LETTERS TO THE EDITOR

EFFECT OF LEVEL DENSITY DISTRIBUTION ON ENERGY AVERAGE OF THE SCATTERING FUNCTION

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The realistic level density distribution recently given by French and Wong is used to evaluate ths energy average of the scattering function and the results are compared with the usual expression which is obtained using Wigner's semi-circle distribution.

In the last couple of years a new kind of Hamiltonian ensemble has been introduced [1-3]. It differs from the usual random-matrix ensemble [4] in the assumption that it is the two-body part of the Hamiltonian which is random rather than each of the many-body Hamiltonian matrix elements. The justification for this assumption is the fact that it leads to level densities which are in agreement with the ones obtained using shell-model spectroscopy. The numerical calculations of French and Wong [1] and Bohigas and Flores [2] have established beyond doubt that the level density distribution is Gaussian in nature and not the semi-circular [4] which one obtains in the usual random matrix ensemble. Because of this French and Wong [1] had cautioned that considerable uncertainty would seem to attach to any results which rely on the assumption of statistical independence of many-body matrix elements.

An important quantity where one uses the eigenvalue distribution is the average of the low energy scattering matrix for the resonance reactions which pass through the formation of the compound nucleus. The purpose of the present note is to study the effect of the density distribution on the well-known relation between the transmission coefficient and the ratio of the average width to the average spacing.

We consider elastic scattering and following Feshbach, Kerman and Lemmer [5] write the energy average of the scattering function $S(E)$ as

$$\langle S(E_0)\rangle = \prod_{\mu} \frac{E_0+i\frac{I}{2}-Z_\mu^*}{E_0+i\frac{I}{2}-Z_\mu}, \tag{1}$$

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where $Z_\mu = \varepsilon_\mu - \frac{i}{2}\,\Gamma_\mu$ are the complex poles of the scattering matrix, I the width and E_0 the center of the Lorentzian energy resolution function. In writing expression (1) a constant phase ($\exp i\,\phi$) arisin~ ~ue to hard sphere scattering has been omitted. We now rewrite expression (1) using its integral representation [6] as

$$\langle S(E_0)\rangle = \exp\left[-\frac{i}{2}\int_0^1 d\lambda \sum_\mu \Gamma_\mu \left\{\frac{1}{(E_0-\varepsilon_\mu)+\frac{i}{2}(I-\lambda\Gamma_\mu)} + \frac{1}{(E_0-\varepsilon_\mu)+\frac{i}{2}(I+\lambda\Gamma_\mu)}\right\}\right]. \tag{2}$$

Making the usual approximations, $I \gg \Gamma_\mu$ and replacing the sums over μ by an integration expression (2) can be recast in the following form

$$\langle S(E_0)\rangle = \exp\left[(-i\langle\Gamma_\mu\rangle)\int \frac{\varrho(\varepsilon_\mu)}{-\varepsilon_\mu+\frac{i}{2}\,I}\,d\varepsilon_\mu\right], \tag{3}$$

where $\langle\Gamma_\mu\rangle$ is the ensemble average of Γ_μ and $\varrho(\varepsilon_\mu)$ is the density of the real parts of the complex poles Z_μ.

From expression (2), one gets the usual expression for $\langle S(E_0)\rangle$,

$$\langle S(E_0)\rangle = \exp\left(-\frac{\pi\langle\Gamma_\mu\rangle}{D}\right), \tag{4}$$

if one uses Wigner's semi-circle distribution for $\varrho(\varepsilon_\mu)$.

$$(\varrho\varepsilon_\mu) = \frac{\pi}{2ND^2}\left(\frac{4D^2N^2}{\pi^2} - \varepsilon_\mu^2\right)^{\frac{1}{2}}, \tag{5}$$

where N is the total number of levels and D their average spacing, the quantity $ND = \Delta E$ is assumed to be much larger than I. The center of the energy resolution function $E_0 = \langle\varepsilon_\mu\rangle$ is taken to be zero.

Now we would like to see what will be the consequences if $\varrho(\varepsilon_\mu)$ is not a semi-circular distribution but is Gaussian in nature as shown by French and Wong.

Let us write this distribution as

$$\varrho(\varepsilon_\mu) = N(2\pi\sigma^2)^{-\frac{1}{2}}\exp\left(-\frac{\varepsilon_\mu^2}{2\sigma^2}\right) \tag{6}$$

where σ is the dispersion and is a function of N, D. Using this ϱ in expression (3) and carrying out the integration over ε_μ we get

$$\langle S(E_0)\rangle = \exp\left\{-N\langle\Gamma_\mu\rangle\left(\frac{\pi}{2\sigma^2}\right)^{\frac{1}{2}}\left[\exp\left(\frac{I^2}{8\sigma^2}\right)\right]\mathrm{erfc}\left(\frac{I}{2\sqrt{2}\,\sigma}\right)\right\}, \tag{7}$$

where erfc is the complimentary error function [7]. As in Wigner's distribution given by expression (5), we assume the dispersion to be much larger than the width I of the Lorentzian energy resolution function. Expression (7) can then be rewritten as

$$\langle S(E_0)\rangle = \exp\left[-N\langle\Gamma_\mu\rangle\left(\frac{\pi}{2\sigma^2}\right)^{\frac{1}{2}}\right]. \tag{8}$$

Therefore, in place of the usual expression (4) for the energy average of the scattering function $S(E)$, one gets the above expression if the distribution is Gaussian in nature.

Now an important property of Wigner's distribution is that the average spacing D is related to $\varrho(\varepsilon_\mu)$ in the following way,

$$\frac{1}{D} \underset{\delta E\to 0}{=} \frac{1}{\delta E}\int\limits_{-\frac{1}{2}\delta E}^{\frac{1}{2}\delta E} \varrho(\varepsilon_\mu)d\varepsilon_\mu. \tag{9}$$

If we impose this condition on French-Wong's distribution given by expression (6), then the dispersion σ is given by

$$\sigma = \frac{ND}{\sqrt{2\pi}}. \tag{10}$$

Putting this value of σ in expression (7) we get

$$\langle S(E_0)\rangle = \exp\left(-\frac{\pi\langle\Gamma_\mu\rangle}{D}\right).$$

Thus we arrive at the important result that if the Gaussian distribution satisfies the condition given by expression (9) then the expression for the average of the scattering function is exactly the same as the one obtained using Wigner's distribution, otherwise the energy average of $S(E)$ will be different than the usual expression.

We would next like to ask if the expression for $\langle S(E_0)\rangle$ would remain unchanged had $\varrho(\varepsilon_\mu)$ been diffrent than the Gaussian distribution but still satisfying the condition given by expression (9). To answer this we consider the following Lorentzian distribution for $\varrho(\varepsilon_\mu)$,

$$\varrho(\varepsilon_\mu) = \frac{Na}{2\pi}\,\frac{1}{\varepsilon_\mu^2 + \frac{a^2}{4}}, \tag{11}$$

where $\frac{1}{2}a$ is the half-width of the Lorentzian. Using the theory of residues the integral in expression (3) can be easily evaluated and we find that $\langle S(E_0)\rangle$ for a Lorentzian distribution is given by

$$\langle S(E_0)\rangle = \exp\left(-\frac{2N\langle \Gamma_\mu\rangle}{a+I}\right). \tag{12}$$

Using the condition expressed by expression (9), we find that

$$a = \frac{4ND}{\pi}. \tag{13}$$

Thus for a Lorentzian distribution satisfying the condition given by expression (9),

$$\langle S(E_0)\rangle = \exp\left(-\frac{\pi\langle \Gamma_\mu\rangle}{2D}\right), \tag{14}$$

taking $ND \gg I$.

Comparing expressions (14) and (4) we find that $\langle S(E_0)\rangle$ is not the same as the one given by Wigner's distribution.

We would further like to remark that from the point of view of edge effects in the theory of average cross sections the choice of the center of the energy interval ΔE in the condition expressed by expression (9) seems to be most suitable.

Lastly we remark that even though we have considered elastic scattering only, it is obvious that if reaction channels are also open, the effect of level density which arises due to summation over resonances remain the same, i. e. all one has to do is to replace the factor π/D by the new factor involving π, σ and N.

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