PROPERTIES OF FISSION ISOMERS

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Properties of fission isomers, like the moment of inertia, pairing energy gap and collective gyromagnetic ratio are investigated or reinvestigated theoretically using the Nilsson potential. In particular, the effect of coupling between the oscillator shells on these properties is researched. The properties are also studied and compared with experiment for the ground state, as a test of the calculations.

1. *Introduction*

There is an ever increasing amount of experimental data on the properties of fission isomers. After the observation [1] of the rotational spectrum of the fission isomer ^{240m}Pu such a spectrum was also observed [2] for ^{236m}U. Also the quadrupole moments have recently been measured [3, 4] for ^{239m}Pu and ^{236m}Pu. An excited state of the two-quasi-particle, *K*-isomeric nature was found [5] in ^{238m}Pu.

This increases interest in the theoretical reproduction and predictions for the above properties. In Ref. [6], the moments of inertia and the pairing energy gaps were calculated using the Nilsson single-particle potential. The moments of inertia were also calculated [7] with the use of the Woods-Saxon potential. Both calculations exploited the cranking approximation. This approximation has been corrected in Ref. [8] for the effect of the modification of the pairing interaction by the rotational motion. The correction consists in the addition of the Migdal term to the cranking term. It increases the moment of inertia. However, a difficult problem here is the value of the strength of the quadrupole component of the pairing interaction, on which the correction strongly depends and for the choice of which there is no direct indication.

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The present paper is closely connected with our previous research [6], which we will refer to as I. In that paper, mixing of the oscillator shells in the wave functions, due to the hexadecapole component of the deformation, has been disregarded for computational reasons. We investigate here the effect of this mixing on the properties of both ground and isomeric states of heavy nuclei considered in I. We also calculate some additional properties of these states and discuss the effects of some changes in the pairing interaction.

2. *Description of the calculations*

The present calculations are very similar to those of our previous paper I. The main difference is that the Nilsson Hamiltonian is diagonalized here exactly rather than within each oscillator shell as in I. In other words, mixing between different oscillator shells is taken into account. The mixing is due to the hexadecapole deformation term in the Hamiltonian, parametrized by ε_4. In the case of a pure quadrupole deformation ($\varepsilon_4 = 0$) there is no mixing (we work in the stretched coordinate system [9], as one usually does).

The second, minor difference is in the solution of the pairing equations. In both calculations $2\sqrt{15Z(N)}$ energy levels are taken to solve the pairing equations. However, the way we choose these states is slightly different. In the present paper, the levels are chosen symmetrically in the number of levels with respect to the Fermi level λ. In other words, the same number of levels ($\sqrt{15Z(N)}$) is taken below and above λ, exactly as done in Ref. [10] where the pairing strengths were fitted to the odd-even mass differences. In paper I, on the other hand, the levels were chosen symmetrically in energy with respect to λ, i. e. they were chosen so as to include the levels closest to λ in energy. Due to the non-uniform distribution of the energy levels, the two choices generally lead to different sets of levels. The effects of these differences on the energy gaps 2Δ are small, 2Δ being slightly smaller in the present paper, as we will see in the next section. Moreover, we can say that the experimental odd-even mass differences are reproduced about equally well in both cases.

We use the "$A = 242$" parameters of the Nilsson potential [10] and the isospin-dependent pairing strength taken from Ref. [10], the same as in I. For each deformation, the number of the oscillator shells N_{max} is taken such that an inclusion of additional shells does not practically change the results. For the deformations corresponding to the second minimum ($\varepsilon \approx 0.60$), N_{max} is 10 for protons and 11 for neutrons.

3. *Results and discussion*

3.1. Results

The calculations are performed for the same nuclei as in I. The results for the first minimum (ground state), treated as a test for the theoretical calculations, are given in Table I and those for the second minimum (shape isomeric state) in Table II. The deformations ε and ε_4 of both these states are taken from Ref. [11]. Both tables are enlarged by the microscopic values of the quadrupole Q_2 and the hexadecapole Q_4 electric moments and of the collective gyromagnetic ratio g_R, as compared with I. The values of Q_2 and Q_4 are taken from Ref. [12] (cf. also Ref. [13]) and those of g_R are calculated in the present paper. For the first minimum, the experimental values of Q_2 taken from Refs [14, 15]

TABLE I

Theoretical quadrupole Q_2 and hexadecapole Q_4 electric moments, energy gaps for protons $2\Delta_p$ and for neutrons $2\Delta_n$, moments of inertia $\mathscr{J}$ and collective gyromagnetic ratios g_R, calculated for the ground state. The theoretical moments Q_2 and Q_4 are taken from Ref. [12]. Experimental values Q_2^{exp}, Q_4^{exp} and $\mathscr{J}^{exp}$ are given for comparison

Nuc-leide	ε	ε_4	Q_2	Q_2^{exp}	Q_4	Q_4^{exp}	$2\Delta_p$	$2\Delta_n$	$\frac{2}{\hbar^2}\mathscr{J}$	$\frac{2}{\hbar^2}\mathscr{J}^{exp}$	g_R
—	—	—	b	b	b²	b²	MeV	MeV	MeV⁻¹	MeV⁻¹	—
^{226}Ra	0.14	−0.055	5.69	7.21±0.36[a]	6.78		1.69	1.46	68	88	0.19
^{230}Th	0.17	−0.060	7.58	9.00±0.06	9.17	10.31±1.39	1.67	1.33	97	113	0.25
^{232}Th	0.19	−0.055	8.41	9.62±0.05	9.72	11.50±1.32	1.70	1.21	112	120	0.23
^{234}U	0.20	−0.055	9.38	10.47±0.05	10.75	13.23±1.89	1.54	1.19	122	138	0.28
^{236}U	0.20	−0.050	9.37	10.80±0.07	10.35	12.29±2.07	1.56	1.26	112	132	0.29
^{238}U	0.20	−0.040	9.28	11.12±0.07	9.43	7.85±2.10	1.60	1.33	101	134	0.29
^{236}Pu	0.20	−0.050	9.81		10.51		1.54	1.22	120	135	0.30
^{238}Pu	0.20	−0.045	9.79	11.27±0.08	10.10	13.03±2.30	1.55	1.27	111	136	0.32
^{240}Pu	0.21	−0.040	10.20	11.58±0.08	10.20	10.82±2.56	1.54	1.29	111	140	0.32
^{242}Pu	0.22	−0.030	10.56	11.64±0.08	9.76	$7.01^{+2.81}_{-3.47}$	1.57	1.22	115	135	0.29
^{244}Pu	0.22	−0.020	10.49	11.70±0.08	8.78	$2.84^{+4.72}_{-2.84}$	1.61	1.18	110	133	0.28
^{240}Cm	0.21	−0.040	10.57		10.09		1.49	1.27	113		0.33
^{242}Cm	0.22	−0.035	10.99		10.18		1.47	1.28	116	142	0.34
^{244}Cm	0.22	−0.025	10.92	12.11±0.08	9.23	$0.00^{+4.73}_{-0.00}$	1.49	1.22	115	140	0.31
^{246}Cm	0.23	−0.015	11.30	12.26±0.08	8.76	$0.00^{+4.73}_{-0.00}$	1.51	1.17	115	140	0.31
^{248}Cm	0.23	−0.005	11.24	12.28±0.08	7.76	$0.00^{+5.67}_{-0.00}$	1.55	1.20	109	138	0.30
^{250}Cm	0.23	0.000	11.23		7.29		1.56	1.32	100		0.32

[a] Ref. [14].

and Q_4 taken from Ref. [15] are also given for comparison. It is worthwhile to mention that the hexadecapole-moment operator $\hat{Q}_4$ is defined here, in accordance with Refs. [12, 13], as $\hat{Q}_4 = 8\int\varrho(r')r'^4P_4(\cos\theta')d\tau'$, i. e. four times larger than that defined in Ref. [15].

We can see in Table I that the calculated values Q_2^{micr} are smaller than Q_2^{exp} by about 11 %, on the average. This means that the experimental quadrupole moments are reproduced rather well by theory. The remaining discrepancy may be due to the dynamic effects which increase the moments and are not taken into account in the static values Q_2^{micr} calculated here. For the hexadecapole moments, the discrepancy is larger, especially for the heavier elements. However, the experimental inaccuracy of Q_4 is also large for these elements.

The calculated proton and neutron energy gaps, $2\Delta_p$ and $2\Delta_n$, are smaller than those calculated in I about 5% and 4%, respectively. Still, they reproduce the experimental gaps $2P_p$ and $2P_n$, given in I and not repeated here, approximately as well as the gaps calculated in I. The moments of inertia $\mathcal{J}$ are larger than those of I by about 7% which

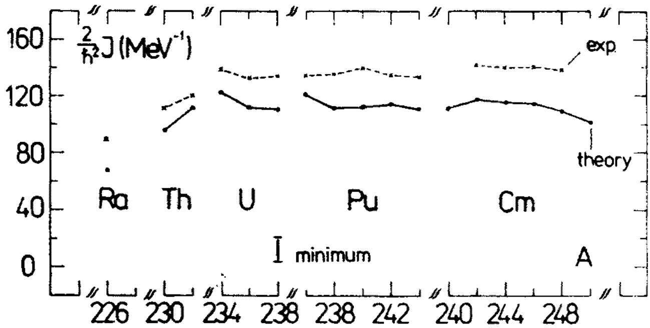

Fig. 1. Theoretical and experimental values of the moment of inertia for the ground state (first minimum)

TABLE II

Theoretical energy gaps $2\Delta_p$ and $2\Delta_n$, moments of inertia $\mathcal{J}$ and gyromagnetic ratios g_R calculated for the fission isomeric state in two cases: G = const. and $G \propto S$. The electric moments Q_2 and Q_4, taken from Ref. [12], are calculated in the G = const. case

Nuc-leide	G = const								$G \propto S$					
	ε	ε_4	Q_2	Q_4	$2\Delta_p$	$2\Delta_n$	$\frac{2}{\hbar^2}\mathcal{J}$	g_R	ε	ε_4	$2\Delta_p$	$2\Delta_n$	$\frac{2}{\hbar^2}\mathcal{J}$	g_R
—	—	—	b	b^2	MeV	MeV	MeV^{-1}	—	—	—	MeV	MeV	MeV^{-1}	—
^{226}Ra	0.41	0.065	18.3	9.3	1.73	1.48	153	0.34	0.41	0.065	1.83	1.60	146	0.34
^{230}Th	0.58	0.055	30.5	37.4	1.88	1.36	287	0.30	0.58	0.055	2.14	1.65	262	0.30
^{232}Th	0.59	0.055	31.5	40.3	1.83	1.10	307	0.30	0.59	0.055	2.10	1.43	280	0.30
^{234}U	0.60	0.060	33.1	42.8	1.83	1.16	309	0.31	0.60	0.060	2.12	1.49	282	0.31
^{236}U	0.60	0.060	33.3	43.3	1.82	0.75	334	0.28	0.60	0.060	2.10	1.26	298	0.29
^{238}U	0.60	0.065	33.3	42.2	1.81	1.22	312	0.31	0.60	0.065	2.09	1.52	287	0.30
^{236}Pu	0.61	0.065	34.8	45.3	1.82	1.22	313	0.33	0.61	0.065	2.11	1.54	287	0.32
^{238}Pu	0.60	0.060	34.3	44.6	1.83	0.77	336	0.29	0.60	0.060	2.11	1.27	300	0.29
^{240}Pu	0.61	0.070	35.0	44.7	1.79	1.21	319	0.33	0.61	0.070	2.08	1.51	293	0.32
^{242}Pu	0.61	0.075	35.0	43.7	1.82	1.40	307	0.35	0.61	0.075	2.11	1.69	283	0.34
^{244}Pu	0.61	0.080	35.0	42.7	1.81	1.49	303	0.35	0.61	0.080	2.10	1.77	279	0.34
^{240}Cm	0.61	0.065	36.0	47.0	1.80	0.82	341	0.30	0.61	0.065	2.08	1.34	304	0.31
^{242}Cm	0.61	0.070	36.0	46.1	1.78	1.21	322	0.33	0.61	0.070	2.06	1.51	296	0.32
^{244}Cm	0.62	0.075	36.9	47.8	1.78	1.45	315	0.36	0.62	0.075	2.08	1.74	290	0.35
^{246}Cm	0.65	0.060	37.0	48.2	1.73	1.54	362	0.33	0.62	0.075	2.02	1.83	288	0.36
^{248}Cm	0.66	0.060	41.1	63.6	1.70	1.58	376	0.32	0.66	0.065	2.00	1.94	336	0.33
^{250}Cm	0.68	0.070	42.0	65.7	1.65	1.65	381	0.34	0.67	0.070	2.02	2.00	339	0.34

makes them closer to experiment. However, they are still too small, by about 17%, to reproduce the experimental energy of the first 2+ state, even if one interprets this state as purely rotational. This is illustrated explicitly in Fig. 1. The result is in line with Refs [16, 17], where the cranking values of the inertial functions are found to be systematically too low.

The larger values of $\mathscr{J}$ obtained in the present paper, as compared with those of I, are partly due to the smaller gaps 2Δ and partly to the effect of coupling between the shells. Regarding the collective gyromagnetic ratio g_R, only the theoretical values are given in the table. There are no experimental values for the moment, for nuclei considered, to compare them with.

Concerning the second minimum, the differences between the present values and those of I are similar to the differences for the first minimum. The proton and the neutron gaps, $2\Delta_p$ and $2\Delta_n$, are smaller by about 5% and 4%, respectively, and the moment of inertia $\mathscr{J}$

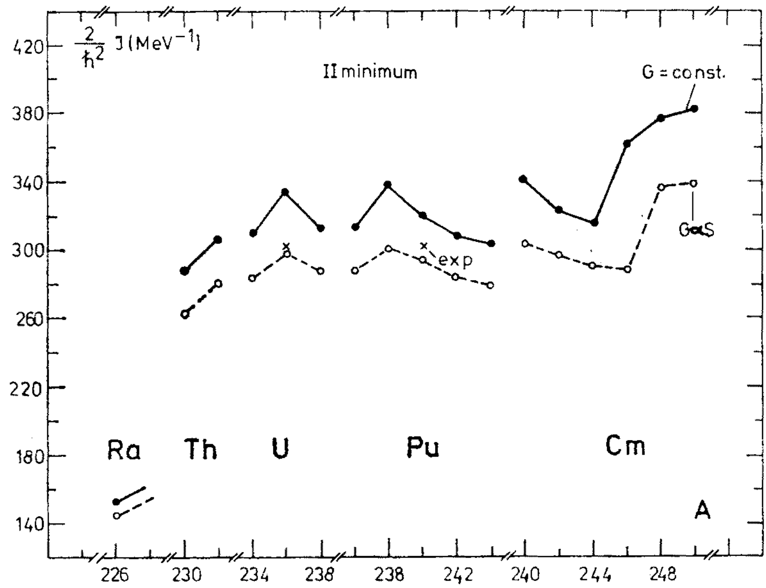

Fig. 2. Moments of inertia calculated for the fission isomeric state (second minimum) for two cases of the pairing force strength: G = const. and $G \propto S$. Two experimental values, available, are also shown for comparison

is larger than that of I by about 6%. This results in the fact that presently $\mathscr{J}^{micr}$ is closer to $\mathscr{J}^{exp}$ in the $G \propto S$ case then in the G = const. case for both nuclei, for which the experimental value is available.

Regarding the quadrupole moment, the experimental value Q_2 = 34–39b, measured recently [3] for the odd nucleus ^{239m}Pu, is rather close to the values calculated for the neighbouring doubly even nuclei ^{238m}Pu and ^{240m}Pu. Also the value $Q_2 = 37^{+14}_{-8}$b, deduced from experiment [4] for ^{236m}Pu with rather large error bars, is close to the calculated one.

Besides Table II, the values of the moment of inertia $\mathscr{J}$, calculated at the second minimum for both variants of the pairing constant G, are illustrated explicitly in Fig. 2. The experimental values, available for two nuclei, are shown for comparison.

The dependence of $\mathscr{J}$ on the deformation is shown in Fig. 3 for ^{240}Pu. It is calculated along the static fission trajectory L, the same as in I. In comparison to I, the values of $\mathscr{J}$ increase slightly faster with the deformation. Position of the two experimental points

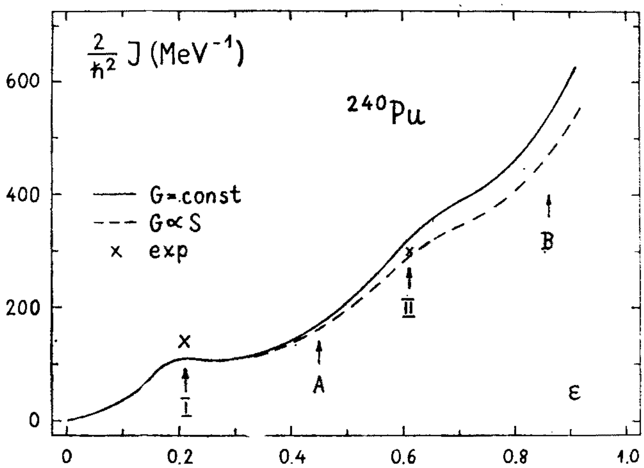

Fig. 3. Dependence of the moment of inertia on deformation for ^{240}Pu

with respect to the theoretical curve suggests that the theoretical values increase too fast with the deformation. This may be partly due to the non-local term, $-\mu(l^2-\langle l^2\rangle_N)$, in the Nilsson potential. This term results in the fact that the moment of inertia, calculated with the help of the Nilsson potential in the limit of non-interacting particles and of no shell effects, is higher than that of the rigid body $\mathscr{J}_{\text{rig}}$ by about 30% [18, 19].

The dependence of $\mathscr{J}$ on the hexadecapole deformation ε_4 is illustrated in Fig. 4, again for ^{240}Pu. The values are calculated for the quadrupole deformation $\varepsilon = 0.61$, corresponding to the second minimum. In comparison to I, the dependence is closer to that obtained in the rigid-body approximation. This is due to accounting for the coupling between all the oscillator shells. The pure effect of this coupling is illustrated in Fig. 4 for the $G \propto S$ case. We can see that starting from $\varepsilon_4 = 0$, for which there is no coupling, the effect increases with increasing ε_4; slowly at the beginning and then more rapidly. At $\varepsilon_4 \approx 0.07$, corresponding to the second minimum, the effect is about 4%, i. e. rather small.

Fig. 5 gives the dependence of the collective gyromagnetic ratio g_R on the deformation. The dependence is calculated along the static fission trejectory L, the same as used in Fig. 3. It is close to that found in Ref. [20] for $\varepsilon_4 = 0$. Rather large shell effects are seen. We can also see that the ratio $\mathscr{J}_p/\mathscr{J}$, where $\mathscr{J}_p$ is the contribution of protons to the total moment of inertia $\mathscr{J}$, is a good approximation to g_R. On the average, g_R slowly increases with increasing deformation.

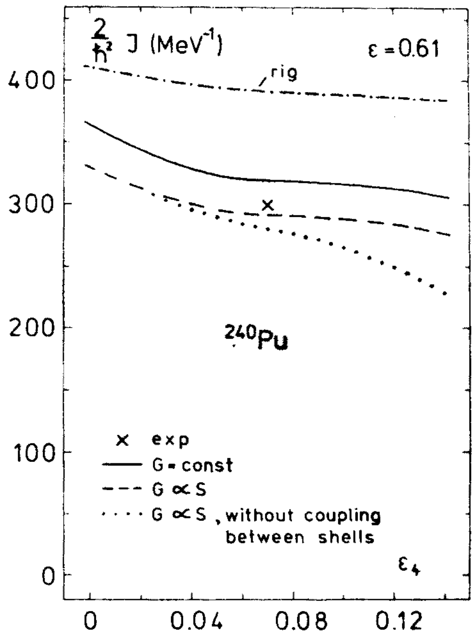

Fig. 4. Dependence of the moment of inertia on the hexadecapole deformation ε_4. Values calculated without coupling between the oscillator shells as well as those obtained in the rigid-body approximation is shown for comparison

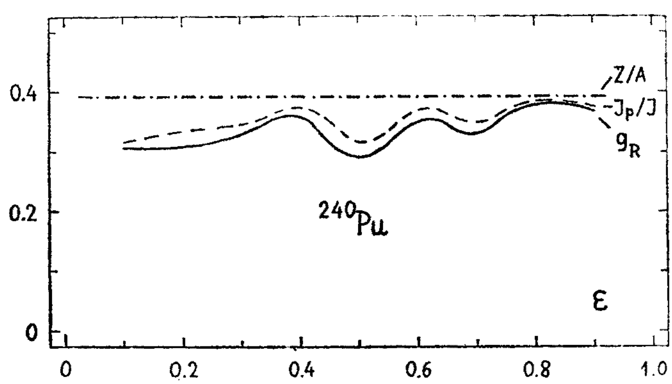

Fig. 5. Dependence of the collective gyromagnetic ratio g_R on the deformation. The ratios $\mathscr{J}_p/\mathscr{J}$ and Z/A are shown for comparison

3.2. Effect of coupling between the oscillator shells

Mixing between the shells is due to the presence of the hexadecapole component ε_4 in the deformation. As already seen in Fig. 4, for the example of the moment of inertia $\mathscr{J}$, the effect of mixing increases with the increase of this component.

For the ground-state deformations, the effect of mixing on the energy gaps $2\Delta_p$ and $2\Delta_n$ is smaller than 1% and it is about 3% on the moment of inertia $\mathscr{J}$.

For the deformations of the second minimum, the effect is smaller than 2% for $2\Delta_p$ and $2\Delta_n$, and it is about 4% for $\mathcal{J}$, for all nuclei considered.

Thus, for both minima, the effect is rather small.

4. *Conclusions*

Summarizing the present research, we can say the following:

1) The theoretical values of both the moments of inertia and the quadrupole moments calculated for the fission isomers are close to the experimental ones. However, the experimental data are too scanty and/or not precise enough to draw conclusions on the accuracy of the theoretical description of these quantities. The theoretical moments of inertia, which are about right (close to experiment) for the isomeric state and are too small for the ground state, seem to increase too fast with increasing deformation.

2) Coupling between different oscillator shells only slightly increases the moment of inertia. The increase is about 3% for the ground state and about 4% for the fission-isomeric state. The coupling affects the pairing energy gap by less than 1–2%.

3) A small change in the pairing interaction, leaving about the same or slightly improving the agreement between the energy gap 2Δ and the experimental odd-even mass difference $2P$ for the ground state, allows the moment of inertia to increase by another 3–4%. Together with the effect of the coupling between the shells, stated above, it results in an 6–7% increase of the calculated moment of inertia in both the ground and the isomeric states.

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