THE x-DEPENDENCE OF THE TRANSVERSE MOMENTUM OF PARTONS*

BY J. A. BARTELSKI**

III. Physikalisches Institut, Technische Hochschule, Aachen***

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We present a naive quark parton model for partons with an arbitrary transverse momentum. In our model, the average value of this momentum for nucleon constituents which contribute to the deep inelastic lepton-nucleon scattering varies with the variable x and is proportional to the difference of the structure functions, i.e. $\langle k_{\perp}^2(x)\rangle \sim F_2^p(x)-F_2^n(x)$.

The distiibution of the transverse momenta of hadron constituents is a very important problem because it has a definite influence on the results of many experiments. The possible dependence of $k_{\perp}$[1] on the variable x was explored in a series of theoretical papers [1–3].

In this paper, we want to discuss the connection between the average value of $k_{\perp}$ and the variable x for partons which are active in deep inelastic lepton-nucleon scattering (DIS). We will study this subject within the framework of the quark parton model [4, 5], with partons on the mass-shell.

Let us start with the N-parton momentum distribution function $G_N(k_1^\mu, k_2^\mu, \ldots, \ldots, k_N^\mu; P^\mu) \prod_{i=1}^{N} d^4k_i\delta(k_i^2-m_i^2)$, where k_i and m_i are respectively the four-momenta and the masses of the partons, and P is the four-momentum of the nucleon.

We introduce for each parton boost invariant, non-negative and dimensionless variables ξ [6] and η defined by

$$\xi = \frac{k^+}{P^+}, \quad \eta = \frac{k^-}{P^-}, \tag{1}$$

where $k^{\pm} = \frac{1}{\sqrt{2}}(k^0 \pm k^3)$ and $P^{\pm} = \frac{1}{\sqrt{2}}(P^0 \pm P^3)$. In the infinite momentum frame

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** On leave of absence from the Institute of Theoretical Physics, Warsaw University, Poland.

*** Address: III. Physikalisches Institut, Technische Hochschule, Physikzentrum, 5100 Aachen, W. Germany.

[1] By $k_{\perp}$ we mean the parton momentum perpendicular to the nucleon-photon axis.

(IMF: $P^3 \to \infty$), the light-cone momentum fraction ξ is approximately equal to the longitudinal momentum fraction x_L.

The element of integration $d^4k\delta(k^2-m^2)$ becomes, in terms of ξ and η,

$$d^4k\delta(k^2-m^2) = \tfrac{1}{4} M^2 d\xi d\eta dk_\perp^2 d\phi\delta(k_\perp^2 - M^2\xi\eta + m^2), \tag{2}$$

where $k^1 = k_\perp \cos\phi$, $k^2 = k_\perp \sin\phi$ and where M is the nucleon mass.

We thus obtain the following relation from the delta function (Eq. (2)) for constituents on the mass-shell

$$k_\perp^2 = M^2\xi\eta - m^2. \tag{3}$$

The condition that partons carry the whole longitudinal proton momentum, i.e. $\sum_{i=1}^{N} k_i^3 = P^3$, gives in the IMF: $\sum_{i=1}^{N} \xi_i = 1$. From this relation and the positivity of $k_\perp^2$ we obtain

$$0 \leqslant \xi \leqslant 1, \tag{4a}$$

$$\frac{m^2}{M^2\xi} \leqslant \eta < \infty. \tag{4b}$$

In order to calculate the deep inelastic electromagnetic structure functions for partons, we consider the following vertex: quark (k) — photon (q) — quark (k'). Four-momentum conservation in this vertex: $(q+k)^2 = k'^2$ and the assumption that both quarks are on their mass-shells gives us a relation between the two variables η and ξ:

$$\eta = \eta(\xi; x, Q^2) = \frac{Q^2}{M^2}\,\frac{\xi - x}{x^2}, \tag{5}$$

where Q^2 and $x = 1/\omega$ are the variables of the virtual photon defined in the usual way. Thus for a parton interacting with such a photon we obtain (inserting (5) into (3))

$$k_\perp^2 = Q^2\,\frac{\xi(\xi-x)}{x^2} = Q^2\omega\xi(\omega\xi-1). \tag{6}$$

Since $k_\perp^2$ is non-negative, we must have $\xi \geqslant x$. Then, for a given x and Q^2 the relations (4) have to be replaced by

$$x \leqslant \xi \leqslant 1, \tag{7a}$$

$$0 \leqslant \eta \leqslant \frac{Q^2}{M^2}\,\frac{1-x}{x^2}, \tag{7b}$$

and hence the transverse momentum must satisfy

$$0 \leqslant k_\perp^2 \leqslant Q^2\omega(\omega-1). \tag{8}$$

In all those formulae ((5)–(8)) we keep the leading terms only, i.e.we omit terms proportional to (mass)^2 in comparison to Q^2 terms. From Eq. (8), one gets a kinematical cut-off for the transverse momenta of partons active in DIS.

The following picture comes out from the above inequalities. A virtual photon described by x and Q^2 interacts not only with those partons which have a light-cone momentum fraction $\xi = x$ but also with all of those which have a ξ between x (corresponding to $k_\perp^2 = 0$) and 1 (corresponding to $k_\perp^2 = Q^2(1-x)/x^2$). So, in DIS, we can probe partons with a big $k_\perp^2$ in the case when x is small or (and) when Q^2 is big. This second result obtained here from the naive on-mass-shell parton kinematics, justifies the intuitive assumption of Kogut and Susskind [7] in their asymptotically free parton model.

Let us now calculate the deep inelastic electromagnetic structure function for spin-$\frac{1}{2}$ quarks. We obtain the formula

$$f_2(\xi, \eta; x, Q^2) = e^2 \frac{x^2+6\xi(\xi-x)}{\xi} \delta\left(\xi-x-\eta x^2 \frac{M^2}{Q^2}\right), \tag{9}$$

where e is the electric charge of the parton.

In order to calculate the structure functions for the nucleon, we must specify the function G_N which depends on the variables ξ, η, $k_\perp^2$ and ϕ (Eq. (2)) of each parton. The integration over $k_\perp^2$ can be easily done because of the delta function. The ϕ-integration is also straightforward since we assume that G_N does not depend on this variable (transverse momentum correlations are omitted for simplicity).

The boost invariant variables ξ and η are not rotation invariant, although their sum is, since it is proportional to a Lorentz scalar $(k \cdot P)$, i.e.

$$\xi+\eta = \frac{2}{M^2}(k \cdot P). \tag{10}$$

In the rest frame of the nucleon, it is: $\xi+\eta = 2\varepsilon/M$ with ε as the parton energy in this frame. Now assume for $G_N(\xi_1, \eta_1, \ldots, \xi_N, \eta_N)$ the following "statistical" form (we wish to have a function of Lorentz scalars only)

$$G_N(\xi_1, \eta_1, \ldots, \xi_N, \eta_N) \sim \exp\left[-\gamma \sum_{i=1}^{N} (\xi_i+\eta_i)\right] = \exp\left[-\beta \sum_{i=1}^{N} \varepsilon_i\right]. \tag{11}$$

The magnitude of γ can now be determined since $\beta \simeq \langle\varepsilon\rangle^{-1}$ for statistical models and hence, $\gamma = \frac{1}{2} M\beta \simeq M/2\langle\varepsilon\rangle$. We obtain $\langle\varepsilon\rangle \simeq \frac{1}{3} M$ from the naive quark counting in the nucleon; so γ is about $\frac{3}{2}$.

Now we can derive the one-parton momentum distribution function, integrating G_N over all the momenta of the other partons. To get the result in an analytical form, we neglect the masses of the partons. We obtain

$$G_N(\xi, \eta)d\xi d\eta = (N-1)(1-\xi)^{N-2}\gamma e^{-\gamma\eta} d\xi d\eta. \tag{12}$$

This leads to a factorization of the function G_N in the variables ξ and η but not in ξ and $k_\perp^2$ (see Eq. (3)). Although G_N does not factorize in those variables, the squared transverse charge radius of the neutron (see e.g. Refs [2, 3]) is zero in this model, since we have flavour independent distribution function.

Integrating the quantity $k_\perp^2 G_N$ over ξ and η, we get the average transverse momentum of partons inside the nucleon:

$$\langle k_\perp^2 \rangle = M^2 \gamma^{-1} \langle 1/N \rangle, \tag{13}$$

where $\langle 1/N \rangle$ means the average value of $1/N$ over all possible N-parton configurations. Using $\gamma = \frac{3}{2}$ and $\langle 1/N \rangle = \frac{1}{3}$ (naive quark counting) we obtain $\langle k_\perp^2 \rangle = (0.44\ \text{GeV})^2$.

To calculate the structure function for the nucleon, we must integrate the parton structure function (Eq. (9)) over ξ and η and sum over N-parton configurations (with the probabilities P_N). Using formulae (9) and (12), we get for $Q^2 \gg M^2$ (we omit the terms of order M^2/Q^2):

$$F_2(x) = x \sum_N P_N e_N^2 (N-1)(1-x)^{N-2}, \tag{14}$$

the well-known result of Bjorken and Paschos [5]. In this formula e_N^2 is the sum of all squared charges in the N-parton configuration, i.e. $e_N^2 = \sum_{i=1}^{N} e_i^2$.

Now calculate the average value of $k_\perp^2$ for partons which interact with a virtual photon. The momentum distribution function G_N for such partons depends on ξ, x and Q^2 since now η is not the independent variable (formula (5)). Integrating $k_\perp^2$ (Eq. (6)) over ξ with the normalized function $G_N(\xi; x, Q^2)$, we obtain for $Q^2 \gg M^2$ the following result:

$$\langle k_\perp^2(x, Q^2) \rangle \simeq 3\gamma^{-1} M^2 [F_2^{\text{p}}(x, Q^2) - F_2^{\text{n}}(x, Q^2)], \tag{15}$$

where superscripts p and n stand for the proton and the neutron, respectively.

Integrating Eq. (15) over x we get

$$\langle k_\perp^2 \rangle \simeq 3\gamma^{-1} M^2 (I_2^{\text{p}} - I_2^{\text{n}}). \tag{16}$$

Using $\gamma = \frac{3}{2}$ and the experimental value of $I_2^{\text{p}} - I_2^{\text{n}} \simeq 0.034 \pm 0.015$ [8] we obtain $\langle k_\perp^2 \rangle \simeq (0.24\ \text{GeV})^2$. The largest value of $\langle k_\perp^2(x) \rangle$ is obtained for $x \simeq 0.35$ [9], which, using experimental figures, gives $\langle k_\perp^2(x \simeq 0.35) \rangle \simeq 3\langle k_\perp^2 \rangle \simeq (0.42\ \text{GeV})^2$. Those results depend on the particular choice of γ which is an independent parameter (of the order of unity) in this model; therefore, the above numbers should not be taken too seriously. If we put $\langle 1/N \rangle = 3(I_2^{\text{p}} - I_2^{\text{n}})$, which is the result [10] of the naive quark parton model with equipartition of the longitudinal momentum of the nucleon, the average values for $\langle k_\perp^2 \rangle$ are equal (for $Q^2 \to \infty$), whether partons interact with the virtual photon (Eq. (16)) or not (Eq. (13)).

If one wants to compare those results with experiment, one has to look for inclusive transverse momentum measurements in the leptoproduction. The mean transverse momentum of the outgoing hadron $\langle p_\perp^2 \rangle$ (measured in a direction perpendicular to the momentum transfer vector) depends on the $k_\perp^2(x, Q^2)$ distribution of the partons and the $h_\perp(z, Q^2)$ distribution of the hadrons produced in the struck quark fragmentation. Gronau and collaborators [11] have shown that this average is the sum of two terms:

$$\langle p_\perp^2 \rangle = \langle h_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle, \tag{17}$$

where z is the fraction of the parton momentum carried by the hadron. Thus from Eq. (17), we see that if one measures $\langle p_\perp^2 \rangle$ as a function of x (for fixed z and Q^2), one can, in principle, extract $\langle k_\perp^2(x) \rangle$ from the experimental data. The authors of Ref. [2] have gotten from a phenomenological analysis of the data a behaviour similar to one obtained in this model. They found that $\langle k_\perp^2(x) \rangle$ increases with x up to a maximum value at $x \simeq 0.4$ and then decreases towards $x = 1$, which is roughly reproduced in our model.

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