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# LEPTON-HADRON RELATION AND pp SCATTERING AT 90°

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The existence of a sharp break in the pp elastic differential cross section is explained here on the basis of the structural aspects of proton. The differential cross section for pp scattering at 90° is found to be in excellent agreement with the theoretical predictions made on the basis of a model of hadrons based on lepton-hadron relation satisfying duality.

An experiment by Akerlof et al. [1] has shown that there exist a sharp break in the pp elastic differential cross-section. It was conjectured by Islam and Rosen [2] that the break might indicate the existence of two different inner regions of the proton. There were also attempts to explain this break on the basis of the singularities in the complex angular momentum plane [3]. However Islam and Rosen [2] have attempted to correlate this break with the structural characteristics of proton. In fact it was pointed out from optical model considerations that the nucleon appears to consist of a number of hadronic density distributions of increasing mean square radii and that the inner distributions which are associated with heavier quanta dominate the large momentum transfer scattering. On the basis of these results, Islam and Rosen attempted to fit the pp scattering differential cross section at 90° and laboratory momentum 5–13 GeV/$c$ in terms of two optical potentials and DWBA of the corresponding amplitudes. However, as the nature of these two hadronic density distributions were unknown, they had to depend on fitting various parameters occurring there. In the present note, we shall try to explain the break observed in the pp

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scattering data on the basis of the configurational characteristics of a proton as indicated by a model of hadron based on a certain lepton-hadron relation developed recently by one of the authors [4].

This model of hadrons envisages five constituents for the structure of a nucleon and the configuration of a proton contains one charged constituent $\mu^+$ and all other constituents are neutral [4]. The configuration of a neutron is taken to be a mixture of the following two configurations (*i*) where all the constituents are neutral, and (*ii*) where two of them have charges $+1$ and $-1$ and the remaining three are neutral. In the previous paper [5] we have shown that this five-parton model for nucleons has the specific characteristics that the structure function for the ep deep inelastic scattering is contributed by the resonances and there is no contribution from the meson cloud. Thus scaling here is found to be a manifestation of resonances as suggested by Bloom and Gilman and is in conformity with the concept of duality. Thus this leptonic theory of hadrons may be taken to represent a dual parton model of hadrons.

In this model of strong interaction, it is considered that baryonic resonance occurs when a constituent pion is excited to the level of a $\varrho$-meson. Also it has been shown that this model can nicely interprete the occurrence or nonoccurrence of dips in the differential cross-sections of various processes in consistency with the experimental results where it is taken that dip is a resonance effect. This also follows from the requirement of duality that amplitude at high energy is contributed by the resonances at low energy. On the basis of this model, we have quantitatively analysed the process like

$$\gamma N \to \pi N \qquad \gamma N \to N\varrho \qquad \pi N \to \pi N$$

$$\pi N \to N\varrho \qquad \pi N \to N\omega$$

and the differential cross-sections are found to be in excellent agreement with experiments [6]. On the basis of this model of hadrons, the configuration of a proton is given by $p = (\mu^+\nu_\mu\nu_\mu\nu_\mu\nu_\mu) = (\pi^+\pi^0\nu_\mu)$. The unbound lepton $\nu_\mu$ cannot take part in strong interaction. Hence we can formulate the pp elastic scattering in terms of the elastic scattering of $\pi^+\pi^0 \to \pi^+\pi^0$ and $\pi^0\pi^0 \to \pi^0\pi^0$ where the two interacting pions are within the structure of the two protons. It should be noted that in pp scattering amplitude the contribution of the $\pi^+\pi^0 \to \pi^+\pi^0$ scattering should be multiplied by 2 as in one case the $\pi^+$ in the incident proton interacts with the $\pi^0$ in the target proton and in the other case, the $\pi^0$ in the incident proton interacts with the $\pi^+$ in the target proton. The scattering of $\pi^+\pi^+ \to \pi^+\pi^+$ does not contribute as demanded by duality. In fact, the absence of any resonance in this channel suggests that the t-channel amplitude contributed by $\varrho$-meson should be cancelled by the contribution of another exchanged particle presumably by $f$-meson indicating the absence of any exchange force. This in Regge term is due to the exchange degeneracy of $\varrho$- and $f$-meson. Also we note that the scattering $\pi^+\pi^0 \to \pi^+\pi^0$ occurs through $\varrho$-exchange. But selection rule forbids the $\varrho^0\pi^0\pi^0$ vertex hence the $\pi^+\pi^0 \to \pi^+\pi^0$ amplitude is not contributed by $\varrho$-exchange in $t$-channel, but the same is possible in the $u$-channel. In fact in this special case duality requires that contribution of $u$-channel exchanges is equivalent to the contribution of $s$-channel resonances. Since $\pi^+\pi^0$ meson gives rise to $\varrho$-meson resonance at low energy, $u$-channel exchange is dominated by $\varrho$-exchange. However the $\pi^0\pi^0 \to \pi^0\pi^0$ must be contributed by $f$-meson as the selection rule forbids the $\varrho^0\pi^0\pi^0$ vertex.

Thus pp elastic scattering, according to our analysis above, can be depicted as the direct interaction between the pion clusters (hadronic distribution), within each interacting protons. There are obviously two such distributions in each proton — one is the $\pi^+$ cluster and the other is the $\pi^0$ cluster. The unbound lepton is a spectator in the whole affair. Again $\varrho$-meson exchange is related with the interaction between $\pi^+$ cluster of one proton and $\pi^0$ cluster of the other and vice versa and $f$-meson is related with the interaction between the two $\pi^0$ clusters in the two interacting protons.

Islam and Rosen [2] studied the pp elastic scattering in terms of two hadronic density distribution within each proton but could not specify them. We, on the basis of our model, have specified it unambiguously. But otherwise our treatment is quite in line with that of Islam and Rosen. Following them we use the relativistic eikonal description and assume that in the energy region of our interest, three different interactions are the most important. Of the three potentials, one is responsible for diffiaction scattering. The other two we take to be of the form

$$V_1(s, r) = \frac{g_{\varrho\pi\pi}(s)\exp\left[-\mu_\varrho(r^2+\beta^2)^{1/2}\right]}{(r^2+\beta^2)^{1/2}}, \tag{1}$$

$$V_2(s, r) = \frac{g_{f\pi\pi}(s)\exp\left[-\mu_f(r^2+\beta^2)^{1/2}\right]}{(r^2+\beta^2)^{1/2}}. \tag{2}$$

The radial dependence of this potential can be understood as due to the two pion interaction, where the pions concerned are in the structure of the two protons through a Yukawa potential $e^{-\mu r}/r$ where finite size is determined by the parameter $\beta$. This parameter $\beta$ is determined by considering that the effective mass of a pion in the structure of a proton is given by $m_p/2$ as the unbound lepton ($\nu_\mu$) is taken to be massless. This gives $\beta = 0.42$ fm. Here we point out that the lepton must have some kinetic energy, but compared to the mass of the pions and their share of the kinetic energy this can be neglected. This is, of course, an approximation; but, indeed, is well within the tolerance limit.

The scattering amplitude for all energies and physical angles is represented by

$$f(s, \Delta) = ik\int_0^\infty b\,db\,J_0(b\Delta)\left[1-e^{2i\delta(s,b)}\right], \tag{3}$$

where $k$ is the c. m. momentum, $\Delta = 2k\sin\theta/2$ is the momentum transfer, $s$ is the square of c. m. energy. The phase shift function $\delta(s, b)$ is related to the optical potential $V(s, r)$:

$$\delta(s, b) = -\frac{1}{2k}\int_b^\infty \frac{V(s, r)r\,dr}{(r^2-b^2)^{1/2}}. \tag{4}$$

If $\delta_0(s, b)$ represents the contribution of the diffraction potential to the phase shift function, then we can write

$$\delta(s, b) = \delta_0(s, b)+\delta_1(s, b)+\delta_2(s, b), \tag{5}$$

where

$$\delta_i(s, b) = -\frac{1}{2k}\int_0^\infty V_i[s, (b^2+z^2)^{1/2}]dz$$

$$= -\frac{g_i(s)}{2k} K_0[\mu_i(b^2+\beta_i^2)^{1/2}], \tag{6}$$

where $i = 1, 2$ represents the contribution due to $\varrho$- and $f$-meson exchanges respectively.

The diffraction term $\delta_0(s, b)$ is independent of $s$ for large values of $s$. Further, if $\mu_i\beta_i$ is much larger than unity, then because of the sharp fall of the modified Bessel function $K_0(z)$ both $\delta_1(s, b)$ and $\delta_2(s, b)$ are going to be small quantities. Treating $\delta_1(s, b)$ and $\delta_2(s, b)$ small, we get

$$f(s, \Delta) = ik \int_0^\infty bdbJ_0(b\Delta)\,[1-e^{2i\delta_0(b)}]$$

$$-g_{\varrho\pi\pi}(s) \int_0^\infty bdbJ_0(b\Delta)e^{2i\delta_0(b)}K_0[\mu_\varrho(b^2+\beta^2)^{1/2}]$$

$$-g_{f\pi\pi}(s) \int_0^\infty bdbJ_0(b\Delta)e^{2i\delta_0(b)}K_0[\mu_f\,(b^2+\beta^2)^{1/2}]. \tag{7}$$

The first term corresponds to the diffraction amplitude, which gives the exponential elastic diffraction peak for small momentum transfer. The second and the third terms correspond to distorted wave Born approximation (DBWA) of the amplitude due to potentials $V_\varrho(s, r)$ and $V_f(s, r)$ respectively. The factoı exp $[2i\delta_0(b)]$ occurring in these terms represents the absorptive correction coming from diffraction scattering.

For the diffraction amplitude, we now use the parametrisation that $\delta_0(b)$ is completely imaginary, and

$$1-e^{2i\delta_0(b)} = \frac{\sigma_T^d}{\pi R^2}\,e^{-2b^2/R}, \tag{8}$$

where $\sigma_T^d$ is the total cross section due to diffraction scattering and $R$ is the optical model radius. Inserting this in Eq. (7) we have

$$f(s, \Delta) = \frac{ik\sigma_T^d}{4\pi}\exp\left(-\tfrac{1}{8}R^2\Delta^2\right)$$

$$-g_{\varrho\pi\pi}(s)\Bigg\{\beta K_1[\beta(\Delta^2+\mu_\varrho^2)^{1/2}]\,(\Delta^2+\mu_\varrho^2)^{-1/2}$$

$$+\frac{\sigma_T^d}{\pi R^2}\int_0^\infty bdbJ_0(b\Delta)\exp\left(-\frac{2b^2}{R^2}\right)K_0[\mu_\varrho(b^2+\beta^2)^{1/2}]\Bigg\}$$

$$- g_{f\pi\pi}(s)\left\{\beta K_1[\beta(\Delta^2+\mu_f^2)^{1/2}](\Delta^2+\mu_f^2)^{-1/2}\right.$$

$$\left. + \frac{\sigma_T^d}{\pi R^2}\int_0^\infty bdbJ_0(b\Delta)\exp\left(-\frac{2b^2}{R^2}\right)K_0[\mu_f(b^2+\beta^2)^{1/2}]\right\}. \tag{9}$$

The elastic diffraction cross section is given by

$$\frac{d\sigma}{dt} = \frac{\pi}{k^2}\frac{d\sigma}{d\Omega} = \frac{\pi}{k^2}|f(s, \Delta)|^2. \tag{10}$$

Excepting $R$ we have the parameters known here. Also $R$ can be approximately estimated from other considerations and we take $R = 1.1$ fm [2]. From experimental information we take $\sigma_T^d = 36.2$ mb. Also we have $g_{\varrho\pi\pi}^2/4\pi = 2.5 \cdot g_{f\pi\pi}^2/4\pi$ can be estimated from the observed decay with $\Gamma_{tot} = 154+25$ with the branching ratio $\Gamma_{f\to\pi^+\pi^-}/\Gamma_{tot} \simeq 0.53$ and is found to be $\simeq 36$ [7]. For convenience, the relevant parameters are tabulated below.

Values of the parameters

| | $\mu$ (GeV) | $\beta$ (fm) | $g^2/4\pi$ | $\sigma_T^d$ (mb) | $R$ (fm) |
|---|---|---|---|---|---|
| $\varrho^0$ | 0.76 | 0.42 | 2.5 | 36.2 | 1.1 |
| $f^0$ | 1.25 | | 36 | | |

Now we come to the point of energy dependence. In our model of strong interactions we have taken that the energy dependence is related with the number of constituents rearranged in the scattering process. For planar and nonplanar diagrams, the rearrangement amplitude has phase factors exp $[2i\pi\gamma]$ and 1 respectively, where $\gamma$ is a suitable constant [8].The rearrangement of constituents involved in these diagrams contributes to the amplitude the following factors.

$$T(s, t) \text{ (planar)} \sim [(p_a+p_b)^2]^{-\gamma}[(p_c+p_d)^2]^{-\gamma} \sim s^{-2\gamma}$$

$$T(s, t) \text{ (nonplanar)} \sim [(p_a+p_d)^2]^{-\gamma}[(p_b+p_c)^2]^{-\gamma} \sim (-u)^{2\gamma} \xrightarrow[s\text{ large}]{} s^{-2\gamma} \tag{11}$$

The factor $[(p_i+p_j)^2]^\gamma$ corresponds to the rearrangement of a constituent from a hadron with momentum $p_i$ to that with $p_j$ and $\gamma$ is suitable constant. It may be relevant here to say that there is a correspondence between $T(s, t)$ and the Regge amplitude $T_{\text{Regge}}$ with strongly degenerate trajectories $\alpha(t)$ and residue $\beta(t)$ in the forward regions if we take $-2\gamma+1 = \alpha(t)$. Now approximately we take here that the energy dependence of the scattering $\pi\pi \to \pi\pi$ via $\varrho$-exchange can be ignored in the energy region of our interest whereas the $\pi^0\pi^0 \to \pi^0\pi^0$ scattering via the tensor meson $f^0$-exchange has the energy dependence of the form

$$\frac{g_{f\pi\pi}^2(s)}{4\pi} \approx \frac{g_{f\pi\pi}^2}{4\pi}\cdot\frac{1}{s^2} \tag{12}$$

Considering these aspects, we now calculate the differential cross section from Eqs (9) and (10). We find the diffraction contribution completely negligible. The contribution of the scattering $\pi^+\pi^0 \rightarrow \pi^+\pi^0$ via $\varrho$-exchange dominates up to the point of break where the contribution of the $\pi^0\pi^0 \rightarrow \pi^0\pi^0$ scattering via $f$-meson dominates and again beyond this point of break, the scattering $\pi^+\pi^0 \rightarrow \pi^+\pi^0$ via $\varrho$-meson dominates. This is explicitly shown in Fig. 1, where the elastic differential cross section ($d\sigma/dt$) at the fixed angle 90°

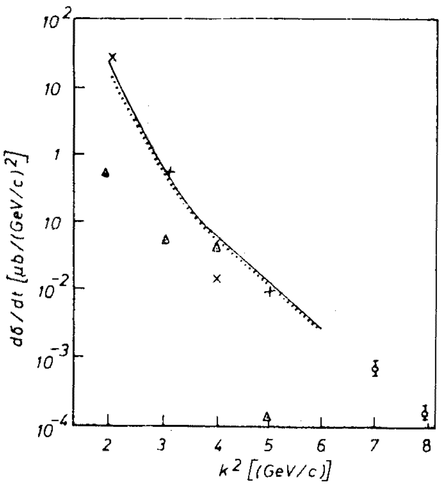


Fig. 1. The differential cross section of pp scattering at 90°. Experimental data are shown by the dots and the predicted cross section is shown by the solid curve

is plotted against the square of the momentum transfer ($k^2$). The asterisk marks represent the contribution of the $\pi^+\pi^0 \rightarrow \pi^+\pi^0$ scattering via $\varrho$-exchange and the triangle marks represent the contribution of the $\pi^0\pi^0 \rightarrow \pi^0\pi^0$ scattering via $f$-exchange. The experimental data are shown by dots and the calculated differential cross section is shown by the solid curve. The results are found to be in excellent agreement with experiments. The interference of the $\pi^0\pi^0 \rightarrow \pi^0\pi^0$ interaction via $f$-meson exchange is found to be strongest near the experimental break and beyond this point, the contribution of this interaction becomes negligible.

The excellent fitting of the calculated differential cross section with the experimental data together with the explanation of the "break" observed in the differential cross section on the basis of this model of hadrons can be taken to provide an indirect test of the model. This is also found to be in conformity with the conjecture by Islam and Rosen that this break might indicate the existence of two different inner regions of the proton.

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