FRAGMENTATION OF QUARK AND DIQUARK JETS

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The quark fragmentation model is extended to describe meson and baryon production from quark as well as diquark jets. The model is applied to e^+e^- inclusive annihilation and the current and target fragmentation region of deep inelastic scattering. An independent test of the model in lepton pair triggered meson nucleon reactions is proposed. The hadron distributions are predicted to depend on $x = \sqrt{M^2/s}$ and x_F only via $x_F/(1-x)$.

1. *Introduction*

Inclusive hadron production in e^+e^- annihilation and the current fragmentation region of deep inelastic scattering is usually studied in terms of so-called quark fragmentation functions $D(z)$. They are also used in parton-model descriptions of large $p_\perp$ production. The introduction of those fragmentation functions rests on factorization of hadron distributions from different jets with large relative angles. Kinematic arguments and model studies, e.g. in the Schwinger model [1], support that factorization.

Possible violations of scaling, i.e. a dependence of the fragmentation function $D(z, Q^2)$ on an intrinsic momentum transfer Q^2, are neglected in this paper. On the bas isof generalized renormalization group equations [2] for the moments of $D(z, Q^2)$ logarithmic scaling violations are expected for an asymptotically free theory, but they are not related in a simple way to those of νW_2. From the experimental side there is no obvious indication for violation of scaling for the quark fragmentation function.

Since there is at present no reliable method of calculating quark fragmentation functions from an underlying theory, say QCD, a simple phenomenological model has been developed [3–8]. In its simplest version $D(z)$ is given as solution of the chain-decay equation [12, 13]

$$D(z) = d(z)+ \int\limits_0^{1-z} \frac{dz'}{1-z'} d(z')D\left(\frac{z}{1-z'}\right), \tag{1.1}$$

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which reflects a planar multiperipheral structure. z is the hadron momentum scaled by the momentum of the quark-like jet. The vertex function $d(z)$, describing the momentum sharing in each emission step, is to be specified by hand. Hence, the model is predictive only at the level of correlations or, to some extend, particle ratios. Therefore, we consider it more as a convenient parametrization.

So far, this model has been used to describe the hadronization of "isolated" quarks, e.g. quarks kicked out of a hadron by some large momentum transfer process. In such reactions there is also a through-going jet containing the remaining valence quarks (plus gluons and sea quarks). The study of those jets is one of the main purposes of this paper.

A thinkable prescription would be to use some quark distribution functions in the jet (possibly those measured in deep inelastic scattering) and fold quark fragmentation functions in. This has been tried [9], and failed. The resulting hadron spectra are much too steep.

Other approaches [9–11] (so-called recombination models) are more successful phenomenologically. Inclusive production is described as resulting from recombination of one or two valence quarks with soft sea quarks. The hadron momentum is taken as the sum of the contributing quark momenta. These analyses revealed that enhanced sea distributions (compared to usual fits to deep inelastic structure functions) are needed to match normalization, while the shape (say of $F_{p\to\pi^+} \sim u(x)$) is nearly automatically in accord with observations. However, this method cannot easily be applied to e^+e^- annihilation or the fragmentation of other "isolated" quarks. This situation is somewhat unsatisfactory since one would expect one common mechanism for the transformation of quarks and gluons into hadrons, independently of how the quark-gluon state is produced (at least for large Q^2 processes a separation into a hard scattering process and a slow hadronization process seems visible).

This unfortunate situation has been a main motivation for the present analysis. In order to obtain a common description of all the mentioned phenomena we start from the above model (Eq. (1.1)) of quark fragmentation and ask for an extension to other types of jets. As simplest generalization we consider the target fragmentation region of deep inelastic scattering. For not too small values of the Bjorken scaling variable x_{Bj} scattering is off a valence quark. Hence the through-going jet has flavour-diquark and color-antitriplet quantum numbers. For given x_{Bj} the total jet momentum is also completely specified. The main point is that we are dealing with the hadronization of this whole jet but not of individual quarks in the jet. This is also supported by the Kinoshita-Lee-Nauenberg argument that for massless theories (which we essentially consider) only those questions are sensible involving the sum over all degenerate quark-gluon states, i.e. the jet.

Introducing fragmentation functions for those diquark (color-antitriplet) jets leads simply to target fragmentation functions in leptoproduction

$$|x_F|\,\frac{dN}{dx_F} = |x_F| D_{qq}(|x_F|), \qquad x_F = \frac{2p^*_{\parallel h}}{W}\,. \tag{1.2}$$

Another test not sensitive to details of the model (e.g. to the choice of the input momentum sharing function), but critical for its general structure, is the relation between deep inelastic lepton scattering and lepton pair production processes in πp collisions. The hadron spectrum in presence of the Drell-Yan trigger ($x = \sqrt{M^2/s}$, $y_{\mu^+\mu^-} = 0$) is expressed by the fragmentation functions of appropriate quark and diquark jets at rescaled momentum $x_F/(1-x)$, $x_F = 2p^*_{\parallel}/\sqrt{s}$. In the pion fragmentation region no such similarity is expected within the recombination model.

The outline of this paper is the following. In Chapter 2 the model is formulated for quark fragmentation into mesons, taking quantum number effects and resonance decay into account. For the convenience of the reader also known results are reviewed. In Chapter 3 the model is extended to diquark jets, including quark and diquark fragmentation into baryons. Chapter 4 contains a comparison with data from e^+e^- annihilation and leptoproduction in the target fragmentation region, as well as a general discussion.

2. *Quark jet fragmentation*

In this section we describe mainly results known also from previous work [3—8] but in a way easily extendable to modifications of the couplings in the chain.

A chain decay model for anisotropic hadronic final states with scaling has been proposed by Krzywicki and Petersson [12], Finkelstein and Peccei [13] some years ago. In this model the final state is envisaged developing itself from a primary excited system by stepwise emission of particles. The scaling distribution of the leading one is put into the model by hand. But then the structure of the model relates all properties of the final state to each other and may be tested unambiguously. We will illustrate the model in terms of the resulting integral equation for the single particle distribution ($x = 2p_{\parallel}/\sqrt{s}$)

$$N_1(x, \vec{p}_\perp, s) = \frac{E}{\sigma_{\rm inel}} \frac{d^3\sigma}{d^3p},$$

$$N_1(x) = \int d^2p_\perp N_1(x, \vec{p}_\perp, s). \tag{2.1}$$

$N_0(x)$ denotes the distribution of the first emitted particle. The following integral equation is established by the inclusive bootstrap assumption. For $x > 0$

$$N_1(x) = N_0(x) + \int_{-1}^{0} \frac{dy}{|y|} N_0(y) N_1(x) + \int_{0}^{1-x} \frac{dy}{y} N_0(y) N_1\left(\frac{x}{1-y}\right). \tag{2.2}$$

The distribution $N_0(x)$ is normalized

$$\int_{-1}^{1} N_0(x) \frac{dx}{|x|} = 1, \tag{2.3}$$

expressing the unit probability for "leading" particle emission. $N_1(x)$ and $N_1(x/(1-y))$ is the invariant distribution of particles with rescaled momentum inside the remaining hadronic system after backward and forward leading particle emission, respectively. This simple form results from neglecting transverse momentum and mass of the produced particles. This approximation allows one to factorize forward and backward production which applies also to many particle distributions. For brevity we assume $N_0(x)$ symmetrical, and define $g(x) = 2N_0(x)$ with

$$\int_{-1}^{0} \frac{dy}{|y|}\, g(y) = \int_{0}^{1} \frac{dy}{y}\, g(y) = 1.$$

Then $f(x) = N_1(x)$ becomes symmetrical, too. For definiteness we consider $x > 0$ and obtain

$$f(x) = \tfrac{1}{2}\, g(x) + \tfrac{1}{2} \int_{-1}^{0} \frac{dy}{|y|}\, g(y) f(x) + \tfrac{1}{2} \int_{0}^{1-x} \frac{dy}{y}\, g(y) f\left(\frac{x}{1-y}\right), \tag{2.4}$$

or, by virtue of normalization of $g(y)$,

$$f(x) = g(x) + \int_{0}^{1-x} g(y) \frac{dy}{y} f\left(\frac{x}{1-y}\right). \tag{2.5}$$

Thus we arrive at independent fragmentation of both a forward and backward jet. This is maintained also when a unsymmetrical

$$N_0(x) = \begin{cases} g_F(x) & x > 0, \\ g_B(|x|) & x < 0, \end{cases}$$

and different probabilities

$$\int_{0}^{1} \frac{dy}{y}\, g_{F/B}(y) = \alpha_{F/B}, \qquad \alpha_F + \alpha_B = 1$$

for forward and backward leading particle emission, respectively, are used.

Now we want to abstract from this scheme a way to parametrize quark or antiquark jet fragmentation. An excited color singlet system with separated color (anti)triplet quark and antiquark with invariant mass W should produce hadrons along a preferred "longitudinal" direction by forming hadrons out of either the quark or antiquark jet in an approximately independent way.

We consider both jets as a stream of momentum carried by the (anti)quark and gluons with total color, flavor and momentum determined by the original (anti)quark. The firstly produced hadron remembers its parent quark's flavor as far as it has to have this quark in its wave function. The bare quark carries only a certain fraction of the total jet momentum. The momentum distribution of the first hadron is essentially determined by this momentum sharing of the bare quark inside its accompanying jet. It must be chosen phenomenologically. We wish to emphasize that we feel not obliged to take it from dimensional counting rules [19] or from comparison with hadron structure functions $\nu W_2(x)$.

We have mentioned the Krzywicki-Petersson-Finkelstein-Peccei model in order to make clear that the notion of independent fragmentation of isolated quark smay be considered as an approximation for the hadronization of an excited color singlet quark anti-quark system. With this caution in mind we define the distribution of hadron h from quark q, $dN/dz|_{h/q} = D_{h/q}(z)$, and emphasize that we wish to use them in the hadron CMS where we have two jets balancing each other in momentum. The scaling variable z is $z = p_{hadron}/p_{jet} = |x_F|$. Usually these D-functions are compared with or extracted from deep inelastic leptoproduction data in the laboratory frame where z is usually defined as $z = p_h/\nu$, $\nu = E_l - E_{l'}$. As far as the chain decay model has been used for predicting hadron distributions [6] the low z region ($z < 0.2$) could not be described successfully.

In this Section we restrict ourselves to fragmentation into mesons.

2.1. Total meson distribution

First we write down the integral equation for the single particle distribution without specification of flavors, $D(z) = \sum_h D_{h/q}(z)$ which represents the total meson distribution and turns out to be independent of the quark flavor. The momentum sharing function of each single decay step is denoted by $d(z)$. Both are related by the equation

$$D(z) = d(z) + \int \frac{dz'}{1-z'} d(z') D\left(\frac{z}{1-z'}\right), \tag{2.6}$$

with nonnegative normalized $d(z)$,

$$\int_0^1 d(z) dz = 1. \tag{2.7}$$

For completeness, the equation for the two-particle distribution is

$$D(z_1, z_2) = d(z_1) \frac{1}{1-z_1} D\left(\frac{z_2}{1-z_1}\right) + d(z_2) \frac{1}{1-z_2} D\left(\frac{z_1}{1-z_2}\right) + \int \frac{dy}{(1-y)^2} d(y) D\left(\frac{z_1}{1-y}, \frac{z_2}{1-y}\right). \tag{2.8}$$

For momentum sharing functions having the form

$$d(z) = (\beta+1)(1-z)^\beta \tag{2.9}$$

the functions

$$D(z) = \frac{\beta+1}{z}(1-z)^\beta \tag{2.10}$$

and

$$D(z_1, z_2) = \frac{(\beta+1)^2}{z_1 z_2}(1-z_1-z_2)^\beta \tag{2.11}$$

are easily found. The distribution of the nonleading meson (i.e. those not emitted in the first step) is

$$D^{NL}(z) = D(z) - d(z) = (\beta+1)(1-z)^{\beta+1}/z. \tag{2.12}$$

Here plateau height, $\beta+1$, the shapes of the leading and nonleading part of the spectrum are related in just the same way as in the bremsstrahlung model [14].

In more general cases the Mellin transformation is used to solve the integral equations (2.6) or (2.8). The transformed distributions

$$\tilde{D}(k) = \int D(z) z^{k-1} dz,$$
$$\tilde{d}(k) = \int d(z) z^{k-1} dz, \tag{2.13}$$

are related by the transformed equation

$$\tilde{D}(k) = \tilde{d}(k) + \tilde{h}(k)\tilde{D}(k), \tag{2.14}$$

with

$$\tilde{h}(k) = \int d(z)\,(1-z)^{k-1} dz. \tag{2.15}$$

From

$$\tilde{D}(k) = \frac{\tilde{d}(k)}{1-\tilde{h}(k)} \tag{2.16}$$

we have to find $D(z)$ by inversion of (2.13)

$$D(z) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{dk}{z^k} \tilde{D}(k). \tag{2.17}$$

Besides the singularities of $\tilde{d}(k)$, the singularities of $\tilde{D}(k)$ are determined by the zeroes of

$$1-\tilde{h}(k) = 0. \tag{2.18}$$

Due to normalization (2.7) there is a pole at $k = 1$. This is the right most singularity which determines the behaviour of $D(z)$ near $z = 0$

$$D(z) \underset{z\to 0}{\sim} \frac{1}{z} \{\int dz d(z)\, |\log(1-z)|\}^{-1}. \tag{2.19}$$

There is a plateau in rapidity $y = \log(z/z_0)$ with height

$$H = \{\int dz d(z)\, |\log(1-z)|\}^{-1} \tag{2.20}$$

and a logarithmic multiplicity

$$\langle n \rangle = \int_{z_0}^{1} D(z) dz \sim H \log \frac{P}{\mu}. \tag{2.21}$$

Here z_0 is a minimum $z_0 = \mu/P$, P is the total jet momentum, and μ is of the order of the transverse mass $m_\perp$ of the produced particles.

We have used this simplified version of the model in order to fix our function $d(z)$ by comparison with charged hadron distributions in e^+e^--annihilation [15] (see Section 4).

2.2. Quantum number structure

Now we want to consider the quantum number structure of the chain decay. Obviously it is sufficient to treat the quark jet fragmentation function $D_{h/q}$ as long as only meson production is considered. Antiquark jet fragmentation is determined by charge conjugation, $D_{h/\bar{q}} = D_{\bar{h}/q}$. We have the Mellin transformed equations

$$\tilde{D}_{h/q}(k) = \tilde{d}_{h/q}(k) + \sum_{h'} \tilde{h}_{h'/q}(k)\tilde{D}_{h/q\bar{h}'}(k). \tag{2.22}$$

The sum runs over those mesons which may be emitted in the first step, i.e. $h' = q\bar{q}'$. Only for these the "primordial" functions $d_{h/q}(z)$ do not vanish identically. $\tilde{h}_{h/q}(k)$ is defined analogously to equation (2.15) in terms of $d_{h/q}(z)$. As long as we consider only quark jet fragmentation into mesons, and according to our view of a quark jet discussed above, we take a universal z distribution,

$$d_{h/q}(z) = d(z)R^{h}_{qq'}, \qquad q' = q\bar{h}$$
$$R^{h}_{qq'} = 0 \quad \text{for} \quad h \neq q\bar{q}'. \tag{2.23}$$

All the couplings $R^{h}_{qq'}$ define a quark transition matrix

$$R_{qq'} = \sum_{h} R^{h}_{qq'}, \tag{2.24}$$

which is normalized according to

$$\sum_{h} \int d_{h/q}(z)dz = \int d(z)dz \sum_{h} R^{h}_{qq\bar{h}} = \sum_{q'} R_{qq'} = 1, \tag{2.25}$$

with

$$\int d(z)dz = 1.$$

The ratios between the couplings $R^{h}_{qq'}$ for h belonging to the same unitary multiplet are given by the corresponding Clebsch-Gordan coefficients in case of complete symmetry. We will consider only SU(3) and keep open the possibility of strange particle production being suppressed by a factor $\kappa \leqslant 1$. The ratios between different multiplets (we will consider directly produced pseudoscalar and vector meson) will be specified by weights α_{PS}, α_{v}, $\alpha_{PS}+\alpha_{v} = 1$. SU(6) symmetry suggests $\alpha_{v}/\alpha_{PS} = 3$.

The solution of equation (2.22) is simply

$$\tilde{D}_{h/q}(k) = \sum_{h=0}^{\infty} (\tilde{h}(k)^{n}\tilde{d}(k)) \sum_{q'} (R^{n})_{qq'}R^{h}_{q'q'\bar{h}}. \tag{2.26}$$

The simplest model of this kind with SU(2) isospin symmetry has been considered in Refs [3, 4]. In the general framework given here and with only $\pi^{\pm 0}$ produced we have symbolically

$$\sum_{h} hR^{h}_{qq'} = \tfrac{1}{3}\begin{pmatrix} \pi^{0} & 2\pi^{+} \\ 2\pi^{-} & \pi^{0} \end{pmatrix}. \tag{2.27}$$

The transition matrix R and its powers are

$$R = \tfrac{1}{3}\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}, \quad R^n = \tfrac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + \tfrac{1}{2}(-\tfrac{1}{3})^n\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}. \tag{2.28}$$

For $d(z) = 2(1-z)$ we find

$$D_{\pi^{\pm}/u}(z) = D_{\pi^{\mp}/d}(z) = \tfrac{2}{3}\frac{1-z}{z} \pm \tfrac{2}{5}(1-z^{5/3}),$$

$$D_{\pi^0/u}(z) = D_{\pi^0/d}(z) = \tfrac{2}{3}\frac{1-z}{z}. \tag{2.29}$$

These quark jet fragmentation functions have been compared to neutrino production data with reasonable success and used for the discussion of models for large transverse momentum processes in Refs [4].

A more realistic model to be used when charm may be neglected is the following SU(3) model. A symmetry breaking factor κ is applied in all emissions which require the creation of a $s\bar{s}$ pair. We allow for direct production both of pseudoscalar and vector mesons. Symbolically we write

$$\sum_{h} hR^{h}_{qq'} = \frac{\alpha_{PS}}{2(2+\kappa)}$$

$$\times \begin{pmatrix} \pi^0+\eta\cos^2\theta+\eta'\sin^2\theta & 2\pi^+ & 2\kappa K^+ \\ 2\pi^- & \pi^0+\eta\cos^2\theta+\eta'\sin^2\theta & 2\kappa K^0 \\ 2K^- & 2\bar{K}^0 & 2\kappa(\eta\sin^2\theta+\eta'\cos^2\theta) \end{pmatrix}$$

$$+\frac{\alpha_V}{2(2+\kappa)}\begin{pmatrix} \varrho^0+\omega & 2\varrho^+ & 2\kappa K^{*+} \\ 2\varrho^- & \varrho^0+\omega & 2\kappa K^{*0} \\ 2K^{*-} & 2\bar{K}^{*0} & 2\kappa\phi \end{pmatrix} \tag{2.30}$$

(θ is the mixing angle, $\sin^2\theta = 0.511$ in case of quadratic mass formulae). Then the quark transition matrix is

$$R = \frac{1}{(2+\kappa)}\begin{pmatrix}1 & 1 & \kappa\\ 1 & 1 & \kappa\\ 1 & 1 & \kappa\end{pmatrix}, \tag{2.31}$$

which reproduces itself under multiplication, $R^2 = R$. The same form for R has been chosen by Seiden [6] and Feynman [5]. We try to describe the data with $d(z) = 2(1-z)$ and $\alpha_V/\alpha_{PS} = 3$ as found to be appropriate by comparison to charged hadron distributions in e^+e^- annihilation (see Section 4). Seiden [6] has used a flat momentum sharing function $d(z) = 1$. The resulting distributions are too flat even modified by resonance decay [6].

TABLE I

h	$\pi^{\pm 0}$	$K^{\pm}K^{0}\overline{K^{0}}$	η/η'
C^{h}	$\alpha_{PS}(2+\kappa)^{-2}$	$\alpha_{PS}\kappa(2+\kappa)^{-2}$	$\alpha_{PS}(2+\kappa)^{-2}(\cos^2\theta+\kappa^2\sin^2\theta)$ $\alpha_{PS}(2+\kappa)^{-2}(\kappa^2\cos^2\theta+\sin^2\theta)$

The selfreproducing property $R^2 = R$ simplifies considerably the calculation of distributions[1]. We obtain from (2.26)

$$\tilde{D}_{h/q}(k) = \tilde{d}(k)R^{h}_{q,q\bar{h}} + \frac{\tilde{h}(k)\tilde{d}(k)}{1-\tilde{h}(k)} \sum_{q'} R_{qq'}R^{h}_{q',q'\bar{h}}. \tag{2.32}$$

If h contains q there is a leading particle contribution $d_{h/q}(z) = d(z)R^{h}_{q,q\bar{h}}$. The nonleading part is independent of the original quark flavor q. Its shape is simply given by $D^{NL}(z) = D(z)-d(z)$, and its magnitude is determined by coefficients

$$C^{h} = \sum_{q'} R_{qq'}R^{h}_{q',q'\bar{h}} = \sum_{q'} P_{q'}R^{h}_{q',q'\bar{h}}. \tag{2.33}$$

(P_q are the probabilities for $q\bar{q}$ pair creation.) For our choice $d(z) = 2(1-z)$ we find the nonleading contribution

$$D^{NL}_{h/q}(z) = C^{h}\,\frac{2(1-z)^2}{z}. \tag{2.34}$$

Thus we have the quark jet fragmentation functions for directly produced pseudoscalar and vector mesons

$$D_{h/q}(z) = R^{h}_{q,q\bar{h}}2(1-z)+C^{h}\,\frac{2(1-z)^2}{z}. \tag{2.35}$$

The coefficients $R^{h}_{q,q\bar{h}}$ can be read off Eq. (2.30). For brevity, we have collected the C^{h} in Table I. They determine the logarithmic growth of the multiplicity

$$\langle n_{h/q}\rangle = \int_{z_0}^{1} dz D_{h/q}(z) \approx 2C^{h}\log\frac{P}{\mu} - 3C^{h}+R^{h}_{q,q\bar{h}}. \tag{2.36}$$

The asymptotic multiplicity ratios for directly produced mesons are therefore simply (see Eq. (2.30))

$$\frac{K}{\pi} = \frac{K^*}{\varrho} = \kappa, \qquad \frac{\varrho}{\pi} = \frac{K^*}{K} = \frac{\alpha_V}{\alpha_{PS}}.$$

For the particle distributions from vector meson decay we have used the known branching

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[1] When choosing R we see no reason to follow Sukhatme [8], who argues for taking R symmetrical, in this probabilistic model.

ratios. The shape of the distributions are calculated analytically with simplified kinematics. The neglect of kaon masses in K* decays is certainly the most serious drawback of this procedure. For two particle decays we have used the parent-child relation

$$D_{\text{indirect}}(z) = \int_z^1 \frac{dz'}{z'} D_{\text{resonance}}(z') \tag{2.37}$$

and three body decays were approximated by

$$D_{\text{indirect}}(z) = \int_z^1 \frac{dz'}{z'} D_{\text{resonance}}(z')2\left(1-\frac{z}{z'}\right). \tag{2.38}$$

We will not reproduce here the resulting distributions of indirectly produced pions and kaons. The suppression factor κ will be fixed by comparison with data for $\pi^{\pm}$ and $K^{\pm}$ spectra from e^+e^- annihilation [16] (see Section 4).

2.3. Multiplicity distribution

For directly produced particles and resonances there are only weak correlations in this model, which may be seen by looking at the predicted multiplicity distribution. The Krzywicki-Petersson-Finkelstein-Peccei model in the kinematic approximation adopted leaves one hemisphere unaffected by leading particle emission into the other one. Within this approximation used to define quark jet fragmentation functions we may also define a multiplicity distribution for either jet separately. Its generating function is denoted as $\psi_q(\{\xi_h\}, P)$ and, according to the recursive principle, fulfills the following integral equation

$$\psi_q(\{\xi_h\}, P) = \int dx d(x) \sum_{q'} \sum_h \xi_h R^h_{qq'} \psi_{q'}(\{\xi_h\}, P(1-x))$$

$$+\text{low momentum input.} \tag{2.39}$$

The low momentum part represents the quasiexclusive component $q\bar{q} \to MM$ and is suppressed by meson formfactors. It is not necessary for our purpose to specify this further. We have only to make sure that it contributes only low lying singularities to the Mellin transform

$$\tilde{\psi}_q(\{\xi_h\}, \lambda) = \int dP P^{-\lambda-1} \psi_q(\{\xi_h\}, P), \tag{2.40}$$

determined by the equation

$$\tilde{\psi}_q(\{\xi_h\}, \lambda) = \int dx d(x)\,(1-x)^{\lambda} \sum_{q'} \sum_h \xi_h R^h_{qq'} \tilde{\psi}_{q'}(\{\xi_h\}, \lambda)$$

$$+\text{transformed input.} \tag{2.41}$$

The asymptotic behaviour of the generating function is represented in the form

$$\psi_q(\{\xi_h\}, P) \propto \left(\frac{P}{\mu}\right)^{\lambda(\{\xi_h\})}, \tag{2.42}$$

where $\lambda(\{\xi_h\})$ is the rightmost singularity to be found from the equation

$$\det(\delta_{qq'} - \int dx d(x)(1-x)^\lambda \sum_h \xi_h R^h_{qq'}) = 0. \tag{2.43}$$

We define α by

$$\alpha \int dx d(x)(1-x)^\lambda = 1, \tag{2.44}$$

and seek for the largest eigenvalue of

$$\det(\alpha\delta_{qq'} - \sum_h \xi_h R^h_{qq'}) = 0. \tag{2.45}$$

For $\alpha = 1$ corresponding to $\xi_h = 1$ we have $\lambda = 0$. From (2.42) we obtain the asymptotic behaviour of the correlation coefficients

$$f^{(n)}_{h_1 \dots h_n} \sim \left. \frac{\partial^n \lambda}{\partial \xi_{h_1} \dots \partial \xi_{h_n}} \right|_{\xi_h = 1} \log\left(\frac{P}{\mu}\right). \tag{2.46}$$

For a momentum sharing function $d(z) = (\beta+1)(1-z)^\beta$ we note that λ is linear in α, $\lambda = (\beta+1)(\alpha-1)$. Then there are no correlation else in the asymptotic multiplicity distribution than due to the quantum number structure of the chain

$$f^{(n)}_{h_1 \dots h_n} \sim \left. \frac{\partial \lambda}{\partial \alpha} \right|_{\alpha=1} \left. \frac{\partial^n \alpha}{\partial \xi_{h_1} \dots \partial \xi_{h_n}} \right|_{\xi_h = 1} \log\left(\frac{P}{\mu}\right). \tag{2.47}$$

Obviously superpositions of different functions $d(z)$ of the kind mentioned above produce dynamical correlations, i.e. λ is no longer linear in α. This applies for instance to Feynman's choice [5]. But also in this case the correlations are weak in the sense that all $f^{(n)} \propto \log(P/\mu)$, i.e. are proportional to the multiplicities.

2.4. Two-particle distributions

At first we want to consider the two-particle distributions for directly produced particles and resonances. $D_{h_1 h_2/q}(z_1, z_2)$ fulfills the integral equation

$$\begin{aligned} D_{h_1 h_2/q}(z_1, z_2) &= d_{h_1/q}(z_1) \frac{1}{1-z_1} D_{h_2/q\bar{h}_1}\left(\frac{z_2}{1-z_1}\right) + (1 \leftrightarrow 2) \\ &+ \sum_h \int \frac{dy}{(1-y)^2} d_{h/q}(y) D_{h_1 h_2/q\bar{h}}\left(\frac{z_1}{1-y}, \frac{z_2}{1-y}\right), \end{aligned} \tag{2.48}$$

which is solved by Mellin transformation with respect to z_1 and z_2

$$\tilde{D}_{h_1 h_2/q}(k_1, k_2) = \int D_{h_1 h_2/q}(z_1, z_2) z_1^{k_1 - 1} z_2^{k_2 - 1} dz_1 dz_2. \tag{2.49}$$

The resulting algebraic equation is solved by the following expression (we suppress here quark indices, matrix multiplication is understood)

$$\begin{aligned} \tilde{D}_{h_1 h_2}(k_1, k_2) = &\{1 + \sum_{n=1}^{\infty} \tilde{h}(k_1 + k_2 - 1)^n R^n\} \{R^{h_1} \tilde{h}(k_1, k_2) \tilde{D}_{h_2}(k_2) \\ &+ R^{h_2} \tilde{h}(k_2, k_1) \tilde{D}_{h_1}(k_1)\}. \end{aligned} \tag{2.50}$$

$\tilde{h}(k)$ has been defined in (2.15), and

$$\tilde{h}(k_1, k_2) = \int dz d(z) z^{k_1-1}(1-z)^{k_2-1}. \tag{2.51}$$

The Mellin transformed two particle distribution has a leading particle contribution

$$\tilde{D}^{L}_{h_1h_2/q}(k_1, k_2) = R^{h_1}_{q,q\bar{h}_1}\tilde{h}(k_1, k_2)\tilde{D}_{h_2/q\bar{h}_1}(k_2)+(1 \leftrightarrow 2) \tag{2.52}$$

if either h_1 or h_2 are directly coupled to the quark q. Using the results (2.35) for the single particle distributions we find

$$\tilde{D}^{L}_{h_1h_2/q}(z_1, z_2) = R^{h_1}_{q,q\bar{h}_1}R^{h_2}_{q\bar{h}_1,q\bar{h}_1\bar{h}_2}d(z_1)\frac{1}{1-z_1}d\left(\frac{z_2}{1-z_1}\right)+(1 \leftrightarrow 2)$$

$$+R^{h_1}_{q,q\bar{h}_1}C^{h_2}d(z_1)\frac{1}{1-z_1}D^{NL}\left(\frac{z_2}{1-z_1}\right)+(1 \leftrightarrow 2). \tag{2.53}$$

For the remaining nonleading contribution we take advantage of the selfreproducing property $R^2 = R$. Thus we obtain

$$\tilde{D}^{NL}_{h_1h_2/q}(k_1, k_2) = \frac{\tilde{h}(k_1+k_2-1)}{1-\tilde{h}(k_1+k_2-1)}\sum_{q_1} R_{qq_1}\tilde{D}^{L}_{h_1h_2/q_1}(k_1, k_2) \tag{2.54}$$

independent of q. We define, in addition to C^h (equation (2.33)), the coefficients

$$C^{h_1h_2} = \sum_q P_q R^{h_1}_{q,q\bar{h}_1}R^{h_2}_{q\bar{h}_1,q\bar{h}_1\bar{h}_2} \tag{2.55}$$

and notice that for $d(z) = (\beta+1)(1-z)^\beta$ the original of the Mellin transform

$$\frac{\tilde{h}(k_1+k_2-1)}{1-\tilde{h}(k_1+k_2-1)}\tilde{D}^{L}(k_1, k_2)$$

is given by the expression

$$(\beta+1)\int_0^{1-(z_1+z_2)}\frac{dz}{(1-z)^3}D^{L}\left(\frac{z_1}{1-z}, \frac{z_2}{1-z}\right).$$

Thus we obtain the two-particle distribution (for directly produced particles)

$$D_{h_1h_2/q}(z_1, z_2) = D^{L}_{h_1h_2/q}(z_1, z_2)+D^{NL}_{h_1h_2/q}(z_1, z_2), \tag{2.56}$$

with

$$D^{NL}_{h_1h_2/q}(z_1, z_2) = C^{h_1h_2}(\beta+1)\int\frac{dz}{(1-z)^2}d\left(\frac{z_1}{1-z}\right)\frac{1}{1-z-z_1}d\left(\frac{z_2}{1-z-z_1}\right)$$

$$+C^{h_1}C^{h_2}(\beta+1)\int\frac{dz}{(1-z)^2}d\left(\frac{z_1}{1-z}\right)\frac{1}{1-z-z_1}D^{NL}\left(\frac{z_2}{1-z-z_1}\right)+(1 \leftrightarrow 2). \tag{2.57}$$

For a study of two-particle distributions within a jet one has to consider explicitly resonance contributions. We plan to do an analysis of correlations elsewhere.

3. *Baryon production and diquark jets*

Now we want to extend the scheme developed for meson production from quark like jets to baryon production. Once a baryon has been emitted one has to deal with the fragmentation of a jet with antidiquark quantum numbers. In this way quark and diquark jet fragmentation functions become coupled. In view of poor data on baryon spectra in leptoproduction and also in e^+e^- annihilation we are going to present here only equations and results concerning total meson ($M_q = M_{\bar{q}}$, $M_d = M_{\bar{d}}$) and baryon distributions ($B_q = \bar{B}_{\bar{q}}$, $\bar{B}_q = B_{\bar{q}}$, $B_d = \bar{B}_{\bar{d}}$, and $\bar{B}_d = B_{\bar{d}}$) from quarks q, antiquarks $\bar{q}$, diquarks d and antidiquarks $\bar{d}$, respectively. These total distributions are then flavor independent. A more detailed description is straightforward. Our Mellin transformed chain decay equations are

$$
\begin{aligned}
\tilde{M}_q &= p_1\tilde{d}_1 + p_1\tilde{h}_1\tilde{M}_q + (1-p_1)\tilde{h}_2\tilde{M}_{\bar{d}},\\
\tilde{B}_q &= (1-p_1)\tilde{d}_2 + p_1\tilde{h}_1\tilde{B}_q + (1-p_1)\tilde{h}_2\tilde{B}_{\bar{d}},\\
\tilde{\bar{B}}_q &= p_1\tilde{h}_1\tilde{\bar{B}}_q + (1-p_1)\tilde{h}_2\tilde{\bar{B}}_{\bar{d}},\\
\tilde{M}_{\bar{d}} &= p_2\tilde{d}_4 + p_2\tilde{h}_4\tilde{M}_{\bar{d}} + (1-p_2)\tilde{h}_3\tilde{M}_q,\\
\tilde{B}_{\bar{d}} &= p_2\tilde{h}_4\tilde{B}_{\bar{d}} + (1-p_2)\tilde{h}_3\tilde{B}_q,\\
\tilde{\bar{B}}_{\bar{d}} &= (1-p_2)\tilde{d}_3 + p_2\tilde{h}_4\tilde{\bar{B}}_{\bar{d}} + (1-p_2)\tilde{h}_3\tilde{\bar{B}}_q.
\end{aligned}
\tag{3.1}
$$

The momentum distributions $d_i(z)$ for firstly emitted mesons from quarks ($d_1(z)$) and antidiquarks ($d_4(z)$), baryons from quarks ($d_2(z)$) and antibaryons from antidiquarks ($d_3(z)$) are normalized: $\int d_i(z)dz = 1$. In a single decay step, p_1 is the probability for meson emission from a quark jet, and $1-p_1$ for baryon emission. From baryon multiplicities in e^+e^- annihilation [16] and in the current fragmentation region of lepton proton interactions [17] p_1 is expected to be slightly less than unity. p_2 is the probability for meson emission from a (anti)diquark jet, and $(1-p_2)$ for (anti)baryon production. From quark combinatorics [18] they are expected to be comparable in magnitude, $p_2 \approx \frac{1}{2}$. Lacking better arguments we have chosen this value and tried to understand the data. The resulting distributions depend critically on this parameter in shape. The transformed functions $\tilde{h}_i(k)$ are related to $d_i(z)$ in the usual way (Eq. (2.15)). The solutions of the system of equations (3.1) are the following Mellin transformed expressions

$$
\begin{aligned}
\tilde{M}_q &= \tilde{M}_{\bar{q}} = \Delta^{-1}\{(1-p_2\tilde{h}_4)p_1\tilde{d}_1 + (1-p_1)\tilde{h}_2 p_2\tilde{d}_4\},\\
\tilde{B}_q &= \tilde{\bar{B}}_{\bar{q}} = \Delta^{-1}(1-p_2\tilde{h}_4)(1-p_1)\tilde{d}_2,\\
\tilde{\bar{B}}_q &= \tilde{B}_{\bar{q}} = \Delta^{-1}(1-p_1)\tilde{h}_2(1-p_2)\tilde{d}_3,\\
\tilde{M}_d &= \tilde{M}_{\bar{d}} = \Delta^{-1}\{(1-p_1\tilde{h}_1)p_2\tilde{d}_4 + (1-p_2)\tilde{h}_3 p_1\tilde{d}_1\},\\
\tilde{B}_d &= \tilde{\bar{B}}_{\bar{d}} = \Delta^{-1}(1-p_1\tilde{h}_1)(1-p_2)\tilde{d}_3,\\
\tilde{\bar{B}}_d &= \tilde{B}_{\bar{d}} = \Delta^{-1}(1-p_2)\tilde{h}_3(1-p_1)\tilde{d}_2,
\end{aligned}
\tag{3.2}
$$

where

$$\Delta = (1-p_1\tilde{h}_1)(1-p_2\tilde{h}_4)-(1-p_1)\tilde{h}_2(1-p_2)\tilde{h}_3. \tag{3.3}$$

We have specified further the shape of the momentum sharing functions $d_i(z)$. $d_1(z) = 2(1-z)$ was found to be suitable for $e^+e^- \to$ mesons, as will be discussed in Section 4. In view of the shape of the antiproton spectra known from e^+e^- annihilation [16] we took $d_2(z) = 3(1-z)^2$ similar to the pion spectra (determined essentially by vector meson decay, see Section 4). Since a diquark is a $\bar{3}$ of colour we expect its momentum sharing within a diquark like jet to be the same as that of a quark in a quark like jet. For the recombination into a baryon we took also $d_3(z) = d_1(z) = 2(1-z)$. Finally we chose for $d_4(z)$ describing the meson emission from a diquark jet $d_4(z) = 3(1-z)^2$ since recombination into meson requires the diquark to split. As a mnemonic for our choice we note a quark counting rule

$$d_i(z) = (n_i+1)(1-z)^{n_i}, \tag{3.4}$$

where n_i is the number of remaining elementary constituents, instead of the dimensional counting rule [19]. There are only two free parameters p_1 and p_2 left. We wish to draw attention to the fact that the ratio of the plateau heights for mesons and baryons is determined only by these probabilities, irrespective of the specific form of the momentum sharing functions. Since $\tilde{h}_i(k=1) = \tilde{d}_i(k=1) = 1$, from Eq. (3.2) follows immediately

$$zB_q(z),\, zB_d(z) \xrightarrow[z\to 0]{} H_B,$$

$$zM_q(z),\, zM_d(z) \xrightarrow[z\to 0]{} H_M, \tag{3.5}$$

and

$$\frac{H_B}{H_M} = \frac{(1-p_1)(1-p_2)}{p_1(1-p_2)+(1-p_1)p_2} = \frac{1}{p_1/(1-p_1)+p_2/(1-p_2)}; \tag{3.6}$$

this ratio is mainly determined by $p_1, p_1 > p_2$. We have used this ratio to fix p_1.

We do not intend to reproduce here the analytic expressions of all the fragmentation functions. For the final calculation of the meson spectra we have used the ratio of directly produced pseudoscalars to vector mesons determined from e^+e^- data (see the following Section 4) and treated the decay of vector mesons according to the approximate method (Eq. (2.37)).

We have assumed that the baryon distributions are not essentially altered due to eventual resonance decay. Baryon resonance decay has to be taken into account more carefully in confrontation with better data.

4. Comparison with data and discussion

In order to test and to extend the applicability of the chain decay model we proceed in several steps. At first we use the momentum spectra of charged hadrons in e^+e^- annihilation below charm threshold at $s = 13\ \text{GeV}^2$ measured by the PLUTO collaboration [15]

(Fig. 1) to fix the momentum sharing function for q-jet → meson. We chose the form $d(z) = 2(1-z)$. When all mesons are assumed to be produced directly from the chain decay, the model gives a too flat distribution (dashed line). In the normalization the experimentally observed neutral energy excess [20] is used to determine here the fraction of

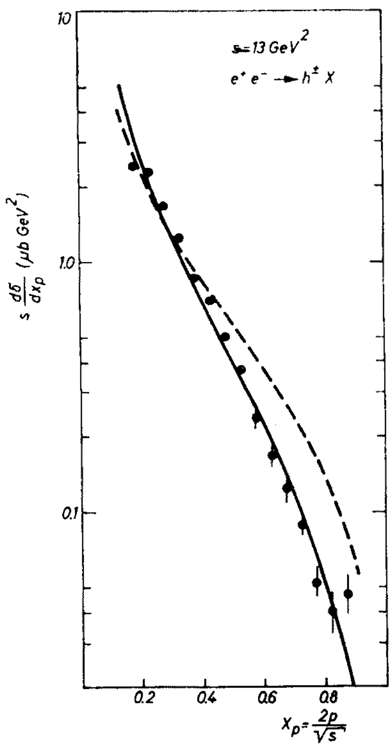

Fig. 1

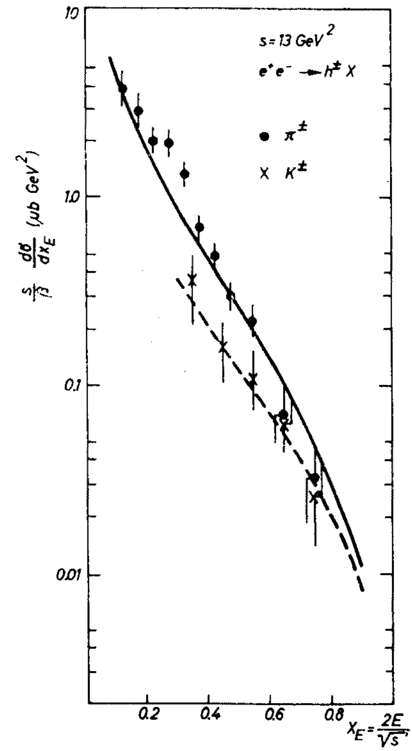

Fig. 2

Fig. 1. Charged hadron spectra in e⁺e⁻ annihilation at s = 13 GeV² from PLUTO [15] compared with the quark fragmentation model. Dashed line: all pseudoscalar mesons are directly produced, full line:

$$\frac{\text{directly produced pseudoscalars}}{\text{directly produced vector mesons}} = \frac{1}{3}$$

Fig. 2. Charged pion and kaon spectra in e⁺e⁻ annihilation at s = 13 GeV² from DASP [16] compared with the quark fragmentation model with SU(3) symmetry. Full line: $\pi^{\pm}$, dashed line: $K^{\pm}$

charged mesons in the total meson distribution. Inducing pseudoscalar meson production partially through vector meson decay we find a very good agreement with data (full line). We have varied the ratio α_{PS}/α_V (tensor mesons are neglected) and have found a ratio $\alpha_{PS}/\alpha_V = 1/3$ to be a optimal choice.

Including quantum numbers we compare this fragmentation model with pion and kaon distributions measured versus $x_E = 2E/\sqrt{s}$ by the DASP collaboration in $e^+e^- \to h^{\pm}X$ also at s = 13 GeV² [16] (Fig. 2). The relative abundance of pions and kaons is reproduced under the assumption that no suppression of $s\bar{s}$ pairs is present ($\kappa = 1$). The observed difference in the spectra is described only by the different contributions from resonance decay.

In e^+e^- annihilation antibaryon spectra are also measured [16]. Therefore we take the model extended to include baryon and antibaryon production from quark and antiquark like jets. As argued in the preceding Section we have chosen the required momentum sharing functions $d_i(z)$ in the specific form

$$d_i(z) = (n_i+1)(1-z)^{n_i} \tag{4.1}$$

where n_i are the numbers of remaining elementary constituents, 1 for a remaining (anti)quark like jet, 2 for a (anti)diquark like jet. We have taken $p_2 = \frac{1}{2}$ and fixed p_1 by Eq. (3.6): $p_1 = \frac{7}{8}$. In Fig. 3 we compare the antiproton spectrum vs. x_E in e^+e^- annihilation at $s = 13\ \mathrm{GeV}^2$ [16] under the assumption that 50% of the antibaryons are antiprotons. With this assumption the normalization is fixed.

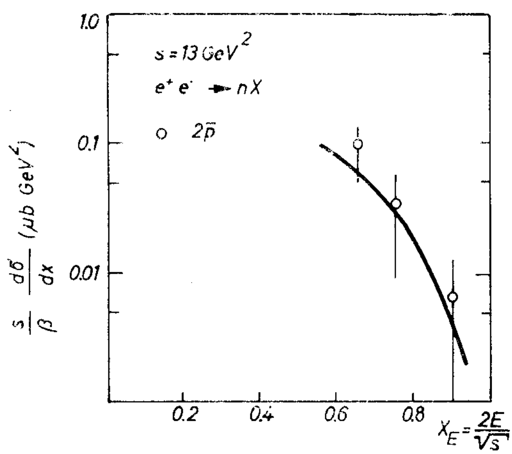

Fig. 3. Twice the antiproton spectrum in e^+e^- annihilation at $s = 13\ \mathrm{GeV}^2$ from DASP [16] compared with the quark fragmentation model with mesons and (anti)baryons

After the extension to baryon production also jets with (anti)diquark quantum numbers appear in the model. It is interesting to test the model predictions for their fragmentation more in detail by confronting them with data from deep inelastic lepton proton scattering. In the quark-parton model there is a space like vector boson sent from the lepton vertex and absorbed by a valence quark at sufficiently large x_{Bj} ($x_{\mathrm{Bj}} > 0.2$). Then there evolve two jets with quark and diquark quantum numbers, respectively, forming an excited color singlet baryonic system with mass W. Particle distributions in the vector boson-nucleon CMS should then scale in x_{F} defined here as $x_{\mathrm{F}} = 2p^*_{\parallel h}/W$ (and not $x_{\mathrm{F}} = p^*_{\parallel h}/p^*_{\mathrm{N}}$). We expect asymptotically

$$|x_{\mathrm{F}}| \frac{dN_{\mathrm{M}}}{dx_{\mathrm{F}}} = |x_{\mathrm{F}}| M_{\mathrm{q/d}}(|x_{\mathrm{F}}|), \qquad x_{\mathrm{F}} \gtrless 0,$$

$$|x_{\mathrm{F}}| \frac{dN_{\mathrm{B}}}{dx_{\mathrm{F}}} = |x_{\mathrm{F}}| B_{\mathrm{q/d}}(|x_{\mathrm{F}}|), \qquad x_{\mathrm{F}} \gtrless 0, \tag{4.2}$$

and compare this with data. In Fig. 4 we show a preliminary Λ distribution versus $x_F = p^*_{\|h}/p^*_{h\|max}$ measured in ep collisions by the DECO collaboration [21]. The normalization was not reported. Thus this figure is only a comparison of the shape of the baryon spectrum from our model with data for baryon production otherwise not available. We find agreement in the general shape. The equal plateau height in the asymptotic model result guarantees a smooth connection between the target and current hemispheres. Of course, the detailled shape near $x_F = 0$ is modified by kinematics as the emitting system becomes less massive, and the factorization assumption breaks then down for heavy particle emission.

Meson and baryon production (full and dashed lines) in the target fragmentation region as described by the model are compared with data from $\bar{\nu}$p charged current interac-

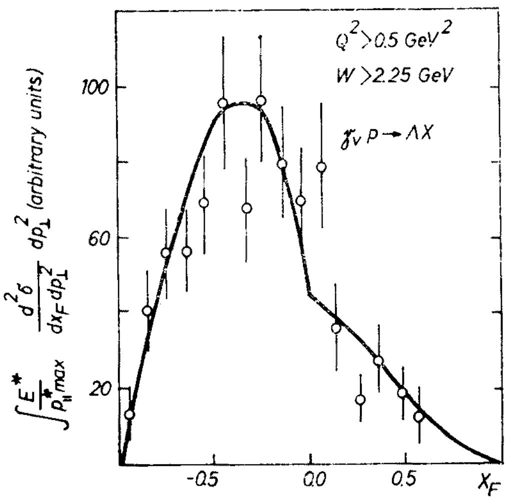

Fig. 4. Feynman $x_F = 2p^*_{\|h}/W$ distribution of electroproduced Λ from the Cornell-DESY collaboration [21] compared with the typical baryon spectrum from quark and diquark jet fragmentation

tions reported from the Fermilab 15 foot hydrogen bubble chamber [22] in Fig. 5. In this experiment hadronic systems in the range $1 < W^2 < 50$ GeV^2 have been investigated, and a charged hadron multiplicity $\langle n_{Ch} \rangle \sim 1.22 \log W^2$ almost independent of q^2 has been found. For this comparison we have assumed: 75% of directly produced mesons to be vector mesons (as suggested by e^+e^- data), 50% of produced baryons are protons, and the π/K ratio vs. x_F analysed before has been applied to find charged pion spectra among the total meson distribution. Experimentally protons with larger $p_{lab} > 1$ GeV/c are not identified. Therefore the model prediction should not be compared in shape with the identified proton distribution. We have included this in Fig. 5 because of the relative good agreement in normalization.

From our point of view $x_F = 2p^*_{\|h}/W$ as used by experimentalists is the natural variable for the distributions to scale in. (Starred quantities refer to the hadronic CMS.) If we take the variable $\hat{x}_F = p^*_{\|h}/p^*_N$ as used from hadron-hadron interactions we should expect, say, in the target fragmentation region

$$|\hat{x}_F| \frac{dN_M}{d\hat{x}_F} = \frac{|\hat{x}_F|}{1-x_{Bj}} M_d\left(\frac{|\hat{x}_F|}{1-x_{Bj}}\right). \tag{4.3}$$

Pokorski and Van Hove [10] have considered hadron fragmentation and leading particle effect in a particular quark recombination model. Specific for their approach is the notion

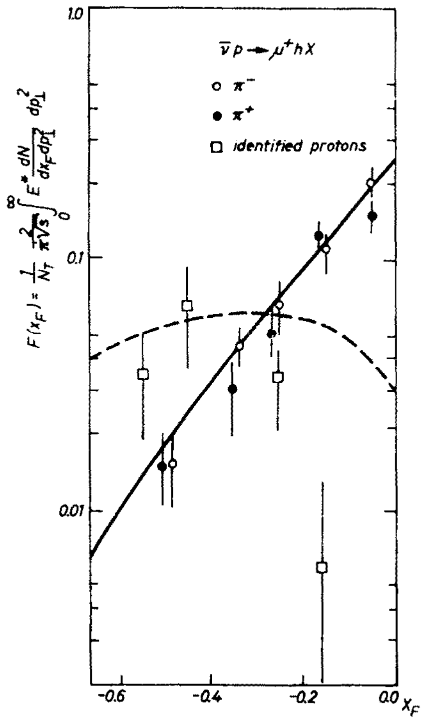

Fig. 5. Feynman x_F distribution of $\pi^{\pm}$ and identified protons in the target fragmentation region in $\nu_\mu p$ interactions [22] compared with the pion (full line) and proton spectrum (dashed line) calculated for a diquark jet

of a joint valence quark distribution $g(x_1, x_2, x_3)$ in the baryon. If in deep inelastic scattering the valence quark q_3 is hit by the probing current ($x_3 = Q^2/2m_p\nu$), the $\hat{x}_F$ distribution of the baryon fragment is obtained via recombination with a wee ($x_{Bj} \approx 0$) quark

$$\frac{dN}{d\hat{x}_F} = \frac{f(\hat{x}_F, x_3)}{\int^{1-x_3} f(x', x_3)dx'}, \tag{4.4}$$

where

$$f(x, x_3) = \int^{x} g(x-x_2, x_2, x_3)dx_2 \tag{4.5}$$

is the unnormalized $x = \hat{x}_F$ distribution of the two valence quarks q_1 and q_2. According to their model one expects scaling in $\hat{x}_F$, whereas the distribution in $\tilde{x} = \hat{x}_F/(1-x_3)$ (= x_F)

should and does not [10] scale at all. $\tilde{x}$ is the scaled baryon momentum relative to the through-going diquark jet momentum, which is our scaling variable found to be the relevant one also in experiment.

We propose to test the adequacy of the notion of jets with diquark and quark quantum numbers in general and their description by the chain model also in hadronic reactions. Triggering on Drell-Yan produced lepton pairs (of relatively high mass, say $M^2/s = x^2 \approx 0.05 \ldots 0.1$) at rapidity $y_{\mu^+\mu^-} = 0$, in meson induced reactions one should be able to study whether forward and backward production may be understood as quark and diquark jet fragmentation. If this concept is reliable meson and baryon distributions in the forward respective backward hemisphere will follow the rescaled distributions (D is either M or B)

$$|\hat{x}_F| \frac{dN}{dx_F} = \frac{|\hat{x}_F|}{1-x} D_{q/d}\left(\frac{|\hat{x}_F|}{1-x}\right), \quad \hat{x}_F = \frac{2p_\parallel^*}{\sqrt{s}} \gtrless 0. \tag{4.6}$$

We summarize our discussion:

1. We have demonstrated that the chain decay model can be successfully used for purely mesonic distributions from (anti)quark like jets with an appropriate amount of vector meson production. This view is shared by others but we give a very convenient formalism and parametrization.

2. This model can be extended to (anti)baryon production and (anti)diquark jets with reasonable assumptions concerning relative probabilities and momentum sharing functions thus providing a suitable parametrization, at least.

3. Model independent but inherent for the concept of quark and diquark jets and supported by deep inelastic experiments is scaling in x_F defined relative to the decaying hadronic system irrespective of the way it is produced.

4. Assigning the notion of diquark like jets an equally universal phenomenological relevance as granted to quark like jets (which we, following Brodsky [19], are inclined to do) one is able to make predictions for certain hadron induced processes, too. The scaling law (4.6) is then an inevitable consequence.

5. We found it necessary to abandon the dimensional counting rules [19] and summarize our attempts to account for deep inelastic data in the counting rule (4.1) for the leading component of particle spectra.

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