ELASTIC p-p CROSS-SECTION AND LONGITUDINAL MOTION OF QUARKS

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We show that quark longitudinal degrees of freedom are important for the shape of the elastic p-p cross-section calculated in the dressed quark model using Glauber multiple scattering expansion. We find also that in this model multiple scattering terms are greater than required by data.

1. Introduction

In this paper we continue the investigation of the dressed quark model of hadrons [1]. According to the experimental information coming from the weak and electromagnetic interactions of hadrons, and also from their spectroscopy, strongly interacting particles are assumed to be structures composed of valence quarks, carrying quantum numbers, and of neutral glue, responsible for binding forces, and possibly of some $q\bar{q}$ pairs. Contrary to lepton-hadron processes both quarks and glue can play the essential role in hadron-hadron interactions. So, hadronic processes give us a chance to study the properties of glue, but, on the other hand, the interpretation of experimental data is ambiguous because of existence of two different types of hadron constituents, which can take part in strong interactions. One way to avoid this ambiguity is to assume that glue is the only strongly interacting part of hadron, not related to the distribution of quarks or related only by conservation laws. This was proposed by Van Hove and collaborators [2, 3].

Another possibility, connected naturally with additivity rules [4], is that all strongly interacting hadron constituents are concentrated around positions of the valence quarks (quarks dressed with glue). The comparison of different models of hadron with experimental data is of great interest as it can be helpful in determining hadron structure and properties of its constituents. The latter model, and its consequences for elastic proton-proton scattering, were discussed in detail in Ref. [1]. It was found that the multiple scattering terms generated by Glauber expansion of the elastic amplitude are by far too strong. In particular, when model parameters are fitted to reproduce experimental total cross section and its value at impact parameter $b = 0$, the position of the first dip in $d\sigma/dt$ is

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distinctly shifted to smaller value of momentum transfer $-t_0 = 0.7-0.8$ GeV^2 than the experimental value $-t_0 = 1.34$ GeV^2 ($\sqrt{s} = 53$ GeV). Furthermore, the calculated differential cross section behind the dip is by 1–2 orders of magnitude too large. These results may indicate the existence of repulsive correlations of quarks inside the proton, suppressing multiple scattering terms. In the present paper we introduce distribution of longitudinal momenta of quarks in proton neglected in [1] and discuss the question whether in this more realistic picture correlations induced by conservation laws only can explain the behaviour of experimental data. The important role of the longitudinal degrees of freedom in description of diffractive processes was already discussed in Ref. [3].

In this paper we keep the q–q interaction independent of relative longitudinal momentum, so we need only quark distribution in the proton integrated over longitudinal momenta. As in Ref. [1] we assume that the quarks are weakly correlated on the "ground level", i.e. the only correlation is caused by kinematics. Nevertheless, the integration over longitudinal degrees of freedom can result in correlated distribution of transverse positions of quarks.

We show that the longitudinal degrees of freedom can be important for the elastic p–p cross-section calculated in the multiple scattering model. Our result is that the correlations introduced by the distribution of longitudinal momenta change the shape of the amplitude shifting the position of the dip in elastic cross-section to the higher values of momentum transfer and thus improve the agreement with data. However, the absolute values of differential cross-section lie above the experimental data indicating that the multiple scattering corrections are still too strong.

In the next Section we specify the model and the parametrization of quark distributions and q–q interactions. In Section 3 we discuss the results of our calculation and compare them with the experimental data.

2. *Details of the model*

As it was already stated in the Introduction we have adopted the picture of hadron as a bound state of the dressed valence quarks. Thus we describe elastic proton-proton scattering by means of Glauber model [5]. The quark-quark interaction is assumed to be purely absorptive. Then the elastic amplitude for collision of two protons A and B with quarks frozen in transverse positions $\{\vec{s}_i\}$, reads

$$\Gamma(\vec{b}; \{\vec{s}_i\}) = 1 - \prod_i \prod_j (1 - \gamma_{ij}(\vec{b} + \vec{s}_i^A - \vec{s}_j^B)), \tag{2.1}$$

where γ_{ij} denotes the elastic amplitude of i-th and j-th quarks interaction and $\vec{b}$ is the impact parameter of incoming protons. The elastic amplitude in impact parameter representation, averaged over quark density distributions $D(\vec{s}_1, \vec{s}_2, \vec{s}_3)$ in both particles, is given by

$$\Gamma(\vec{b}) \equiv \langle \Gamma \rangle_{AB} = \int d^2 s_1^A \dots d^2 s_3^B D_A(\{\vec{s}_i^A\}) D_B(\{\vec{s}_j^B\}) \Gamma(\vec{b}; \{\vec{s}_i\}) \tag{2.2}$$

and in momentum transfer representation it reads

$$T(\vec{\Delta}) = \int d^2 b \langle \Gamma(\vec{b}; \{\vec{s}_i\}) \rangle_{AB} e^{i\vec{\Delta}\cdot\vec{b}}. \tag{2.3}$$

With this normalization the cross-sections are given by the formulae:

$$\text{total cross section} \qquad \sigma_{\text{tot}} = 2 \operatorname{Re} T(0), \tag{2.4}$$

$$\text{elastic cross section} \ \frac{d\sigma}{dt} = \frac{1}{4\pi} |T(\Delta)|^2, \qquad t \simeq -\Delta^2. \tag{2.5}$$

To calculate the elastic amplitude we have to specify the q–q amplitudes γ and quark densities $D(\{\vec{s}_i\})$. We have already discussed in Ref. [1] quite a wide class of these functions, defined by the following requirements:

(*i*) quarks are weakly correlated (at most by kinematical $\delta^{(2)}(\vec{s}_1+\vec{s}_2+\vec{s}_3)$ factor, removing unphysical transverse motion of quark CMS),

(*ii*) all the quarks interact with the same amplitude,

(*iii*) distribution of quarks is consistent with the electromagnetic forfmactor of the proton.

It was pointed out that the correlations of quarks induced by conservation laws are of great importance for the large $|t|$ behaviour of the differential cross-section. However, the multiple scattering terms are in any case too large; may be some stronger correlations, giving larger average distances of quarks in the hadron, could help. The main features of the results were almost independent on the details of the shape of the functions from that class. Representative examples of the functions belonging to the above defined class are as follows

$$\gamma(\vec{b}) = \gamma(0)e^{-b^2/r^2}, \tag{2.6}$$

$$D(\{\vec{s}_i\}) = \int\limits_0^\infty \frac{dx}{x} e^{-1/x} \frac{3}{(\pi A)^2} \exp\left[-\frac{x}{A}(s_1^2+s_2^2+s_3^2)\right] \delta^{(2)}(\textstyle\sum \vec{s}_i). \tag{2.7}$$

(Note that the magnitude of inelastic diffraction requires $\gamma(0) = 1$, so $\gamma(0)$ is not really a free parameter, and that $D(\{\vec{s}_i\})$ (Eq. (2.7)) correctly reproduces experimental dipole shape of the proton electromagnetic formfactor; x in (2.7) is the integration variable without physical interpretation in our model.)

Following the program of our work described in previous section we are going to extend the presented scheme introducing longitudinal degrees of freedom for quarks in the proton [6]; this will change the correlations caused by the conservation laws. In the case of longitudinal momentum distribution of the quarks the centre of particle momentum (transverse position of the particle) is defined as

$$\vec{\beta} = \sum_i x_i\vec{s}_i, \quad \sum x_i = 1, \quad 0 \leqslant x_i \leqslant 1.$$

(x_i is the longitudinal momentum fraction of i-th dressed quark). Then proton-proton impact parameter is defined by:

$$\vec{b} = \vec{\beta}_A - \vec{\beta}_B.$$

In the following we will use the density distribution function for proton A (and B) defined individually in the reference frame in which $\vec{\beta}_A = 0$ ($\vec{\beta}_B = 0$). Thus, because in our model there are no other proton constituents than the dressed valence quarks, the kinematical constraints take the form $\delta^{(2)}(\sum_i x_i\vec{s}_i) \cdot \delta(\sum_i x_i - 1)$. We adopt q−q amplitude given by Eq. (2.6), so, for the sake of further calculations, we need the quark density distribution integrated over x_i. Our choice, suggested in a natural way by Eq. (2.7), is

$$D(\{\vec{s}_i\}) = \int dx_1 dx_2 dx_3 \delta(\sum x_i - 1)\delta^{(2)}(\sum x_i\vec{s}_i)e^{-\Sigma x_i s_i^2/A} f(\{x_i\}). \tag{2.8}$$

In general $f(\{x_i\})$ is almost free function of longitudinal degrees of freedom, consistent with normalization condition

$$\int D(\{\vec{s}_i\}) \{d^2 s_i\} = 1. \tag{2.9}$$

However, let us emphasize that we are interested mainly in investigation of consequences of the new form of conservation constraints. Thus, the simplest choice is $f(\{x_i\}) = \text{const.}$ Such a choice does not change the influence of the longitudinal degrees of freedom on the dip position (discussed in the next section). It leads, however, to infinite radius of quark distribution, because the quarks with small x have very broad $\vec{s}$ distribution, as can be easily seen from Eq. (2.8). To obtain one-quark distribution exponentially vanishing for large quark distances from the centre of particle (in order to avoid singularities in form-factor derivatives at $t = 0$) one has to suppress the probability of small x values by the factor of the form

$$f(\{x_i\}) \sim \exp\left(-\alpha \sum_i \frac{1}{x_i}\right). \tag{2.10}$$

(Note that analogical factor in (2.7) results in dipole formfactor.) In the next section we present the results for that specification of $f(\{x_i\})$ (Eq. (2.10)).

Before the rather complicated calculation of the full elastic p−p amplitude it is interesting to see how much our parametrization of quark distribution differs from that previously [1] used (Eq. (2.7)). As it is seen from Fig. 1 one-quark distributions in the transverse posi-

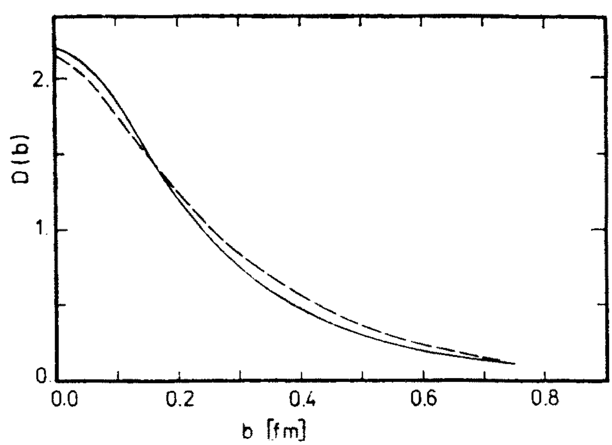

Fig. 1. Comparison of one-quark densities resulting from the discussed parametrizations: with (Eqs. (2.8) and (2.10)) (solid line) and without (Eq. (2.7)) longitudinal momentum distribution (dashed line)

tion plane are almost the same for (2.7) and (2.8); our parametrization (2.8) reproduces the dipole shape of proton electromagnetic formfactor, at least for $|t| \lesssim 10\ \mathrm{GeV}^2$. However, the correlations of quarks in the distributions (2.7) and (2.8) distinctly differ. As a measure of the correlations we present (Fig. 2) the mean distance squared d^2 of two quarks in the proton versus position $\vec{s}$ of one of the quarks, for both cases. (N.b. $d^2(s)$ calculated for

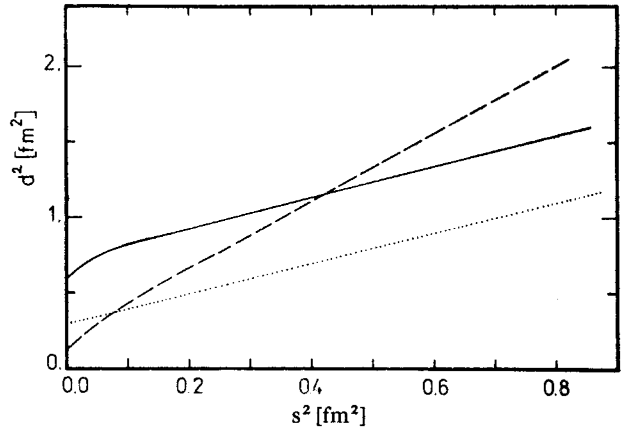

Fig. 2. Comparison of the dependence of mean distance squared $d^2(s)$ of two quarks on the position s of one of them in the proton for parametrizations with (solid line) and without longitudinal momentum distribution (dashed line)

the distribution (2.7) coincides with the result for simple Gaussian parametrization $D \sim \exp(-\sum s_i^2/A)\delta^{(2)}(\sum \vec{s}_i)$ discussed in [1]. The result for uncorrelated distribution $D \sim \exp(-\sum s_i^2/A)$ is also shown for comparison.) As is clearly seen, the largest separations of quarks for their typical configurations inside hadron are given by the discussed distribution (2.8). The parameters used here are set by the requirements (3.2).

3. *Results*

In this section we present results of numerical calculations of the elastic proton-proton differential cross-section. For quark density distribution in the proton we use the function $D(\{\vec{s}_i\})$ given by Eqs (2.8) and (2.10). The q – q scattering amplitude is taken as (see (2.6) and discussion below it):

$$\gamma(b) = e^{-b^2/r^2}. \tag{3.1}$$

Three parameters α, A and r^2 are set by the following experimental constraints (data for $\sqrt{s} = 53$ GeV):

$$\sigma_{\mathrm{tot}} = 42.5\ \mathrm{mb}, \tag{3.2a}$$

$$\Gamma(b = 0) = 0.753, \tag{3.2b}$$

$$-t_{\mathrm{dip}} = 1.34\ \mathrm{GeV}^2. \tag{3.2c}$$

The above conditions result in the following values of the parameters:

$$A = 0.11 \text{ fm}^2, \quad r^2 = 0.097 \text{ fm}^2, \quad \alpha = 0.1. \tag{3.3}$$

As was already stated in the previous section the form of $f(\{x_i\})$ (Eq. (2.10)) was chosen in order to obtain the correct behaviour of proton formfactor. Nevertheless we have checked the influence of correlations induced by kinematical δ factor only, taking $f(\{x_i\}) = \text{const}$. In this case (after fulfillment of conditons (3.2a, b) we get the dip in $d\sigma/dt$ at $-t_0 = 1.6$ GeV^2. This is to be compared with $-t_0 = 0.7 \div 0.8$ GeV^2 for all cases discussed in [1]. Thus we see that the longitudinal degrees of freedom are really important in the multiple scattering model.

In figure 3 we present differential elastic proton-proton cross-section together with the experimental data for $\sqrt{s} = 53$ GeV [7]. As can be seen, the model gives too large values of $d\sigma/dt$ behind the first dip. This means that the higher order multiple scattering terms

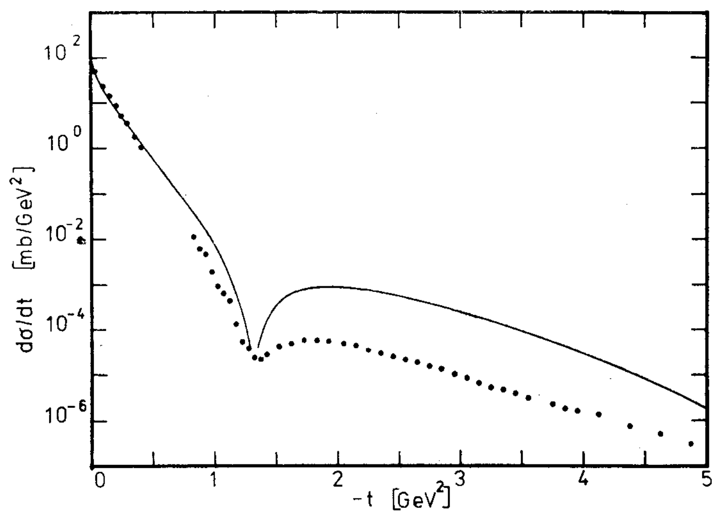

Fig. 3. Elastic p-p cross section calculated using quark density with longitudinal momentum distribution (Eqs. (2.8), (2.10)) (solid line) and the experimental data (dots)

are overestimated. For the smaller values of $|t|$ out fit to the data is satisfactory; in particular for the slope in forward direction we obtain the value $B \cong 10$ GeV^{-2}. The multiple scattering contributions for $|t| \lesssim 2$ GeV^2 are smaller than in the version without x distribution and consequently they result in correct dip position (see Fig. 4). Nevertheless, even in small $|t|$ region the multiple scattering effects are non negligible, e.g. they give corrections amounting about 20% of the single scattering amplitude at $t \doteq 0$.

The results of this paper and of Ref. [1] seem to suggest that the multiple scattering model without introducing substantially new features of hadron structure or interaction of quarks is not likely to reproduce well the experimental behaviour of $d\sigma/dt$. One possibility which could distinctly change the presented picture is suggested by the current ideas about colour structure of strong interactions based on QCD [9]. The colour degrees of

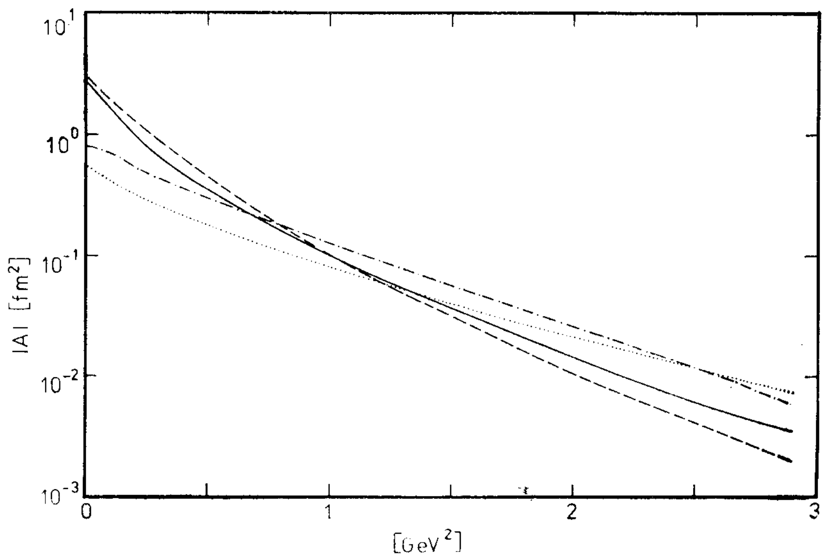

Fig. 4. Single (solid line) and sum of the all multiple (dotted line) scattering contributions to the elastic amplitude for the quark distributions with longitudinal degrees of freedom (Eqs. (2.8), (2.10)). Analogous contributions for the distribution without longitudinal degrees of freedom (Eq. (2.7)) are represented by dashed-dotted line respectively

freedom are likely to result in much more complicated interference of interaction amplitudes than hitherto considered in our model. It would be thus interesting to extend the model by introducing these colour interference effects and to examine the consequences for the elastic proton-proton scattering.

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REFERENCES

[1] A. Białas, K. Fiałkowski, W. Słomiński, M. Zieliński, *Acta Phys. Pol.* **B8**, 855 (1977).
[2] S. Pokorski, L. Van Hove, *Acta Phys. Pol.* **B5**, 229 (1974); L. Van Hove, S. Pokorski, *Nucl. Phys* **B86**, 243 (1975); L. Van Hove, *Acta Phys. Pol.* **B7**, 339 (1976); L. Van Hove, *Nucl. Phys.* **B122** 525 (1977); S. Pokorski, L. Van Hove, *Hadron fragmentation and leading particle effect in the quark-gluon model*, CERN preprint TH. 2427, Nov. 1977.
[3] L. Van Hove, K. Fiałkowski, *Nucl. Phys.* **B107**, 211 (1976).
[4] H. J. Lipkin, F. Scheck, *Phys. Rev. Lett.* **16**, 71 (1966); E. M. Levin, L. L. Frankfurt, *Zh. Eksp. Teor. Fiz. Lett.* **2**, 105 (1965).
[5] R. J. Glauber, *Lectures in Theoretical Physics*, Vol. 1, Interscience, New York 1959.
[6] J. D. Bjorken, J. B. Kogut, D. E. Soper, *Phys. Rev.* **D3**, 1382 (1971); H. Cheng, T. T. Wu, *Phys. Rev. Lett.* **24**, 1456 (1970).
[7] H. De Kerret et al., *Phys. Lett.* **62B**, 363 (1976); G. Barbiellini et al., *Phys. Lett.* **39B**, 663 (1972).
[8] H. I. Miettinen, *Acta Phys. Pol.* **B6**, 625 (1975) and private communication.
[9] F. E. Low, *Phys. Rev.* **D12**, 163 (1975); S. Nussinov, *Phys. Rev. Lett.* **34**, 1238 (1975); J. Gunion, D. Soper, *Phys. Rev.* **D15**, 2617 (1977).