LETTERS TO THE EDITOR

ON THE SUPPRESSIONS OF THE DECAYS $\psi \to \pi^+\pi^-$, K^+K^- AND $\psi' \to \pi^+\pi^-$, K^+K^-

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As $\psi(3095)$ and $\psi'(3684)$ can couple to the photon (as evidenced from the production of the particles concerned in the e^+e^--annihilation experiments), therefore, the decays $\psi, \psi' \to \pi^+\pi^-$, K^+K^- are theoretically expected to occur but these decays have not been experimentally observed. In this note an explanation for the suppressions of the decays concerned has been given in terms of a dimensionality-based selection rule introduced in a previous paper.

Assuming $\psi(3095)$ and $\psi'(3684)$ to be hadrons, the isospin invariant coupling between ψ or ψ' and the pions is only possible for odd number of pions. However, both ψ and ψ' are produced in the e^+e^--annihilation experiments suggesting that these particles can couple to the photon and as such they can suffer indirect hadronic decays (via one photon intermediate states) into an even number of pions. Therefore, the decays $\psi(3095) \to \pi^+\pi^-$, K^+K^-, $2\pi^+2\pi^-$, $3\pi^+3\pi^-$, ... and $\psi'(3684) \to \pi^+\pi^-$, K^+K^-, $2\pi^+2\pi^-$, $3\pi^+3\pi^-$, ... are theoretically possible. It may be recalled that the decays $\psi(3095) \to 2\pi^+2\pi^-$, $3\pi^+3\pi^-$ have been observed [1] but, surprisingly enough, the decays $\psi(3095) \to \pi^+\pi^-$, K^+K^- have not been seen [1]. Needless to mention, the phase space restrictions are much less serious for the $\pi^+\pi^-$, K^+K^- modes compared to the 4π and 6π modes of the particle concerned. No less surprising is the non-observation [1] of the decays $\psi'(3684) \to \pi^+\pi^-$, K^+K^-, $2\pi^+2\pi^-$, $3\pi^+3\pi^-$. This is so because, as we have already noted, both ψ and ψ' can couple to the photon implying that the indirect hadronic decays should be similar for these particles [1]. Obviously, we cannot rule out the decays $\psi'(3684) \to \pi^+\pi^-$, K^+K^-, $2\pi^+2\pi^-$, $3\pi^+3\pi^-$ by arguing that the decay of $\psi'(3684)$ is purely strong (for which the G-parity conservation can be demanded) unlike the decay of $\psi(3095)$ for which the value of the coupling constant is different from the value of the same for typical strong decays. This is so because an unstable particle,

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if it is a resonance, can have, in principle at least, the electromagnetic and weak decay channels apart from the strong ones. In fact, the non-observations [1] of the decays $\psi(3095) \to \pi^+\pi^-$, K^+K^- and $\psi'(3684) \to \pi^+\pi^-$, K^+K^- are indeed baffling because the phase space restrictions are much less serious compared to the higher modes of the particles concerned. Needless to mention, the suppressions of the decays concerned cannot be understood with the help of the conventional selection rules. In this note we offer an explanation for the suppressions of the decays ψ, $\psi' \to \pi^+\pi^-$, K^+K^- in terms of the dimensionality-based selection rule introduced in an earlier paper [2].

In the earlier paper [2] we have examined the feasibility of obtaining a new selection rule for particle decays by considering the dimensions of the fields involved in decay processes. The problem of obtaining a new selection rule for particle decays in terms of the dimensions of the fields involved may be much simplified by avoiding the use of "anomalous dimensions" [3] by exploiting Gell-Mann's philosophy that Nature "reads the free-field theory books" and, accordingly treating the fields involved in decay processes as free fields. Obviously, Gell-Mann's "free-field" criterion enables us to use the canonical dimensions of the fields of the particles taking part in decay processes. Unfortunately, however, the considerations of the canonical dimensions of the fields of the particles involved in decay processes do not lead to a selection rule of practical utility [2]. This difficulty was bypassed in the previous paper [2] by assigning a pseudo-dimension d to a free field carrying the spin S; the pseudo-dimension d was defined [2] by the following relation

$$d = -KS, \tag{1}$$

where K is a positive odd integer. In order that d can be looked upon as a pseudo-dimension, it has to possess some (but not all) of the properties of the canonical dimensions. This is achieved by the negative sign in Eq. (1) and by a positive odd integral value of K. Obviously, an odd integral value of K implies, through Eq. (1), that the fermion fields will have odd-half integral d values and the boson fields integral d values like the canonical dimensions of the fields concerned. It may be noted that the signs of canonical dimensions are arbitrary and the negative signs often used in the literature are simply due to convention (stemming out of the particular choice of the metric). The negative sign in Eq. (1), however, suggests that it is included in the definition of the pseudo-dimension and it can be used to assign a non-zero value of d to a spin-zero field. Needless to mention, a field cannot be a dimensionless quantity. A similar remark is also true for the pseudo-dimension of a field. Using the properties of negative integers (for assigning a non-zero value of d to a spin-zero field) and also the fact that the photon is a class by itself as it is distinctly different from all other (massive) vector bosons (due to its well known peculiarities such as, for example, two states of polarization), the following relations have been obtained in the previous paper [2].

$$d\,(\text{magnitude}) = 3S, \quad S \neq 0, \tag{2a}$$

$$d\,(\text{magnitude}) = 1, \quad S = 0, \tag{2b}$$

$$d\,(\text{magnitude}) = 2 \quad (\text{for a photon}). \tag{2c}$$

It may be emphasized that Eqs (2a)—(2c) give the magnitudes of the pseudo-dimensions of *free* fields in terms of which the following selection rule was suggested in the earlier paper [2]: All the allowed decays (not occurring through subreactions) of an unstable particle must be governed by *one and only one* of the two constraints

$$d_u \geqslant D, \tag{3a}$$

$$d_u \leqslant D, \tag{3b}$$

where d_u is the magnitude of the pseudo-dimension of the field of the unstable particle and D is the sum of the magnitudes of the pseudo-dimensions of the fields of the particles constituting a decay mode (not occurring through a subreaction). For a given unstable particle d_u is fixed whereas D can take, in general, a finite spectrum of discrete values by virtue of Eqs (2a)—(2c) corresponding to a finite number of allowed decay modes. It may be emphasized that the D values appearing in relations (3a) and (3b) refer to the decay modes *not* occurring through subreactions which involve final state interactions necessitating interacting fields whereas Eqs (2a)—(2c) refer to free fields for which, even in principle, subreactions cannot occur. In reality, however, the fields involved in decay processes can at best enjoy the "asymptotic freedom" and as such the constraints, given by relations (3a) and (3b), are expected to be reliable to the extent the free-field approximation is justified. In the previous paper [2] it has been demonstrated that the free-field approximation is indeed very reliable.

To exhibit how the selection rule discussed above reduces the number of theoretically not forbidden but experimentally unseen decay modes of unstable particles, we first consider the decay of $\omega(784)$. The observed decays [4] $\omega(d_u = 3) \to 2\pi(D = 2)$, $3\pi(D = 3)$, $\pi^0\gamma(D = 3)$, $e^+e^-(D = 3)$ indicate that the ω-decay is governed by the constraint $d_u \geqslant D$ since $d_u = 3$, which follows from Eq. (2a), for the decaying particle ω which is a spin-one particle and the 2π mode $D = d_\pi + d_\pi = 1+1 = 2$ as $d_\pi = 1$ given by Eq. (2b). For the $\pi^0\gamma$ mode, $D = d_\pi + d_\gamma = 1+2 = 3$ since $d_\gamma = 2$ from Eq. (2c) and for the e^+e^- mode $D = d_{e+} + d_{e-} = 3/2+3/2 = 3$ as $d = 3/2$ for a spin-$\frac{1}{2}$ particle as evident from Eq. (2a). As stated earlier, the selection rule under considerations implies that one and the same constraint must be valid for all the allowed decays (not occurring through subreactions) for a given unstable particle. Since the constraint $d_u \geqslant D$ is found to be valid in the observed decays of ω, therefore, the same constraint must be satisfied by all other allowed decays of the particle concerned. It is interesting to note that for the decay of ω the theoretically allowed decay modes $\pi^0\mu^+\mu^-(D = 4)$, $\pi^+\pi^-\gamma$ $(D = 4)$, $3\gamma(D = 6)$ clearly fail to satisfy the constraint concerned as $d_u = 3$ for ω and, needless to mention, these modes have not been observed [4]. As another example, we consider the decay of $A_3(1640)$ for which the appropriate constraint is $d_u < D$ as evident from the observed decay [4] $A_3(d_u = 6) \to \to f\pi(D = 6+1 = 7)$. Obviously, the constraint $d_u < D$, valid for the A_3-decay, rules out the theoretically not forbidden but unobserved [4] decay modes $\eta\pi(D = 2)$, $K\bar{K}(D = 2)$, $3\pi(D = 3)$, $K\bar{K}\pi(D = 3)$, $\varrho\pi(D = 4)$, $KK^*(D = 4)$, $5\pi(D = 5)$, $\omega\pi\pi(D = 5)$. It may be emphasized here that we cannot throw away by hands the unseen decay modes of A_3 simply because they are not the parity or the G-parity conserving modes. Needless to mention, the phrase "theoretically allowed decay modes" do not necessarily imply strong decay modes (for the reasons already discussed). A look into Section 3 of Ref. [2] will reveal the impressive success of the selection rule under investigations in reducing the number of theoretically allowed but unseen decay modes of unstable particles.

We now examine the feasibility of the occurrence of the $\pi^+\pi^-$ and K^+K^- modes in the decays of $\psi(3095)$ and $\psi'(3684)$ in the light of the selection rule under considerations. The observed decays [1, 4] $\psi(3095, d_u = 3) \rightarrow e^+e^-(D = 3)$, $\mu^+\mu^-(D = 3)$, $4\pi(D = 4)$, $5\pi(D = 5)$, $6\pi(D = 6)$, $7\pi(D = 7)$ and $\psi'(3684, d_u = 3) \rightarrow e^+e^-(D = 3)$, $\mu^+\mu^-(D = 3)$, $\psi(3095)\eta(D = 4)$, $\psi(3095)\pi\pi(D = 5)$ indicate that the appropriate constraint for the decays of both ψ and ψ' is $d_u \leqslant D$ according to which the modes $\pi^+\pi^-(D = 2)$, $K^+K^-(D = 2)$ are forbidden as these modes cannot satisfy the constraint concerned as $d_u = 3$ for both ψ and ψ' (which are spin-one particles). Needless to mention, the $\pi^+\pi^-$ and K^+K^- modes have been searched for [1] but not seen in the decays of ψ and ψ'. It may not be out of place if we mention that $\psi'(3684)$ is very often compared with $\varrho'(1600)$ the decay of which is controlled by the constraint $d_u < D$ as suggested by the observed decay $\varrho'(d_u = 3) \rightarrow$ $\rightarrow 4\pi(D = 4)$. It is interesting to note that the decays $\varrho' \rightarrow \pi^+\pi^-(D = 2)$, $K^+K^-(D = 2)$ are forbidden according to the constraint governing the ϱ'-decay and as such the $\pi^+\pi^-$, K^+K^- modes must be suppressed relative to the 4π mode. It is well known that a forbidden mode is suppressed (or slowed down) compared to the allowed mode(s). It may be recalled that the $\Delta T = \frac{1}{2}$ rule is not satisfied in the decay $K^+ \rightarrow \pi^+\pi^0$ and, as expected, this forbidden decay is much slow compared to the allowed decay $K^0 \rightarrow \pi^+\pi^-$ [5]. From what has been said so far we can conclude that the $\pi^+\pi^-$ and K^+K^- modes are suppressed in the decays of ψ, ψ' and ϱ' as the modes concerned fail to satisfy the appropriate constraints governing the decays of these particles.

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