KINEMATIC POLARIZATION OF LARGE-MASS FINAL STATES IN DIFFRACTION DISSOCIATION

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We point out that the mass-dependent polarization effects observed in diffraction dissociation can be attributed largely to two simple causes: (1) the increasing importance of higher partial waves with increasing final mass; (2) a kinematic suppression of helicity flip for dissociation into large-mass states. These lead respectively to (a) strong violation of s-channel helicity conservation and (b) a much weaker violation of t-channel helicity conservation for large diffractively-produced mass. This is observed experimentally. The kinematic suppression of helicity flip for large mass is a new result which we prove in this article. Recent data is discussed, and the implications of our result for the triple-pomeron vertex are mentioned.

One of the long-standing problems in diffraction dissociation is to understand the strong polarization effects which characterize this type of process. The observed polarization depends on the mass M of the diffractively-produced system. To describe the experimental results it is conventional to use the terminology s-channel/t-channel helicity conservation (SCHC/TCHC, respectively), not because these expressions have any deep theoretical meaning, but because they correspond to the two extreme cases which encompass all possible polarizations. For M near the real mass threshold M_{th} both SCHC and TCHC are strongly "broken" (in the sense they are very poor approximations to the data). As M increases TCHC improves noticeably while SCHC does not. One observes this in certain azimuthal-angle distributions which must be flat when helicity is conserved along the relevant axis, and which flatten with increasing M in the t-channel case but not in the s-channel case. TCHC seems to become a fairly good approximation to the data when $M-M_{th}$ is large enough. On the other hand, SCHC may improve with increasing mass, but only very slowly, and in the mass region presently explored SCHC remains strongly broken.

In this article we point out that mass-dependent polarization effects of exactly this sort are to be expected on the basis of kinematics alone. Indeed, it would be remarkable if such effects were *not* observed in the high-mass region. This does not mean that polariza-

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tion in diffraction dissociation can be attributed entirely to kinematics. It cannot. The observed polarization is due partly to kinematics and partly to dynamics. In the low-mass region dynamics plays the leading role, while kinematics largely determines things for higher masses.

Qualitatively one can easily understand why TCHC is always a better approximation than SCHC for diffractive dissociation into states with *large angular momentum*. In such cases it cannot be otherwise, because of the pronounced asymmetry between the s- and t-channel helicity amplitudes. The t-channel amplitudes can be approximated by pomeron exchange which limits t-channel helicity flip to one unit. But *all* values of s-channel helicity flip are allowed (this follows from crossing). For example, exact TCHC implies that *every* s-channel helicity amplitude is important (except in the forward direction). Generalizing slightly to allow one unit of t-channel helicity flip cannot affect this situation much. In particular, there is no way to arrange that all the s-channel amplitudes with large helicity flip become unimportant. Therefore when systems with large angular momentum are diffractively produced, one can be sure that TCHC will be better than SCHC.

This argument implies that SCHC should remain strongly broken even when the final mass is well above threshold. For, it is a known experimental fact that the average angular momentum $\langle J \rangle$ of the final system increases with increasing mass M. To get an idea how rapid this increase is, we quote some data from Bosetti et al. [1] for the reaction $\pi N \rightarrow (3\pi)N$;

$$\langle J \rangle = 0.55 + 1.1(M - M_{\text{eff}}),$$

where M_{eff} is the effective mass threshold. Presumably this rather rapid increase of $\langle J \rangle$ with M is characteristic of diffraction dissociation generally. If so, then because of the steadily-growing number of s-channel helicity-flip amplitudes it seems unlikely that an increase in M (by a limited amount) will lead to an improvement in SCHC. Therefore we have a simple qualitative explanation for one of the two major polarization effects in diffraction dissociation.

This does not tell us why TCHC becomes a better approximation with increasing masses. The following theorem answers this question, at least at the qualitative level.

THEOREM: For diffraction dissociation into a state with very large mass M and definite angular momentum the helicity of this state is preferentially the same as the helicity of the initial state because there is a strong kinematic suppression of helicity flip. Roughly speaking, each unit of helicity flip costs a factor $\sqrt{\bar{t}}/M \ll 1$.

This theorem is easy to prove within the context of two-body scattering processes. To apply it to diffraction dissociation we observe that the dissociation amplitude can be decomposed into a partial-wave series with respect to the final diffractively-produced system. The theorem holds for each partial wave separately. In Appendix A we prove the theorem above for the reaction $\pi + \pi \rightarrow \pi^* + \pi$ where the π^* spin is arbitrary. In Appendix B the proof is given for $N + \pi \rightarrow N^* + \pi$ with arbitrary N^* spin. Because of factorization, these results apply directly to more complicated reactions such as $\pi + N \rightarrow \pi^* + N$, $N + N \rightarrow N^* + N$ and $N + N \rightarrow N^* + N^*$.

We cannot conclude rigorously that helicity flip is forbidden for large mass, because constraints among invariant amplitudes *could* exist such that the kinematic suppression of helicity flip is overcome and some other polarization results. The theorem tells us that, barring this very special dynamical circumstance, helicity flip is suppressed. The existence of such constraints is a purely-dynamical question which can only be decided by experiment. However, for large diffractively produced mass, where many partial waves are excited, it seems rather unlikely that the kinematic suppression of helicity flip is overcome for each partial wave separately. More likely, the invariant amplitudes are independent or only weakly dependent for each partial wave, in which case the kinematic suppression takes effect.

In the statement of our theorem no mention is made of a particular channel because at the diffractive vertex there is practically no difference between the s- and t-channel helicity labels when the final mass is extremely large. To make this explicit we shall have to introduce some notation. Diffraction dissociation amplitudes will be decomposed into two-body partial-wave amplitudes; thus, symbolically,

$$T(\pi N \rightarrow (3\pi)N) = \sum T_J(\pi N \rightarrow \pi^* N) D_J(\pi^* \rightarrow 3\pi),$$

where π^* is the three-pion partial-wave state with angular momentum J. The partial-wave amplitudes $T(\pi N \rightarrow \pi^* N)$ control helicity flip at the $\pi \rightarrow \pi^*$ vertex. These partial-wave amplitudes are two-body amplitudes which we call $f^{(s)}_{cdab}$. The direct or s-channel is $ab \rightarrow cd$ with $s = (p_c+p_a)^2$, $t = (p_c-p_a)^2$. The t-channel is $Db \rightarrow cA$ ($D = \bar{d}$, $A = \bar{a}$). The dissociation vertex is $a \rightarrow c$ and the transition $b \rightarrow d$ is elastic.

The kinematic region we are going to consider is

$$s \gg m_c^2 \gg m_a^2,\, m_b^2,\, m_d^2,\, |t|, \tag{1}$$

where m_c is the diffractively produced mass which is taken to be large compared with $|t|^{1/2}$ and the other three masses. In this region the s-t crossing equations are diagonal in the helicities c and a,

$$f^{(s)}_{cdab} = \sum d_{D'd}(\chi_d) d_{b'b}(\chi_b) f^{(t)}_{caD'b'} \tag{2}$$

because the crossing angles [2] χ_c, χ_a are both zero. The other crossing angles χ_d, χ_b do not concern us here.

In the mass region (1) there is no distinction between SCHC and TCHC. Clearly the present data is far from this asymptotic region, and one may wonder how the theorem above applies to this data. The answer is simple. Already for rather small masses (say $m_c \sim 2$ GeV and even smaller) the kinematic suppression of helicity flip begins to influence the helicity amplitudes in both channels. In the t-channel, where there are only nonflip and single-flip amplitudes present, even a weak suppression of helicity flip can have a noticeable effect. But for the s-channel amplitudes this will certainly not be the case when the angular momentum J is large and many helicity-flip amplitudes are present. For these partial waves m_c will have to be quite large before the kinematic effect is strong enough to enable the s-channel nonflip amplitude to dominate all the other ones.

Admittedly this is only a qualitative argument. Nevertheless, it is striking that the predicted behaviour is exactly what is observed in diffraction dissociation. There is little doubt that the kinematic suppression of helicity flip implied by the above theorem exists. The question is, what is its numerical effect in the mass region presently explored? We suggest that this numerical effect is large for the partial waves with $J \geqslant 2$ for pion and kaon dissociation, and $J \geqslant 5/2$ for nucleon dissociation. For these partial waves there is a substantial asymmetry between the s- and t-channel helicity amplitudes and SCHC should be definitely worse than TCHC. Furthermore, these partial waves are not dominant in the lowest mass region; rather, they become important for somewhat larger mass where the kinematic suppression of helicity flip is becoming stronger. The choice of angular momentum $J = 2$ ($J = 5/2$) for bosons (fermions) as the division between dynamical and kinematical polarization is somewhat arbitrary, of course. In the lowest partial waves there is no kinematic preference for SCHC or TCHC and any polarization is dynamical. In the higher partial waves we expect TCHC to hold for kinematic reasons. For some J there must be a change-over. The data suggest that the angular momentum values we have chosen are appropriate ones (see below).

If our conjecture is correct then polarization in diffraction dissociation arises from two main sources in the large-M region:

(*i*) the kinematic suppression of helicity flip in each partial wave, which strengthens with increasing M and leads eventually to TCHC;

(*ii*) the fact that the higher partial waves become more important with increasing M, which sustains the SCHC breaking in spite of the spin-flip suppression.

There is no way to rule out dynamics as a third cause of polarization effects until diffraction dissociation is understood dynamically, at the quantitative level. It may well be that dynamics continues to play some role even in the large-mass region. What we are suggesting is that this role is only a minor one.

In the resonance region the proceeding argument makes it clear that the observed polarization is mainly dynamical. This is the case for diffractive production of the 1^+ mesons and the N* (1520), for example. An exception to this statement is the N*(1690); TCHC seems to characterize the production of this $J = 5/2$ resonance. The large spin and mass of the N* (1690) make it likely that kinematic effects are partly responsible for the observed polarization. Two other exceptions are the $J = 2$ enhancements A_3 and L in pion and kaon dissociation, for which TCHC is also observed.

In the remainder of this article we should like to discuss the data on pion, kaon and nucleon dissociation in more detail than we have done so far. Following this, the implications of our theorem for the triple pomeron vertex are mentioned.

(1) $N\pi \to (\pi N)\pi, \quad NN \to (\pi N)N$

Let us first consider the data on nucleon dissociation into a πN system. For this reaction the effective mass threshold M_{eff} and the real mass threshold $M_{th} = m_\pi + m_N$ are the same. Near threshold both SCHC and TCHC are strongly broken [3-5]. For increasing mass $M_{(\pi N)}$ the strength of the TCHC violation decreases noticeably while SCHC violation remains strong [3–5]. An early experiment [6] concluded that TCHC becomes a rather good approximation in the region $1.6 < M_{(\pi N)} < 1.8$ GeV. But more recent data [5] indicates that TCHC violation persists in this region and extends to even higher masses. The mass region $1.6 < M_{(\pi N)} < 1.8$ GeV is dominated by the $J = 5/2 N^*$ (1690) partial wave. Evidently TCHC is only a rough approximation in N^* (1690) production, at least in the $N \to (\pi N)$ channel. The choice $J = 5/2$ for the separation of kinematic and dynamical polarization seems to be reasonable in this case. For $m_c \approx 1.7$ GeV the kinematic suppression of helicity flip is substantial but not overwhelming, and some TCHC breaking is to be expected. From Appendix B we find that one unit of helicity flip (in either channel) costs a factor

$$\sin \chi_c \approx (2\sqrt{|t|}/m_c)\left[(1-m_a^2/m_c^2)^2 + \frac{2|t|}{m_c^2}\left(1+\frac{m_a^2}{m_c^2}\right) + \frac{|t|^2}{m_c^4}\right]^{-1/2} \tag{3}$$

where $m_a = m_N$. For $m_c \approx 1.7$ GeV, $\sin \chi_c$ is approximately

$$\sin \chi_c \approx (1.2)\,|t|^{1/2}[0.5+(0.87)\,|t|+(0.12)\,|t|^2]^{-1/2},$$

and when $|t|$ is a few tenths of a GeV^2 then $\sin \chi_c$ is not very small. Therefore, if TCHC were almost exact for N* (1690) production then a dynamical effect would clearly be implied.

(2) $N\pi \to (\pi\pi N)\pi$, $NN \to (\pi\pi N)N$

The data on nucleon dissociation into $\pi\pi N$ is quite different. First of all, the effective mass threshold $M_{eff} \approx 1.4$ GeV is substantially larger [7] than the real mass threshold $M_{th} = 2m_\pi + m_N$. The reason for this is well-known [8]; two-body intermediate states (πN^* in this case, with $N^* \to \pi N$) dominate diffraction dissociation into three-body systems. This pushes the effective mass threshold up because the lowest N* state, the P_{11} (1470), has a mass nearly 0.4 GeV larger than $m_\pi + m_N$. $M_{(\pi\pi N)}$ is therefore generally several hundred MeV larger than M_{th}. Having seen the general trend in the $N \to (\pi N)$ data towards TCHC with increasing mass, it is not surprising that a similar trend is observed in the $N \to (\pi\pi N)$ data. (See Refs [9, 10] for πN data and Ref. [11] for NN data). However, the region near M_{th} is missing from the $(\pi\pi N)$ mass spectrum, and correspondingly no strong violation of TCHC is observed, although there is some indication that TCHC may improve with increasing mass in $N \to (\pi\pi N)$ [9]. The $J = 5/2$ partial wave corresponding to the N* (1690) is very important, and for this partial wave TCHC seems to be a fair approximation. The same is true for higher mass [9], and therefore presumably, for the higher partial waves. The onset of TCHC seems to occur in the N*(1690) mass region [9], and presumably this occurs in the $J = 5/2$ partial wave. It seems reasonable to conclude that $J = 5/2$ is the proper choice for the J-value separating (mainly) kinematic polarization from (mainly) dynamical polarization. One unit of helicity flip costs a factor $\sin \chi_c$ given by Eq. (3). If the polarization of the N*(1690) partial wave is mainly kinematic then some violation of TCHC is expected. However, this should by relatively weak, as the mass $m_c \approx 1.7$ GeV is large enough to induce a suppression of helicity flip.

(3) p+p → p+(charged particle+anything)

Next we mention an ISR experiment on diffraction dissociation into systems with very large invariant mass, in which polarization was studied in the following averaged fashion [12]. Elastically scattered protons were detected in coincidence with other charged particles in the inclusive reaction pp → p+H, H → charged particle+anything. The polarization of H was probed (not measured) by transforming the data to the rest frame of the final system H and comparing it with predictions obtained from the hypotheses of SCHC and TCHC for the production of H. The data was seen to be *consistent* with TCHC and *inconsistent* with SCHC. This is, of course, the sort of polarization we have already encountered in nucleon dissociation in the low-mass region. But here masses of 10 GeV and larger are involved! For such large final mass one might think that the kinematic region (1) has been reached, so that the crossing equations are diagonal in the relevant helicities (see Eq. (2)) and there is no difference between SCHC and TCHC. Obviously this is not the case. There is still a difference. The reason for this difference can only be that states with extremely large angular momentum are involved, and the asymmetry between the s- and t-channel amplitudes is so great that the kinematic spin-flip suppression (which is very strong in this mass range) cannot overcome it. (The only alternative is that dynamical constraints among invariant amplitudes are operative in many partial waves, and this possibility we reject.) If this is true then there ought to be some indication that the difference between SCHC and TCHC decreases with increasing mass. (Otherwise one would have to doubt that SCHC and TCHC ever coincide, no matter how large the mass is.) Fortunately, the data in Ref. [12] clearly show such a trend through the mass range $m_c^2 = 42-228\ \mathrm{GeV}^2$. Even for the largest masses there is a real difference, however. This indicates that the kinematic region (1) is an asymptotic region in the true sense of the word. Even for diffractively produced masses of 5 to 10 GeV the kinematic suppression of helicity flip is not overwhelmingly strong, the crossing equations are not completely diagonal, and TCHC is better than SCHC.

(4) πN → (3π)N, KN → ($\pi\pi$K)N

All experiments on the diffractive dissociation of pions and kaons agree that TCHC is not strongly broken in the diffractive transitions $\pi \to (3\pi)$, $\pi \to (\bar{\mathrm{K}}\mathrm{K}\pi)$ and $\mathrm{K} \to (\pi\pi\mathrm{K})$. (An exception: SCHC is observed in a specific $\mathrm{K} \to (\pi\pi\mathrm{K})$ partial wave which we discuss later.) In pion and kaon dissociation the effective mass thresholds M_{eff} are well above the real mass thresholds M_{th}. For $\pi \to (3\pi)$, $\pi \to (\bar{\mathrm{K}}\mathrm{K}\pi)$ and $\mathrm{K} \to (\pi\pi\mathrm{K})$ these thresholds are roughly 1.0 GeV, 1.5 GeV and 1.2 GeV (see Refs [1, 13]). These rather large values of M_{eff} clearly indicate that two-body intermediate states dominate the production mechanism. (This is especially clear in $\pi \to (3\pi)$ where the intermediate state with lowest mass is $\pi \to \pi\varrho \to 3\pi$.) The situation here is rather like the $\mathrm{N} \to (\pi\pi\mathrm{N})$ case. Therefore it may not be surprising that TCHC holds to a good approximation in pion and kaon dissociation [1, 13] since the region near M_{th} where TCHC breaking should be maximum is unpopulated. The situation is nevertheless quite complicated, with very prominant dynamical polarization effects in the mass region near M_{eff}. In the dominant partial waves corresponding to $\pi \to A_1$ and $K \to Q$ there is no asymmetry between the s- and t-channel helicity amplitudes, and therefore any observed polarization is dynamical. The fact that TCHC holds to within ten percent in $\pi \to A_1$ [13] can only have a dynamical explanation. It is interesting to note that for $\pi \to (3\pi)$ the kinematic suppression of helicity flip is rather strong even in the A_1 mass region. The crossing angle χ_c which determines the invariant amplitude coefficients (see Appendix A) and therefore the spin-flip suppression is given by

$$\cos \chi_c \approx \frac{1-|t|/m_c^2}{1+|t|/m_c^2} \tag{4}$$

because the pion mass $m_a = m_\pi$ can be neglected. Each unit of helicity flip costs a factor $\sin \chi_c$ (see Appendix A) and for $m_c > 1$ GeV and limited $|t|$ this is a small factor. Therefore, in the higher partial waves, TCHC should be rather a good approximation. These partial waves with $L \geqslant 2$ should be strongly influenced by kinematics. This applies to the $\pi \to A_3$ partial wave, for example.

In the case of Q production the situation in the low-mass region is very complicated indeed. Recent data [14, 15] have shown that two resonances Q_1, Q_2 with different properties are produced in the reaction $K+N \to Q_{1,2}+N$. Q_1 and Q_2 are $J^P = 1^+$ strange mesons with masses 1300 MeV and 1400 MeV respectively. Their primary decay modes are $Q_1 \to \varrho K$ and $Q_2 \to \pi K^*$. Q_1 production conserves s-channel helicity while Q_2 production conserves t-channel helicity. In this sense Q_2 production is typical while Q_1 production in atypical. The observed polarization in Q_1 and Q_2 production has a dynamical origin. Even in this mass region the kinematic suppression of helicity flip is rather strong, however. For larger masses where partial waves with $L \geqslant 2$ are dominant, the kinematic polarization should be quite strong. This applies to the $K \to L$ partial wave in particular.

(5) Triple-pomeron vertex

The behaviour of the dissociation vertex for large final mass determines the nature of the triple pomeron vertex. In general, as pointed our by Sakai and Uschersohn [16], this vertex depends on five variables. These can be chosen to be the masses squared t_1, t_2, t_3 of the three pomerons and two angular variables ϕ_1, ϕ_2 which describe the polarization of the undetected final system in $a+b \to (\text{any})+d$. After a moderately involved analysis of the six-point function which determines the inclusive cross section, Sakai and Uschersohn were able to show that TCHC implies the triple-pomeron vertex is independent of ϕ_1 and ϕ_2. The converse is also true; if the triple-pomeron vertex depends only on $t_{1,2,3}$ then TCHC holds in inclusive diffraction dissociation. Now we have argued in the present article that TCHC will always characterize diffraction dissociation into large invariant mass for simple kinematic reasons. If this is true then the triple-pomeron vertex is a function only of the pomeron masses, for essentially kinematic reasons.

We should like to present a much simpler derivation of the result found by Sakai and Uschersohn. First we notice that the triple-pomeron vertex can be regarded as a function of t_1, t_2, t_3 and two of the pomeron helicities (as for any three-particle vertex, one of the helicities is not independent). Second we notice that TCHC means that the pomeron has zero helicity in the Gottfried–Jackson frame. To be specific, let us consider the reaction $ab \to cd$ for which the Gottfried–Jackson frame is defined by

$$\vec{p}_c = 0, \quad \hat{n}_c = \hat{p}_a, \quad \vec{k} = \vec{p}_a - \vec{p}_c = \vec{p}_a,$$

where $\vec{k}$ is the pomeron three momentum and $\hat{n}_c$ is the spin quantization axis for the diffractively-produced state c. For simplicity we give the particles at the elastic vertex $b \to d$ spin zero. Then the scattering amplitude in the Gottfried–Jackson frame coincides with the continued t-channel helicity amplitude [17],

$$f_{c0a0}(\mathrm{G}J) = f^{(t)}_{ca00}.$$

TCHC means that $c = a$, and by angular momentum conservation ths pomeron must have zero helicity in this frame. This means that the two "external" pomerons at the triple pomeron vertex have helicity zero, and the vertex therefore depends only on the three pomeron mass variables.

APPENDIX A

Pion dissociation

Consider the pion dissociation process $\pi\pi \to \pi^*\pi$ with spins $0+0 \to L+0$, where π^* is a multiparticle system with angular momentum L and the quantum numbers of the pion. Typically $\pi^* = 3\pi$, $\overline{\mathrm{K}}\mathrm{K}\pi$, 5π etc. The formulas for the s- and t-channel helicity amplitudes describing $\pi\pi \to \pi^*\pi$ (with π^* helicity $c \geqslant 0$) are

$$(m_c\sqrt{2})^L G^L_c f^{(s)}_c$$
$$= \sum_{N=0}^{L-c} (-)^{L-N}(L-N)![(L-N+c)!(L-N-c)!]^{-1/2} S^N_{cd} T^{L-N}_{ca} d^{L-N}_{0c}(\chi_c) A_{1+N}, \tag{A1}$$

$$(m_c\sqrt{2})^L G^L_c f^{(t)}_c$$
$$= \sum_{N=c}^{L} (-)^N N![(N+c)!(N-c)!]^{-1/2} S^N_{cd} T^{L-N}_{ca} d^N_{0c}(\chi_c) A_{1+N}, \tag{A2}$$

where the following notation is used:

$$G^L_c = [(2L)!/(L+c)!(L-c)!]^{1/2},$$

$$S^2_{cd} = s^2 - 2s(m^2_c + m^2_d) + (m^2_c - m^2_d)^2,$$

$$T^2_{ca} = t^2 - 2t(m^2_c + m^2_a) + (m^2_c - m^2_a)^2,$$

$$S_{cd}T_{ca}\cos\chi_c = (s + m^2_c - m^2_d)(t + m^2_c - m^2_a) - 2m^2_c(m^2_c - m^2_d - m^2_a + m^2_b),$$

$$S_{cd}T_{ca}\sin\chi_c = 2m_c\sqrt{\phi}\,.$$

ϕ is the usual physical boundary function. The amplitudes A_n are invariant amplitudes defined by the M-function

$$M_{\mu_1 \cdots \mu_L} = (p_a \cdots p_a)_{\mu_1 \cdots \mu_L} A_1 + p_{d\mu_1}(p_a \cdots p_a)_{\mu_2 \cdots \mu_L} A_2 + \cdots + (p_d \cdots p_d)_{\mu_1 \cdots \mu_L} A_{L+1}.$$

The calculations of the invariant amplitude coefficients in Eqs (A1) and (A2) is described in Ref. [18].

Pomeron exchange limits the t-channel helicity flip to zero or one. This means that the helicity amplitudes $f_c^{(t)}$ with $c \geqslant 2$ are unimprortant. We can arrange this by setting all of the invariant amplitudes $A_3, A_4 \ldots A_{L+1}$ equal to zero (note that both of Eqs (A1) and (A2) are triangular). The remaining two t-channel amplitudes are

$$(m_c \sqrt{2})^L G_0^L f_0^{(t)} = T_{ca}^L A_1 - S_{cd} T_{ca}^{L-1} \cos \chi_c A_2, \tag{A3}$$

$$(m_c \sqrt{2})^L G_1^L f_1^{(t)} = -\tfrac{1}{2} S_{cd} T_{ca}^{L-1} \sin \chi_c A_2. \tag{A4}$$

All of the s-channel helicity amplitudes are still nonzero,

$$(m_c \sqrt{2})^L G_c^L f_c^{(s)} = (-)^L L![(L+c)!(L-c)!]^{-1/2} T_{ca}^L d_{0c}^L(\chi_c) A_1$$
$$-(-)^L (L-1)![(L+c-1)!(L-c-1)!]^{-1/2} S_{cd} T_{ca}^{L-1} d_{0c}^{L-1}(\chi_c) A_2, \tag{A5}$$

and away from the forward direction (where $\chi_c = 0$) all of them are important.

In the kinematic region $s \gg m_c^2 \gg |t|,\ m_a^2.\ m_b^2, m_d^2$ things are different. One easily verifies that both Eqs (A1) and (A2) reduce to

$$(\sqrt{2}/m_c)^L G_c^L f_c \approx \frac{1}{c!} \left(\frac{\sqrt{|t|}}{m_c} \right)^c \sum_{N=c}^{L} (-)^N \frac{N!}{(L-N)!} \left(\frac{s}{m_c^2} \right)^N A_{1+N}. \tag{A6}$$

Here we have used the approximation $\phi \approx s^2\ |t|$. As promised, the crossing equations have become diagonal.

Eq. (A6) proves the theorem in the text for diffractive $\pi \to \pi^*$ and $\mathrm{K} \to \mathrm{K}^*$ transitions. In the kinematic region (1) every unit of spin flip costs a factor $\sqrt{t}/m_c$. Furthermore, if pomeron exchange is still dominant then only the amplitudes f_0, f_1 can be important, and the spin-flip amplitude f_1 is smaller than the spin-nonflip amplitude f_0 by a factor $\sqrt{t}/m_c \ll 1$. Helicity is conserved unless the invariant amplitudes A_1, A_2 satisfy the condition

$$A_1 - L(s/m_c^2) A_2 \approx 0. \tag{A7}$$

This is an example of the kind of dynamical constraint which can overcome the kinematic suppression of helicity flip.

The discussion above applies to π^* with unnatural parity. Diffraction dissociation into π^* with natural parity is also possible. However, for such partial waves the nonflip amplitudes in either channel are zero if factorization holds (because of parity invariance) and so helicity conservation is impossible. For the natural-parity waves one can show that the same kinematic suppression of helicity flip holds, and the minimum flip of helicity by one unit is therefore preferred.

APPENDIX B

Nucleon dissociation

Consider the process $N\pi \to N^*\pi$ with spins $1/2+0 \to J+0$; where N^* is a multiparticle system with angular momentum J and the quantum numbers of the nucleon. Typically $N^* = \pi N$, $\pi\pi N$, etc.... The t-chanel helicity amplitudes for $N\pi \to N^*\pi$ are (for N^* helicity $c \geqslant 1/2$)

$$(m_c\sqrt{2})^{J-1/2}G_c^J\,T(-)\,(-)^{1/2-A}f_{cA}^{(t)}$$

$$= \sum_{N=c-1/2}^{J-1/2} N![(2N)!]^{-1/2}(-S_{cd})^N(T_{ca})^{J-1/2-N} \sum_{\sigma=\pm 1/2} G_{c-\sigma}^N d_{0,c-\sigma}^N(\chi_c)$$

$$\times\{\delta_{\sigma A}[T(-)^2A_{1+2N}+Q(-)A_{2+2N}]+(-)^{1/2-A}\delta_{-\sigma A}\sqrt{\phi}\,A_{2+2N}\}, \tag{B1}$$

where

$$T(-)^2 = t-(m_c+m_a)^2,$$

$$Q(-) = -m_c(s+m_a^2-m_b^2)-m_a(s+m_c^2-m_d^2).$$

All other notations are as in Eqs (A1) and (A2). A corresponding formula can be given for the s-channel helicity amplitudes, but it is too lengthy to present it here. The invariant amplitudes A_n are defined by the M-function

$$M_{\mu_1\cdots\mu_{J-1/2}} = (p_a\cdots p_a)_{\mu_1\cdots\mu_{J-1/2}}[A_1+\gamma\cdot QA_2]$$

$$+p_{d\mu_1}(p_a\cdots p_a)_{\mu_2\cdots\mu_{J-1/2}}[A_3+\gamma\cdot QA_4]+\ \cdots$$

$$+(p_d\cdots p_d)_{\mu_1\cdots\mu_{J-1/2}}[A_{2J}+\gamma\cdot QA_{2J+1}],$$

where $Q = p_a+p_b$ in the s-channel and $Q = -p_A+p_b$ in the t-channel.

For pomeron exchange only the amplitudes $f_{1/2,1/2}^{(t)}$, $f_{1/2,-1/2}^{(t)}$ and $f_{3/2,1/2}^{(t)}$ are important because t-channel helicity cannot flip by more than one unit. This means that only the three invariant amplitudes A_1, A_2 and A_3 are important. These contribute to all the s-channel helicity amplitudes, so that helicity flip in the s-channel can be arbitrarily large. Therefore, when J is large many s-channel amplitudes are nonzero in comparison with the three in the t-channel.

In the kinematic limit $s \gg m_c^2 \gg |t|$, m_a^2, m_b^2, m_d^2 Eq. (B1) simplifies to

$$(\sqrt{2}/m_c)^{J-1/2}G_c^J m_c(-)^{1/2-A}f_{cA}$$

$$\approx \sum_{\sigma=\pm 1/2}\left(\frac{\sqrt{|t|}}{m_c}\right)^{c-\sigma}\frac{1}{(c-\sigma)!}\sum_{N=c-1/2}^{J-1/2}(-)^N\frac{N!}{(N-c+\sigma)!}\left(\frac{s}{m_c^2}\right)^N$$

$$\times\{\delta_{\sigma A}[-m_c^2A_{1+2N}-m_csA_{2+2N}]+(-)^{1/2-A}\delta_{-\sigma A}s\sqrt{|t|}\,A_{2+2N}\}. \tag{B2}$$

Again we have used the approximation $\phi \approx s^2|t|$. The crossing equations are diagonal and so Eq. (B2) also gives the s-channel helicity amplitudes.

From Eq. (B2) it is clear that N* helicity $c > 1/2$ is suppressed relative to $c = 1/2$ by the factor $(\sqrt{t}/m_c)^{c-1/2}$ Barring constraints among invariant amplitudes, this means that only the helicity amplitudes with $c = \frac{1}{2}$ can be important. Keeping only, A_1, A_2 A_3 we find

$$(\sqrt{2}/m_c)^{J-1/2}G^J_{1/2}f_{1/2\,1/2} \approx -m_cA_1 - sA_2 + (s/m_c)A_3, \tag{B3}$$

$$(\sqrt{2}/m_c)^{J-1/2}G^J_{1/2}f_{1/2,-1/2} \approx \frac{\sqrt{|t|}}{m_c}[-sA_2 + (s/m_c)A_3]. \tag{B4}$$

The nonflip amplitude is larger than the flip amplitude by a factor $\sqrt{t}/m_c$.

The preceding discussion applies to N* with natural parity. Diffraction dissociation into N* with unnatural parity is also possible, of course. Only minor changes in Eqs (B1)—(B4) are needed to convert these formulas into the corresponding ones for unnatural parity N*. Nothing changes in the argument leading to the suppression of helicity flip.

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