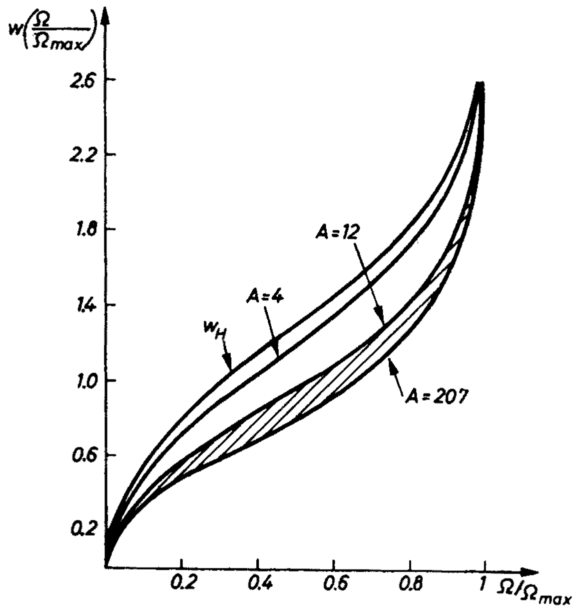

Fig. 3. Average multiplicity vs eikonal

description of all dynamical variables. However, similarly as for hydrogen, the dependence of w on Ω is not linear and therefore some simple conjectures [10, 11] are ruled out.

In conclusion, we calculated average multiplicity as function of impact parameter in hadron-nucleus collisions, using geometrical model. The obtained results show important differences with the situation found previously for hydrogen. Thus it seems that geometrical model does not provide a universal picture of hadron-nucleon and hadron-nucleus interaction.

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