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ON THE RELATIVE IMPORTANCE OF THE ALPHA-CLUSTER EXCHANGE AND ONE NUCLEON EXCHANGE IN THE ANOMALOUS LARGE ANGLE ALPHA SCATTERING*

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The possibility of interpretation of anomalies in backward-angle alpha scattering is investigated in terms of the alpha-cluster exchange and one nucleon exchange. The alpha-exchange model alone is insufficient to explain the ALAS effect. Both the alpha-exchange and one nucleon exchange reproduce better the anomalies. Some information on alpha-clusters in calcium nuclei is obtained.

1. *Introduction*

The characteristic feature of scattering of the alpha particles on some nuclei is the substantial growth of the differential cross-section at angles greater than 90 deg [1–8]. This so called Anomalous Large Angle Scattering (ALAS) has been observed for a wide range of nuclei. According to Eberhard's criteria [9] the strongest ALAS appears in the alpha-^{40}Ca scattering. The ALAS shows distinct isotopic and energy dependence [4, 8, 10], especially for calcium. The strongest effect is observed for ^{40}Ca, the significant ALAS for ^{42}Ca, no ALAS for ^{44}Ca and again the ALAS for ^{48}Ca. The anomalies exist for the 20–45 MeV energy range of alpha-projectiles and disappear above 55 MeV. There exists a great amount of experimental data illustrating the ALAS phenomenon for calcium nuclei.

The natural attempt to explain the ALAS effect is the potential scattering. It has been shown that it is possible to fit the alpha-^{40}Ca angular distribution with the optical model in a wide range of energy [11, 12]. However, the optical potentials reproducing

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alpha-^{40}Ca scattering differ significantly from those reproducing alpha-^{44}Ca angular distributions, where there is no ALAS. Also the optical potential for the alpha-^{40}Ca scattering found in the global search for a very wide energy range [11] does not reproduce the magnitude of backward cross-section in the cases of strongest ALAS. Various explanations of the ALAS effect were proposed. They were based on correlations between other reaction channels and angular momentum mismatch [10, 13], on intermediate structure resonances [14–18], on rotations of the alpha + target quasimolecule [19–22] and on exchange effects [23–27]. The aim of this paper is to investigate the possibilities of the explanation of the ALAS effect in terms of the alpha-cluster exchange and one-nucleon exchange.

As mentioned before, the strongest ALAS appears in the scattering of alpha particles from ^{40}Ca. This nucleus, pertaining to the group of the light $A = 4n$ nuclei may be treated as partially constructed of alpha-clusters [28, 29]. The alpha-clusters in calcium, if they do exist as distinguishable structures, may exist only near the surface of the nucleus [30, 31]. Therefore, the target nucleus A may be treated as the two-particle system: the $A-4$ core and the alpha-cluster bound to the core in the shell-model potential well. The cluster may be exchanged with the incoming alpha particle.

The one nucleon exchange is a very important process in the scattering of composite structure on nuclei [32–34]. In spite of the extraordinary stability of the alpha particle, the one-nucleon exchange plays an important role in explaining the ALAS [26]. In this paper the meaning of these two exchanges for reproducing the differential cross-section in the cases of ALAS is analysed.

2. *The alpha-cluster exchange*

According to Agassi and Wall [25] the alpha-cluster exchange effect was calculated in the DWBA. The scattering matrix element has an usual form

$$T = T_{\text{optical model}} + T_{\text{DWBA}}. \tag{2.1}$$

In the description of the scattering of composite structures on nuclei the total wave function must obey the antisymmetry relation. Introducing into DWBA the antisymmetrization [35] for scattering of the four-particle system splits T_{DWBA} into five terms

$$T_{\text{DWBA}} = T_{\text{direct}} + T^1_{\text{ex}} + T^2_{\text{ex}} + T^3_{\text{ex}} + T^4_{\text{ex}}. \tag{2.2}$$

T_{direct} term represents the scattering of the incoming alpha on the alpha-cluster bound in the target nucleus. It was shown by Agassi and Wall [25] and Thompson [36], that this term is negligible. T^i_{ex}, $i = 1, \ldots, 4$ terms represent the exchange of one, two, three or four nucleons between the target and the projectile. In the Agassi–Wall model it is assumed, that T^4_{ex} is the most important term in the T_{DWBA}. In the model it is also assumed, that these four exchanged nucleons are correlated together to form the alpha-cluster. This alpha-cluster, formed near the surface of the target nucleus is "knocked-on" by the incoming alpha-projectile. The knock-on amplitude is treated as a perturbation to the elastic scattering amplitude. The total scattering amplitude has the form

$$f(\theta) = f_{\mathrm{el}}(\theta)+f^{4}_{\mathrm{ex}}(\theta), \tag{2.3}$$

$$f_{\mathrm{el}}(\theta) = f_{\mathrm{c}}(\theta)+\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)e^{2i\sigma_l}(\eta_l-1), \tag{2.4}$$

$$f^{4}_{\mathrm{ex}}(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)e^{2i\sigma_l}a_l. \tag{2.5}$$

The "exchange phase shifts" a_l are given by formulae

$$a_l = -\frac{4i\mu}{\hbar^2 k}\sum_{nL\lambda}S_{nL}\begin{pmatrix} l & \lambda & L \\ 0 & 0 & 0\end{pmatrix}^2 I(l,\lambda,L), \tag{2.6}$$

$$I(l,\lambda,L) = \iint dR_1 dR_2 f_l(R_1,k)\Phi_{nL}(R_2)v_\lambda(R_1,R_2)\Phi_{nL}(R_1)f_l(R_2,k). \tag{2.7}$$

$f_l(k, R)$ are the distorted waves, i.e. the wave functions corresponding to the optical model potential reproducing experimental cross-sections up to 70−90 deg in cases of ALAS and in the full angular range in the no-ALAS cases. $\Phi_{nL}(R)$ are the shell-model wave functions of the alpha-cluster bound in the $A-4$ core potential well with quantum numbers n, L and proper binding energy. The alpha-cluster should be constructed of nucleons from the highest occupied shells n_i, l_i of the target nucleus and should have the angular momentum L as high as possible [25]. The alpha-cluster quantum numbers n, L must obey the sum rule [37]

$$2(n-1)+L = \sum_{i=1}^{4} 2(n_i-1)+l_i. \tag{2.8}$$

$v_\lambda(R_1, R_2)$ is the element of the multipole expansion of the alpha-alpha interaction, which is chosen to be of the Gaussian shape

$$V(\boldsymbol{R}_1,\boldsymbol{R}_2) = -V_0\exp\left(-\gamma^2(\boldsymbol{R}_1-\boldsymbol{R}_2)^2\right) = \sum_\lambda v_\lambda(R_1,R_2)P_\lambda(\cos\theta_{R_1R_2}). \tag{2.9}$$

S_{nL}, the only free parameter of the model may be treated as the "spectroscopic factor" of the alpha-cluster in the target nucleus. It differs from the standard spectroscopic factor in the normalization and it does not contain angular momentum coefficients [25]. It must not be too large because of the unitarity. The maximum value of the S_{nL}, corresponding to the complete clusterization of the target nucleus A, is [25]

$$S^{\max}_{nL} = \sum_{nL}S_{nL} = \frac{1}{4!}\binom{A}{4}. \tag{2.10}$$

The exchange phase shifts a_l are the small corrections to the elastic scattering phase shifts η_l, but they are spiked at the values of l corresponding to some surface-grazing partial waves. The exchange amplitude is added to the normal elastic scattering amplitude and S_{nL} factor is varied to get the best possible agreement with experiment. The addition of the exchange amplitude changes the predicted cross-section for backward angles and does not destroy the agreement with experiment for forward angles. The probability of finding an alpha-cluster in the target nucleus is given by the ratio of the S_{nL} obtained from fitting the experimental data to its maximum value

$$p_\alpha = \frac{S_{nL}}{S_{nL}^{\max}} . \qquad (2.11)$$

The differential cross-sections for the elastic alpha scattering on ^{40}Ca in the energy range $18-50$ MeV and on $^{42,44,48}Ca$ in the energy range $18-29$ MeV were calculated using the presented model. The binding energies for the alpha-cluster in various Ca isotopes are listed in the first row of Table I. They were calculated by subtracting the $A-4$ core and the intra alpha-cluster binding energies from the target A binding energy

$$B_{\text{clust}} = B_A - B_{A-4} - B_\alpha^{\text{intra}} . \qquad (2.12)$$

The cluster quantum numbers calculated from (2.8) for the $s-d$ shell in calcium are: $n = 1$, $L = 8$. The geometry of the shell-model potential well was assumed to be of the Woods–Saxon shape with parameters $r_0 = 1.6$ fm, $a = 0.585$ fm and V_0 listed in Table I.

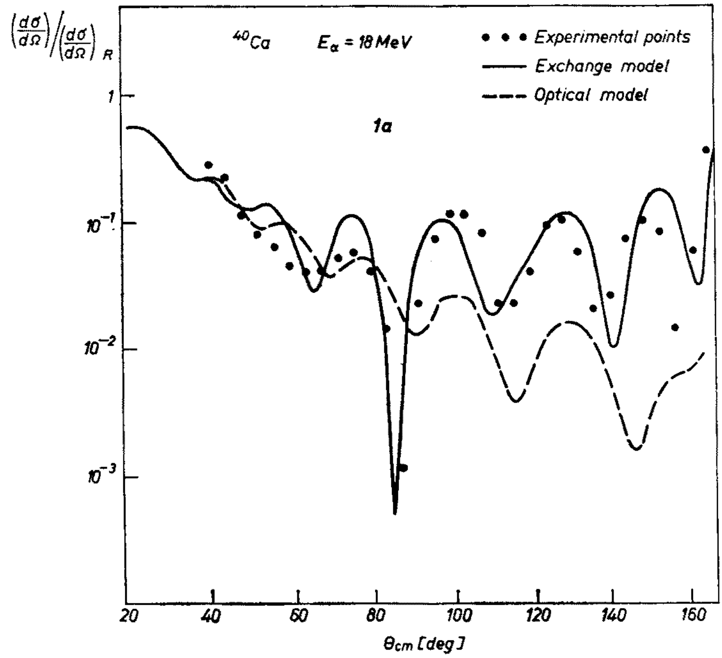
TABLE I

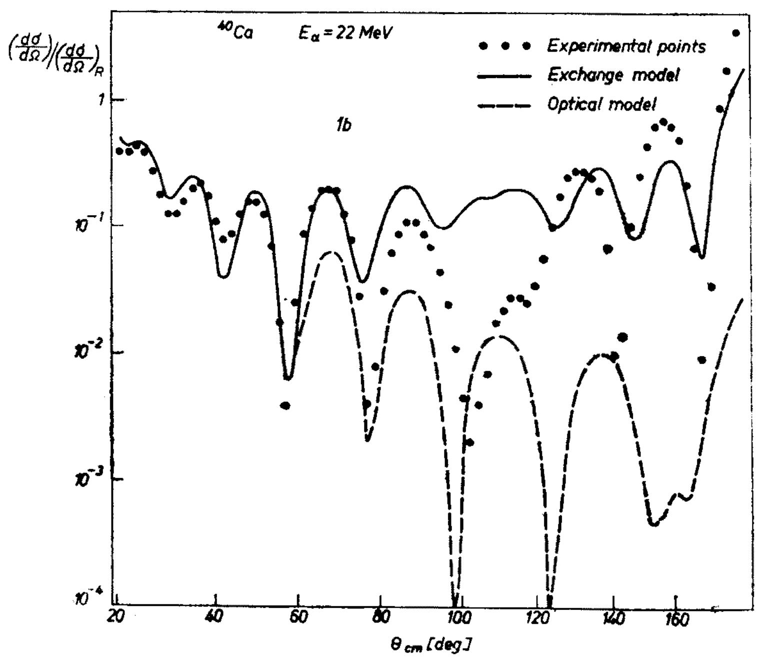
The alpha-cluster binding energies and the shell-model well-depth parameters used in the calculations

Target	^{40}Ca	^{42}Ca	^{44}Ca	^{48}Ca
B_{clust} [MeV]	7.1	6.25	8.84	12.0
V_0 [MeV]	58.78	55.83	57.37	58.67

The optical model potential generating distorted waves has volume absorption and parameters: $U = 50.0$ MeV, $r_{0U} = 1.6$ fm, $a_U = 0.585$ fm, $W = 12.4$ MeV, $r_{0W} = 1.652$ fm, $a_W = 0.53$ fm. According to reference [25] the parameters of the alpha-alpha interaction are: $V_0 = 125$ MeV, $\gamma = 0.467\ \text{fm}^{-1}$.

The set of a_l's for each case was calculated and the $S_{1,8}$ factor was adjusted to give the best possible agreement with the experiment. As can be seen from Fig. 1 for ^{40}Ca the agreement with the experiment is rather qualitative. However, the alpha-cluster exchange model represents correctly one of the basic features of ALAS — its isotopic dependence (see Fig. 2).


$\left(\frac{d\sigma}{d\Omega}\right)/\left(\frac{d\sigma}{d\Omega}\right)_R$
^{40}Ca
$E_\alpha = 18\ MeV$
Experimental points
Exchange model
Optical model
1a
1
10^{-1}
10^{-2}
10^{-3}
20
40
60
80
100
120
140
160
θ_{cm} [deg]


$\left(\frac{d\sigma}{d\Omega}\right)/\left(\frac{d\sigma}{d\Omega}\right)_R$
^{40}Ca
$E_\alpha = 22\ MeV$
Experimental points
Exchange model
Optical model
1b
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
20
40
60
80
100
120
140
160
θ_{cm} [deg]

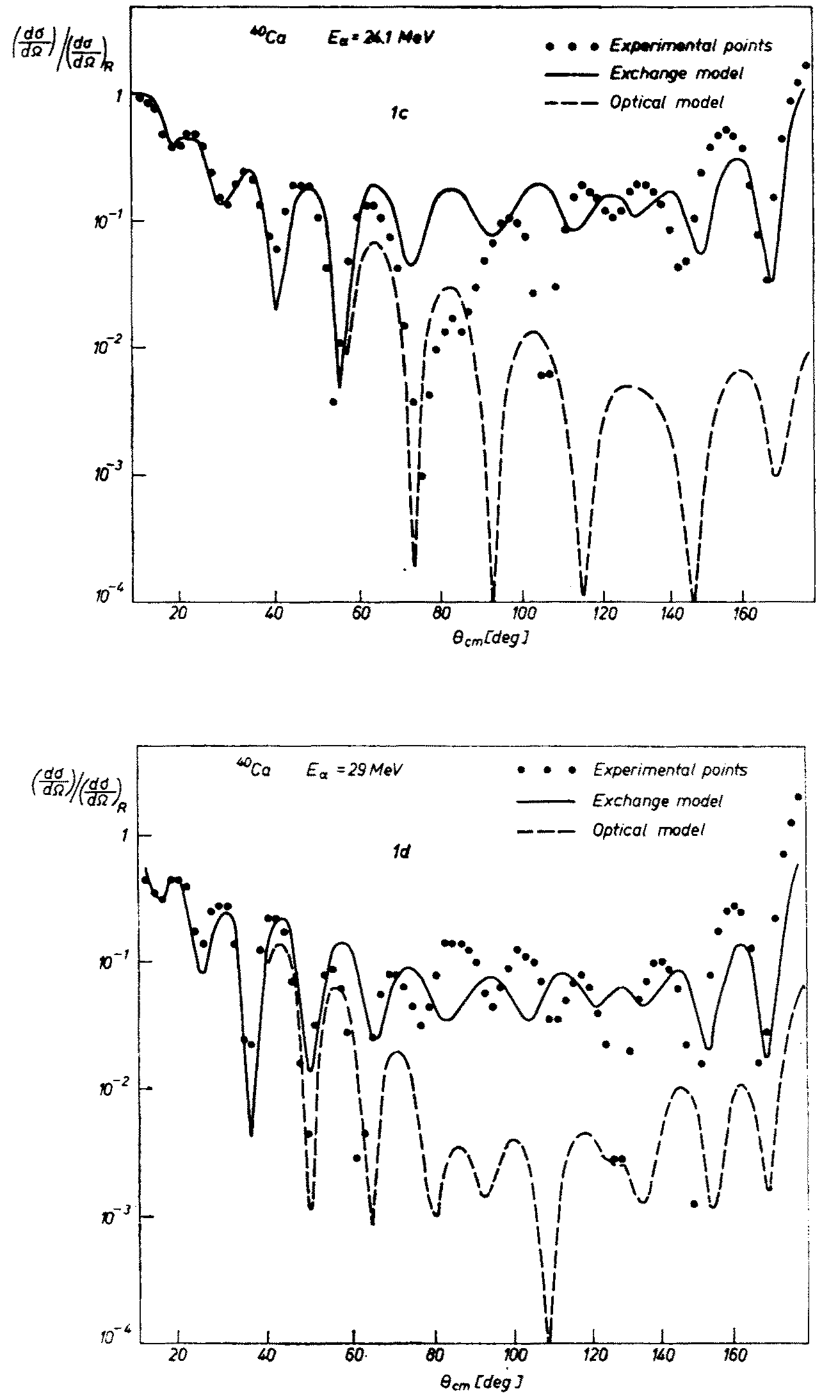
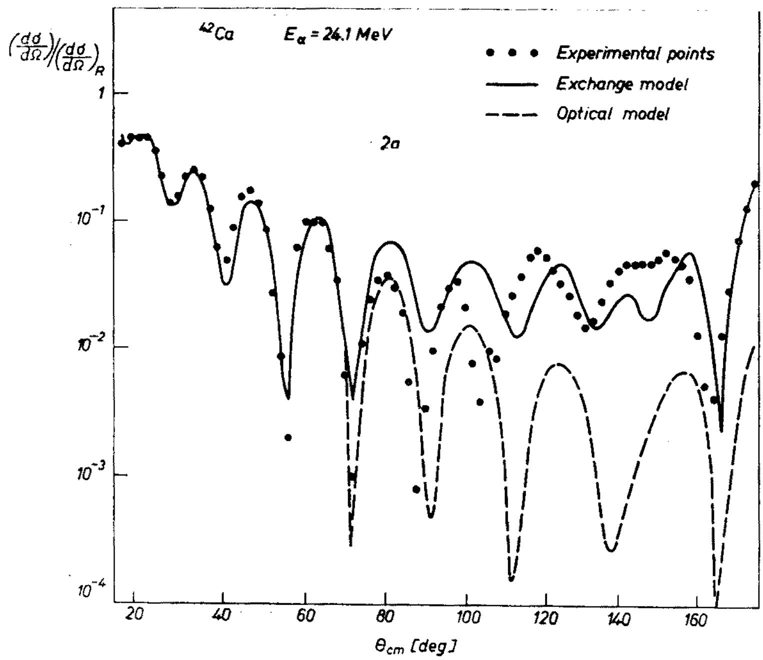
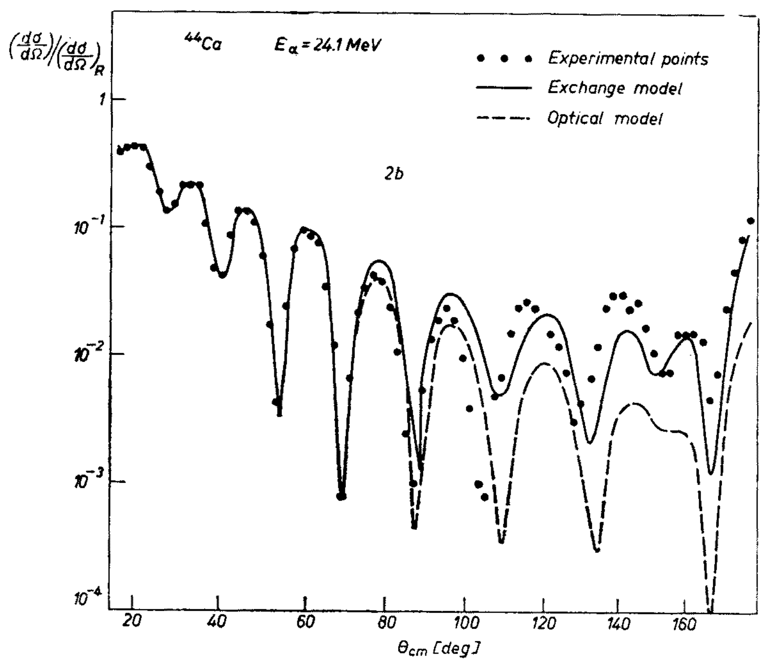


Fig. 1. The alpha-cluster exchange model calculations for ^{40}Ca. a) E_α = 18.0 MeV, b) E_α = 22.0 MeV, c) E_α = 24.1 MeV, d) E_α = 29.0 MeV



$\left(\frac{d\sigma}{d\Omega}\right)/\left(\frac{d\sigma}{d\Omega}\right)_R$
^{42}Ca
E_α = 24.1 MeV
Experimental points
Exchange model
Optical model
2a
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
20
40
60
80
100
120
140
160
θ_{cm} [deg]



$\left(\frac{d\sigma}{d\Omega}\right)/\left(\frac{d\sigma}{d\Omega}\right)_R$
^{44}Ca
E_α = 24.1 MeV
Experimental points
Exchange model
Optical model
2b
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
20
40
60
80
100
120
140
160
θ_{cm} [deg]

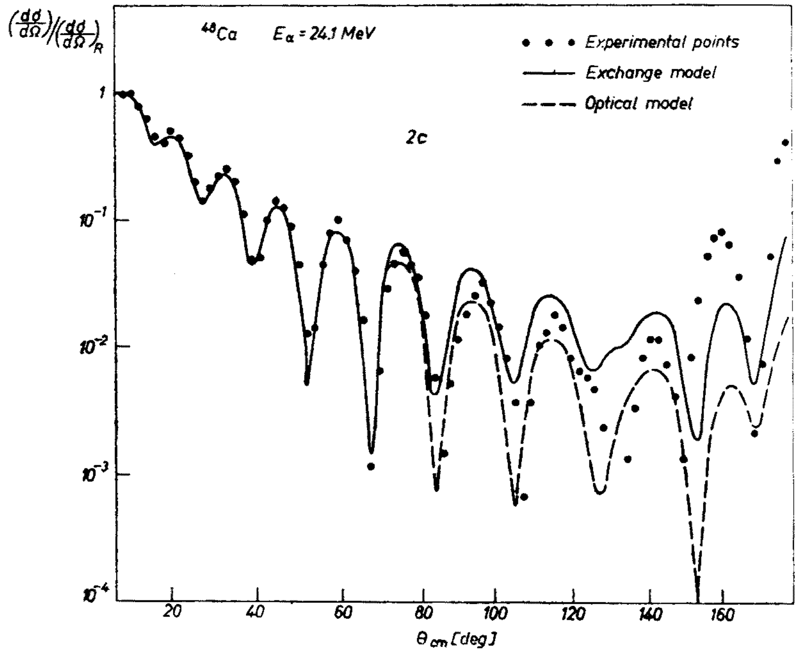


Fig. 2. The alpha-exchange model calculations for E_α = 24.1 MeV. a) ^{42}Ca, b) ^{44}Ca, c) ^{48}Ca

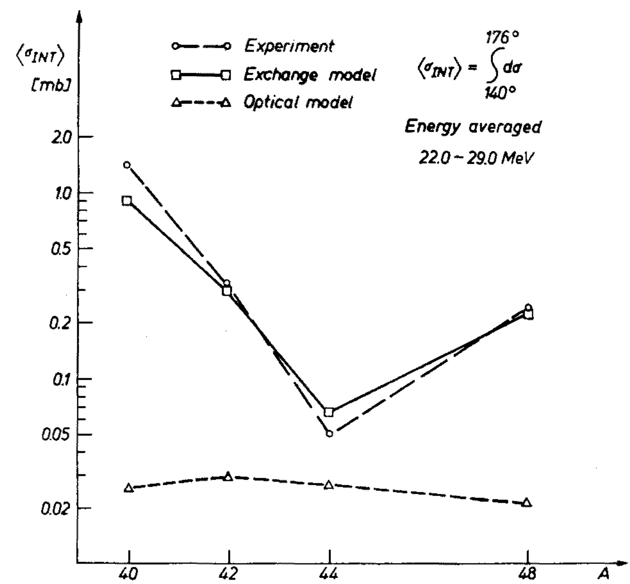


Fig. 3. The back-angle integrated and energy-averaged cross-section for ^{40,42,44,48}Ca obtained from alpha-exchange model compared with the experiment and the optical model predictions

In Fig. 3 back-angle integrated and energy-averaged cross-sections obtained from the exchange model are compared with the experiment and with the optical model predictions. The agreement with experiment is quantitative. The obtained values of p_α (cf. (2.11)) are listed in Table II.

TABLE II

The energy-averaged alpha-cluster spectroscopic factors and the probabilities of finding alpha-clusters in various Ca isotopes

Target	^{40}Ca	^{42}Ca	^{44}Ca	^{48}Ca
$S_{1,8}$	77.4	27.3	17.5	41.0
p_α	0.022	0.0059	0.0031	0.0051

The sensitivity of the model to various factors was also investigated. Shell model wave functions appearing in the formula (2.7) depend: on the cluster binding energy B_{clust}, on the quantum state n, L and on the potential well used to generate them. B_{clust} was also calculated according to other formula [39] and was varied from 2.0 to 35.0 MeV, but that did not influence the quality of fits.

When the alpha-cluster is constructed of the nucleons from the inner shells of target nucleus its angular momentum is lower than the maximum possible value according to (2.8). It leads to the disagreement with experimental angular distributions in the whole angular range. This is in agreement with previous suggestions that alpha-clusters may exist only near the nuclear surface [25, 30, 31]. The alpha-cluster may exist in the core potential well in several n, L states. It gives to the model additional degrees of freedom (cf (2.6)). However, for the values of cluster angular momentum L, lower than the maximum possible, the spike in the exchange phase shifts, a_l moves towards the lower values of l and does not influence the surface grazing partial waves. The corresponding S_{nL} values are fast vanishing. For example, for ^{40}Ca and for the nucleons from the highest occupied shell we have

$$2(n-1)+L = 8, \quad S_{1,8} = 50, \quad S_{2,6} = 20, \quad S_{3,4} = 5.$$

The shell-model potential well depth was also varied from 50 MeV to about 120 MeV with simultaneous changing the geometry parameters appropriately to the continuous ambiguity. The alpha-cluster exchange effect was calculated combining the deep and shallow shell-model potentials with the deep and shallow optical model potentials. The shell-model potential for alpha-cluster similar to this for nucleons only gives correct agreement with experiment.

The influence of the optical model potential on the exchange model predictions was also tested. There is a significant preference of the two classes of optical potential in the model: the shallow one similar to the shell-model potential well and the deep one similar to the unambiguous optical potential for 100 MeV alpha-^{40}Ca scattering [40]. However, the best fit optical potentials were not searched being outside the aim of this work. For simplicity, our optical potential was also energy independent.

The alpha-cluster spectroscopic factors in Table II are somewhat ambiguous. They are influenced both by the optical model potential and by the alpha-core interaction. Such difficulties in calculating the alpha-particle spectroscopic factors are typical for other DWBA calculations [41, 42].

Calculations were also made for the alpha-^{40,44}Ca scattering in the energy range 40 – 50 MeV. Model predicts disappearing of ALAS with increasing projectile energy. It was not possible to reproduce the differential cross-sections above 40 MeV with the same spectroscopic factor as for lower energies. To give the correct predictions of ALAS the spike in a_l must influence the phase shifts η_l in the vicinity of the grazing value of l. As seen from (2.7), the angular momentum of the cluster L, of the partial wave l and of the alpha-alpha interaction multipole must obey the triangle rule. The multipoles v_λ vanish fast with increasing λ, so the position of the spike in the exchange phase shifts a_l is determined by the cluster angular momentum L. The sum (2.8) is 10 for $(sd)^{-2}\,(fp)^2$ configuration and is 12 for $(sd)^{-4}\,(fp)^4$ configuration in ^{40}Ca. When the exchange phase shifts were calculated with $L = 10$ or $L = 12$, it was possible to reproduce the anomalies up to about 50 MeV without changing the spectroscopic factor, Audi et al. [41] found, that the alpha-cluster spectroscopic factor for the ground state measures only a part of the alpha-clustering probability in the target nucleus. The alpha-cluster spectroscopic factor for the excited states is sometimes greater than that for the ground state. This may occur also in the case of alpha-Ca scattering.

3. The one-nucleon exchange

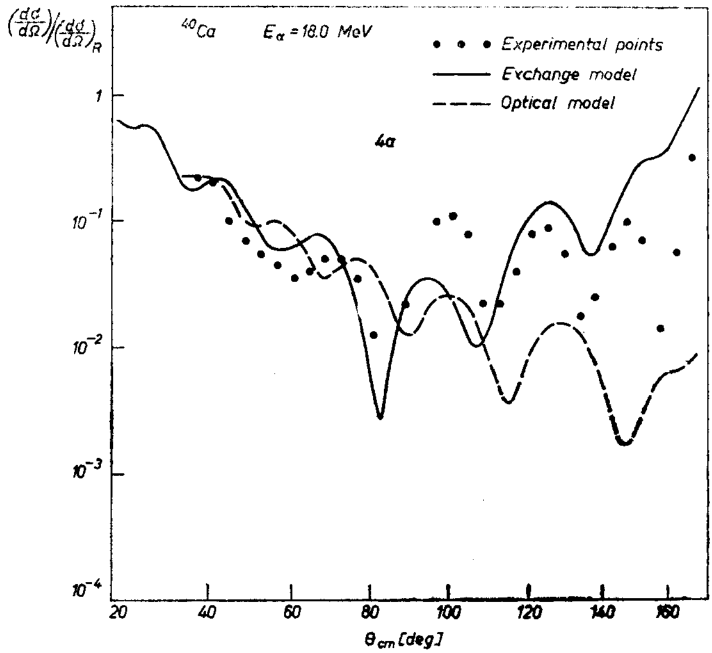
It was pointed out [26], that the one-nucleon exchange plays also significant role in explaining the ALAS. The single-particle exchange has been introduced into these calculations in the simplified way, neglecting the spin and structural effects. According to Schaeffer [33] and Boridy [26] the T-matrix element for one-nucleon exchange has the form

$$T(1\mathrm{PE}) \propto \langle \Psi_0(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A)\chi^{(-)}(\boldsymbol{r}_\alpha) \mid \sum_{i=1}^{A} V(\boldsymbol{r}_\alpha - \boldsymbol{r}_i) \mid \chi^{(+)}(\boldsymbol{r}_\alpha)\Psi_0(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A)\rangle. \tag{3.1}$$

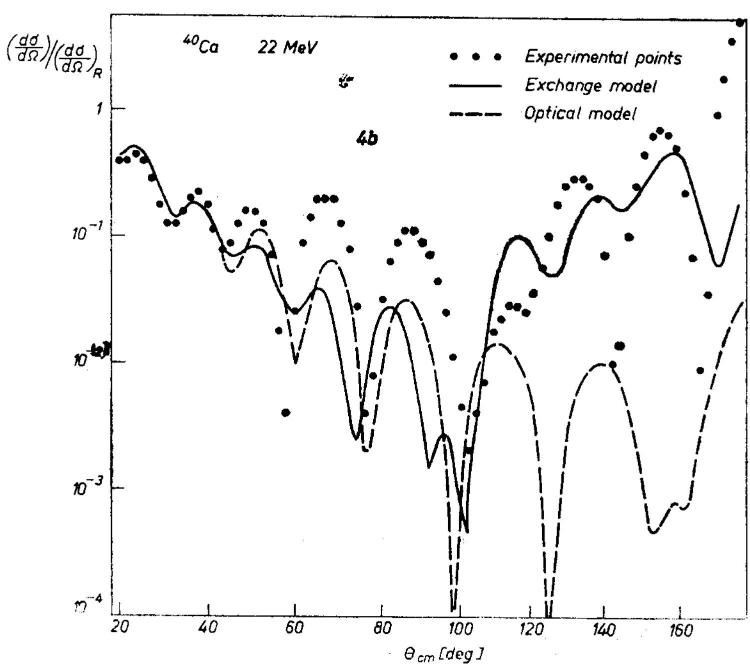
The alpha-nucleon $V(\boldsymbol{r}_\alpha - \boldsymbol{r}_i)$, is assumed to be non-local and of the Gaussian shape [43]. The integrals are calculated approximately, taking into account only the zero-order element of the multipole expansion of the $\alpha - \mathrm{n}$ potential and assuming, that the potential is the slowly varying function as compared with the distorted waves χ and the target wave functions Ψ_0. Similarly, as in the case of alpha-cluster exchange, one gets in this case

$$f^1_{\mathrm{ex}}(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) e^{2i\sigma l} a_l, \tag{3.2}$$

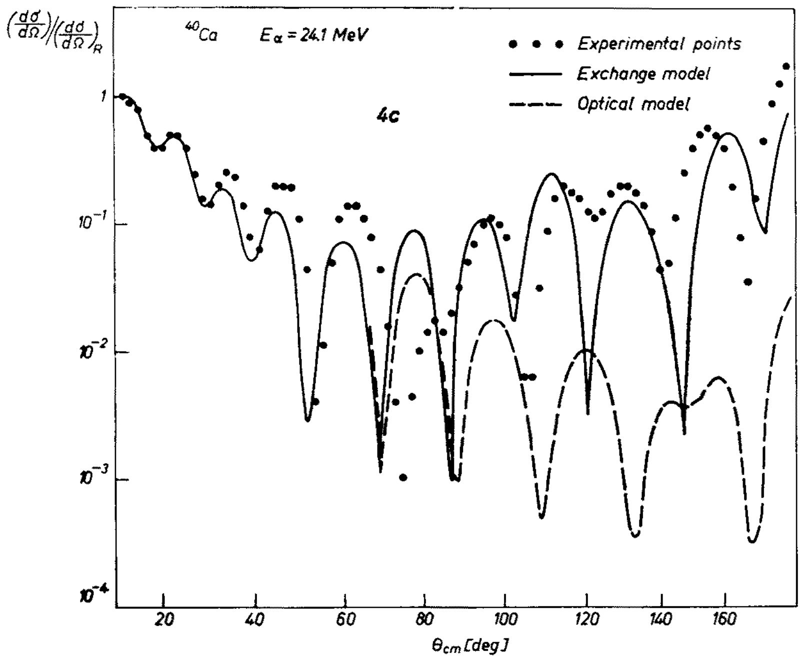
$$a_l = -\frac{64 i\mu}{\hbar^2 k} \frac{A}{A+4} \int_0^\infty f_l^2(r_\alpha) dr_\alpha \int_0^\infty \varrho(r) V_0(r_\alpha, r) r^2 dr. \tag{3.3}$$



$(\frac{d\sigma}{d\Omega})/(\frac{d\sigma}{d\Omega})_R$
^{40}Ca $E_\alpha = 18.0$ MeV
Experimental points
Exchange model
Optical model
4a
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
20
40
60
80
100
120
140
160
θ_{cm} [deg]



$(\frac{d\sigma}{d\Omega})/(\frac{d\sigma}{d\Omega})_R$
^{40}Ca 22 MeV
Experimental points
Exchange model
Optical model
4b
1
10^{-1}
10^{-3}
20
40
60
80
100
120
140
160
θ_{cm} [deg]



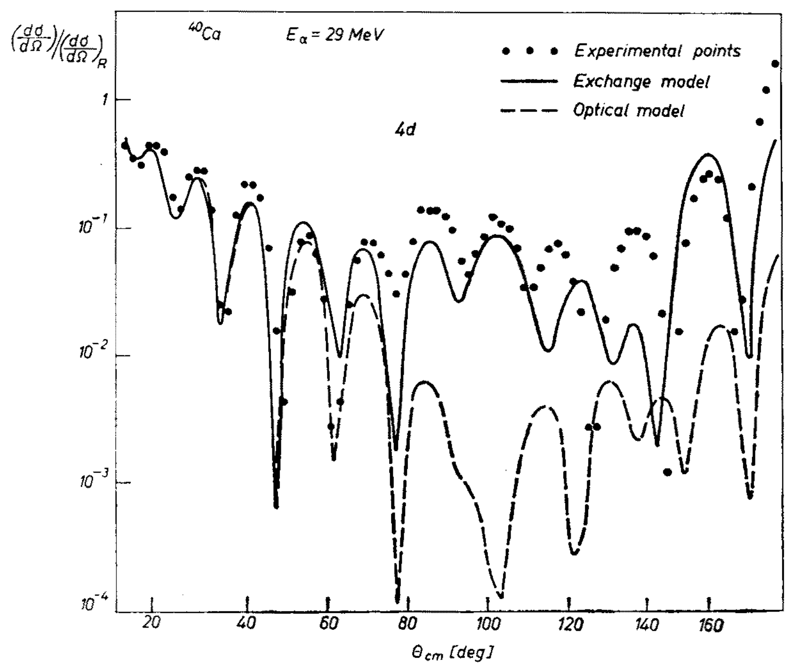


Fig. 4. The one-nucleon exchange model calculations for ^{40}Ca. a) $E_\alpha = 18.0$ MeV, b) $E_\alpha = 22.0$ MeV, c) $E_\alpha = 24.1$ MeV, d) $E_\alpha = 29.0$ MeV

The optical wave function f_l is generated in the same way as for the alpha-cluster exchange. The nuclear matter density $\varrho(r)$ appears here as the effect of the integration of the target wave functions without extracting the variables appearing in the interaction $V(\boldsymbol{r}_\alpha - \boldsymbol{r}_i)$ (cf. Ref. [25]). It has a Woods–Saxon shape and is normalized as usual

$$\varrho(r) = \varrho_0[1+\exp((r-R)/a_d)]^{-1}, \tag{3.4}$$

$$R = r_d A^{1/3}, \quad r_d = 1.03 \text{ fm}, \quad a_d = 0.52 \text{ fm}, \quad 4\pi \int_0^\infty \varrho(r) r^2 dr = A. \tag{3.5}$$

In Fig. 4 the results of one-nucleon exchange calculations for ^{40}Ca in the energy range 18.0–29.0 MeV are compared with the experiment and optical model predictions. The optical model parameters are the same as for the alpha-exchange calculations. The agreement with experiment is also qualitative only. The model does not reproduce the isotopic dependence of ALAS as it averages the structure of the target nucleus. It also does not reproduce the magnitude of the backward-angle cross-sections for alpha energies above 40 MeV. It is clear, that the one-nucleon exchange effects play a very important role in explaining the ALAS phenomenon, but they need more exact treatment including spin effects and introducing spectroscopic factors.

4. *The alpha-cluster and one-nucleon exchange calculations*

The investigation of the total effect of the exchange of alpha-particle and single nucleon in the anomalous large angle alpha scattering is very promising. The total scattering amplitude has the form

$$f(\theta) = f_{e_l}(\theta) + f_{ex}^4(\theta) + f_{ex}^1(\theta). \tag{4.1}$$

$f_{ex}^4(\theta)$ is taken from (2.5), $f_{ex}^1(\theta)$ is given by (3.2).

The one-nucleon exchange model used here is rather simplified, so the results obtained from adding one-nucleon exchange and alpha exchange amplitudes to the elastic scattering amplitude did not agree quite well with the experiment. To improve the results the distorted waves "adjusted to the distortion" have been generated. After calculating the total exchange effect with the given optical potential, the optical model parameters were adjusted to improve the results. Then the exchange phase shifts a_l were calculated for both exchanges with the new optical potential. This method was self-consistent after two steps. Only the depth of the real part and the geometry parameters of the imaginary part of the optical model potential had been changed by a few per cent. The results of the final calculations are shown in Fig. 5. It should be pointed out, that it was not possible to improve the alpha-exchange model predictions with this method.

The addition of the amplitude (3.2) to the amplitude (2.3) has reduced the adjusted spectroscopic factor for alpha-clusters in ^{40}Ca by the factor of two. It should be stressed, that even taking into account the one-particle exchange overestimated did not reduce completely the alpha-cluster exchange. These both exchange effects are very important in explaining the ALAS effect.

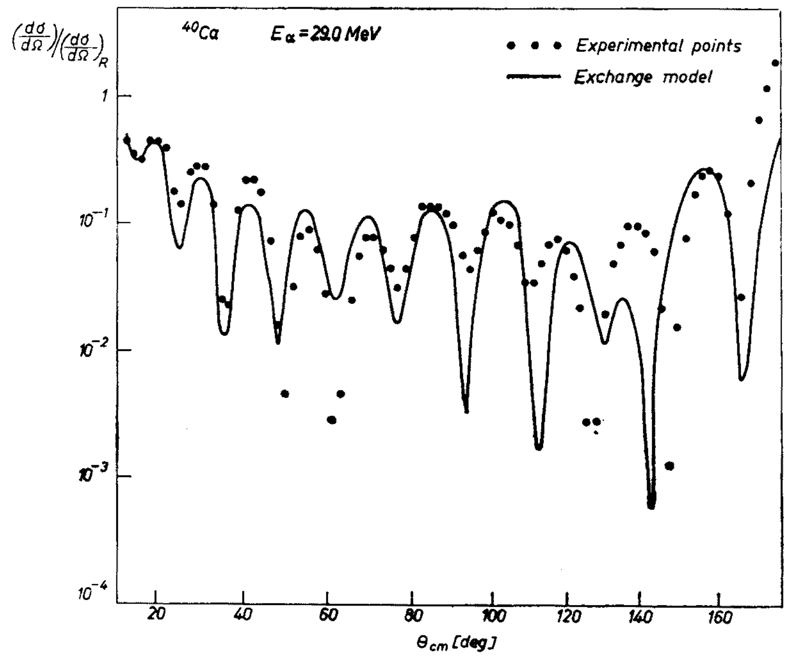


Fig. 5. The total effect of the one-nucleon exchange and alpha-cluster exchange for ^{40}Ca $E_\alpha = 29.0$ MeV

5. Conclusions

The results of calculations of the total exchange effect (cf. (4.1)) may be compared with the results of Kondo et al. [27] and Wall [44]. In the quoted papers all possible exchange effects in scattering of alpha particles on calcium are taken into account by adding to the optical potential real [27] or complex [44] Majorana term. Comparing the results shown in Fig. 5 with the results obtained in precited papers one can conclude that both alpha-cluster exchange and one-nucleon exchange exhaust the exchange effects in alpha-^{40}Ca scattering.

The probability of forming the alpha-cluster in ^{40}Ca is low, it is about one per cent. In spite of this the clusterization effects play an important role in explaining the ALAS effect, especially its isotopic dependence. The alpha-clusters existing in Ca nuclei are bound in the potential well, which is very similar to that for nucleons with a binding energy of about 10 MeV.

The both exchange models work in a limited projectile energy range — up to 40 MeV.

The model of one-nucleon exchange needs improvements by taking into account the target nucleus structure and probably the spin of the exchanged nucleon.

As seen from the last section both the alpha-cluster exchange and the one-nucleon exchange are necessary in order to explain the anomalous large angle alpha scattering on ^{40}Ca.

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