PRODUCTION OF DILEPTONS DURING THE SPACE-TIME EVOLUTION OF HADRONIC COLLISIONS

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The quark-parton model description of the space-time evolution of hadronic collisions is shown to lead to a two component model of dilepton production. The Drell-Yan mechanism takes into account the possible annihilations of those quarks and antiquarks which are present already in the incoming hadrons. This process is fast (operative at the beginning of the collision) and hard (long range in rapidity) and it is responsible for the production of dimuons with large masses. The second component describes the annihilations of quarks and antiquarks created during the collision. Due to the constraints imposed by the space-time evolution of the collision, only quarks and antiquarks separated by small rapidity gaps can annihilate what leads to the production of dimuons with low masses. Monte Carlo model containing both components is shown to be in good qualitative agreement with data available at FNAL energies. The y- and p_T-distributions of quarks and antiquarks entering the calculations were fixed by data on multiparticle production.

1. *Introduction*

During the past decade considerable attention has been given to the production of leptons in hadronic collisions. The history of the subject and the state of the art are described in recent reviews [1–3]. At present, the interest in lepton production is motivated by rather surprising experimental results as well as by the hope to obtain a more direct information about the dynamics of hadron collisions.

To make the point more clear let us suppose that a hadron consists of quarks (Q' s) and antiquarks ($\overline{\text{Q}}$' s) moving relatively freely within some region of confinement. Any Q which, as a result of collision, tends to leave the system is transformed by the confinement forces into a jet of hadrons. In contradistinction to that, a lepton pair created in the interior of the hadron system is not influenced by the confinement forces and brings out an immediate snapshot of the situation inside. Because of that the dilepton production may be a rather stringent test of models of hadronic collisions.

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Basic features of the data on lepton production can perhaps be summarized into the following points:

(*i*) lepton/pion ratio at large p_T ($p_T > 1$ GeV/c) and high energies is about 10^{-4} and it is roughly independent of both p_T and s [1–3],

(*ii*) lepton/pion ratio at low p_T decreases with increasing longitudinal momentum [4, 5],

(*iii*) lepton/pion ratio increases with decreasing p_T within the range $0.3 < p_T < 1.0$ GeV/c [6],

(*iv*) the cross section $d\sigma/dM_{\mu\mu}$ shows a prominent enhancement at low dimuon masses [5],

(*v*) The production of dileptons with large masses (above the J/ψ region) seems to be well described [7–10], by the Drell-Yan mechanism [11].

As shown by the Chicago-Princeton data and calculations [5], a substantial part of single leptons comes from lepton pairs. A particularly important contribution is provided by copiously produced low-mass continuum. The origin of this continuum is not yet clear.

The points (*i*) to (*v*) listed above are not independent. The production of low mass dilepton continuum seems to be responsible for (*ii*), (*iii*) and (*iv*). To clarify item (*i*), the data on the production of dileptons with low mass and large p_T are required.

In this paper we present a dynamical model of the origin of low mass dimuon continuum (for a brief exposition of the main point and a few results see Ref. [12]) produced in hadronic collisions. The model is based on the three following assumptions:

(*i*) The annihilations of Q's and $\overline{\mathrm{Q}}$'s created during the collision are responsible for important part of dileptons being produced (Bjorken and Weisberg [13]),

(*ii*) The creation of these Q's and $\overline{\mathrm{Q}}$'s is regulated by the Bjorken [14] – Gribov [15] picture of the space-time evolution of hadronic collision. This point is particularly important, since the constraints coming from space-time evolution permit only the annihilation of a Q and an $\overline{\mathrm{Q}}$ separed by a small rapidity gap. In fact, due to the dilatation of time, different rapidity regions get excited at different moments and a Q and an $\overline{\mathrm{Q}}$ separated by a large rapidity gap do not exist simultaneously,

(*iii*) The distribution of Q's and $\overline{\mathrm{Q}}$'s created during the collision has to be closely related to the observed features of multiparticle production, since hadrons in multiparticle final state are supposed to come from the recombinations of Q's and $\overline{\mathrm{Q}}$'s. We have recently been involved in the construction of such a model [16, 17] and we have used here the procedures for the Monte Carlo generation of the exclusive configurations of Q's and $\overline{\mathrm{Q}}$'s borrowed from it. These procedures are somewhat model-dependent, but the differences among the possible models of this type can hardly influence qualitative conclusions.

In our model the total production of dileptons in hadronic collisions is thus the sum of two components. One of them describes the annihilation of Q's and $\overline{\mathrm{Q}}$'s created during the collision and it is responsible for the production of dileptons with low masses (hereafter referred to as LM component). The second one gives the contribution of the Drell-Yan mechanism (DY component) and it is responsible for the production of dileptons with large masses. Dynamics behind these components is rather different. The former (LM) is in action during the whole evolution of the collision, while the latter (DY) is operative only at the early stage of the collision.

We calculate here both components by taking into account the transverse momenta and effective masses of quarks. The sum of LM and DY components is then compared with the data at FNAL energies [5–10]. Due to the ignorance of nuclear effects this comparison serves only for illustrative purposes. Still, we have found it quite encouraging.

The paper is organized as follows. In the next Section we briefly review the method for generating the exclusive configurations of quarks and antiquarks used in our Monte Carlo quark-parton model [16, 17] of multiparticle production in hadron collisions. The Q's and $\overline{Q}$'s generated here contain both original valence quarks, the original sees of both hadrons and the $Q\overline{Q}$ pairs created during the collision. In Section 3 we briefly review the Bjorken-Gribov description of the space-time evolution of hadronic collisions. The constraints imposed by this space-time evolution are essential for the shape of the LM dimuon continuum. This section contains also the derivation of formulae for the LM component in the dimuon production.

Although the main topic of the present paper is the origin of the low mass continuum we have found it useful to calculate also the contribution of the Drell-Yan process, so that the contributions of both DY and LM components can be compared. The DY contribution is calculated in Section 4. The whole procedure is rather standard, except perhaps for the way in which the transverse momenta and effective masses of Q's and $\overline{Q}$'s are introduced. This point is of importance in discussing the dependence of $\langle p_T \rangle$ of dimuons on $M_{\mu\mu}$.

The results of our calculations are presented and compared with the data in Section 5. The concluding Section 6 contains a few comments and suggestions of further tests of the proposed origin of the low-mass dilepton continuum.

2. The distribution of quarks and antiquarks in the system formed by the two colliding hadrons

In an attempt to explain the origin of dimuons produced in hadronic collisions by the annihilation of Q's and $\overline{Q}$'s created during the collision [13] one needs first to know, or to assume something about the distribution of such Q's and $\overline{Q}$'s. In the present paper we shall use the distributions of Q's and $\overline{Q}$'s following from our study [16, 17] of multiparticle production in hadronic collision. The obvious advantage of such a procedure is that all the free parameters of the distribution are completely fixed by the data on multiparticle production.

In the model of Refs [16] and [17] it is assumed that a hadron-hadron collision proceeds in a way assumed by the simple quark-parton model. The collision is initiated by the interaction of the wee partons and the two colliding hadrons form in a sense a compound system. During the evolution of collision the gluons present in the incoming hadrons are converted to $Q\overline{Q}$ pairs and finally the neighbouring (in rapidity) $Q\overline{Q}$ pairs recombine to mesons and QQQ ($\overline{QQQ}$) triplets to baryons (antibaryons).

We do not try to make a quantitative description of the whole process. Instead, we construct an explicit model for the distribution of Q's and $\overline{Q}$'s just prior to the recombination. This distribution has to fulfill the following simple criteria

(*i*) the transverse momenta of Q's and $\overline{\text{Q}}$'s should be limited,
(*ii*) the valence quarks should keep large momentum fractions,
(*iii*) the distribution, completed by an explicit prescription for the recombination should lead to (at least) a qualitative description of the multiparticle production.

The simplest way how to respect the item (*i*) is to assign to a configuration specified by the momenta $\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_N$ of Q's and $\overline{\text{Q}}$'s a probability dictated by the cylindrical phase space

$$dC(\vec{p}_1, \ldots, \vec{p}_N) = \exp\left(-\sum \vec{p}_{\mathrm{T}i}^{\,2}/R^2\right)\delta\left(\sum \vec{p}_i\right)\delta\left(\sqrt{s}-\sum E_i\right)\prod \frac{d^3p_i}{2E_i},$$

where the symbols are selfexplanatory and we are using the c.m. system of the hadronic collision.

In order to fulfill the requirement (*ii*) we have to multiply $dC(\vec{p}_1, \ldots, \vec{p}_N)$ by a factor which gives a larger probability to configurations where valence quarks keep larger momentum fractions $x_i = 2p_{\mathrm{L}i}/\sqrt{s}$. Following the work by Kuti and Weisskopf [19] we choose

$$V(x_1, \ldots, x_6) = \prod_1^6 \sqrt{|x_i|},$$

where the indices 1, ..., 6 denote the six valence quarks in a nucleon-nucleon collision.

In order to have a correct average multiplicity of hadrons produced by the recombination we have to weight appropriately the configurations with various numbers of the $\text{Q}\overline{\text{Q}}$ pairs. This is accomplished by a suitable choice of the "coupling" constant G. Putting all this together we have that the probability of a particular configuration of 6 valence quarks plus n additional Q's and n $\overline{\text{Q}}$'s can be written in the following way

$$dP_N(y_1, \vec{p}_{\mathrm{T}1}, \ldots, y_N, \vec{p}_{\mathrm{T}N}) = KW_{\mathrm{id}}G^n\left(\prod_1^6 \sqrt{|x_i|}\right)\exp\left(-\sum_1^N p_{\mathrm{T}i}^2/R^2\right)$$
$$\times\,\delta\left(\sum \vec{p}_i\right)\delta\left(E-\sum_i E_i\right)\prod_1^N (dy_i d^2p_{\mathrm{T}i}). \qquad (1)$$

Here $N = 2n+6$, K is a constant, W_{id} is a factor for identical particles, G is the "coupling constant", x_i are the longitudinal momenta, R is the cut-off on transverse momenta, $E = (s)^{1/2}$ and y_i are rapidities of Q's and $\overline{\text{Q}}$'s. Quantum numbers of the Q's and $\overline{\text{Q}}$'s which are not valence are determined at random, with strange Q's being suppressed by a phenomenological parameter λ specifying the ratio of probabilities for a particular Q to be an s-quark or to be an u-quark[1].

In studying the multiparticle production we have been using [16, 17] the distribution (1) and a procedure for recombination of neighbours in rapidity to mesons, baryons and antibaryons. In our opinion the results [16, 17] show that a good qualitative description of the p_{T}- and y-dependence of various single particle spectra in pp collisions with $100 < p_{\mathrm{lab}} < 1500$ GeV/c is obtained with $G = 1.15$, $R^2 = 0.20$ (GeV/c)^2 and $\lambda = 0.22$.

[1] In writing up the computer program we found Jadach's work [20] very useful.

We shall not discuss here further details concerning the multiple production, since all what we need here is the distribution of Q's and $\overline{\text{Q}}$'s. We have to stress however that the distribution given by Eq. (1) is to be considered as a sum of the distributions which are in fact materialized only gradually during the space-time evolution of the collision. For multiparticle production, where one recombines only the neighbours in rapidity, the explicit implementation of constraints required by the space-time evolution of the collision is not necessary. This is however not so for the production of dimuons. This topic is discussed in the next Section.

3. The space-time evolution of hadronic collision and the origin of low mass dileptons

We start with reviewing briefly the Bjorken-Gribov [14, 15] picture of the space-time evolution of hadronic collision following closely the Bjorken pictorial presentation [14]. Then on the basis of this picture we shall derive the formula for the production of low mass lepton pairs.

The whole picture is based on the short range (in rapidity) character of interactions which are responsible for the dynamics of hadron collisions, and on Lorentz invariance. The two colliding hadrons at the very beginning of the collision are shown in Fig. 1a (in

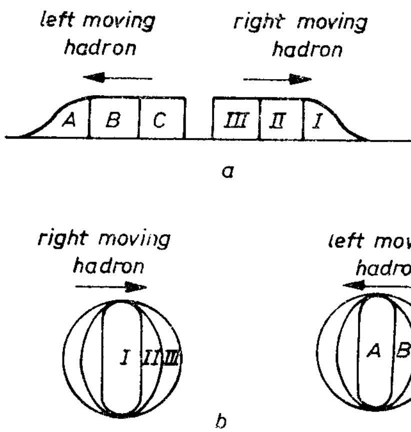

Fig. 1. Two colliding hadrons at the beginning of a collision: (a) in rapidity, (b) in space

rapidity) and in Fig. 1b (in space). Each of the regions I, II, III, A, B, C occupies about $\Delta y = 1$ in rapidity units (RU). The spatial extension of these regions is given by the corresponding Lorentz contraction, for instance, $V_B = V_0/\gamma(y_B)$, where $\gamma(y) = \cosh(y)$ is the Lorentz factor corresponding to the rapidity y and V_0 is the volume occupied by the hadron at rest, $V_0 \approx m_\pi^{-3}$.

Immediately after the situation shown in Fig. 1 the two hadrons pass through each other and the evolution of the collision is initiated by the interaction of "wee" partons (regions C and III). The Drell-Yan annihilation understood in the usual sense (annihilation of a Q from one hadron with an $\overline{\text{Q}}$ from the other one) can happen only during this short period when hadrons overlap in space and time, and quark distribution functions are still equal to those in free hadrons.

When the interaction starts (at $t = 0$), the two hadrons form a compound system excited at $y = 0$ (the c.m. system of hadron collision is used). The excited region around $y = 0$ cools down at a time scale $t_0 \sim m_\pi^{-1}$ by emitting hadrons and by heating up neighbouring regions in rapidity. The evolution of the collision is shown in Fig. 2. The excitation and cooling down of the region around y evolves at a dilated time scale $t \approx t_0 \cosh(y)$.

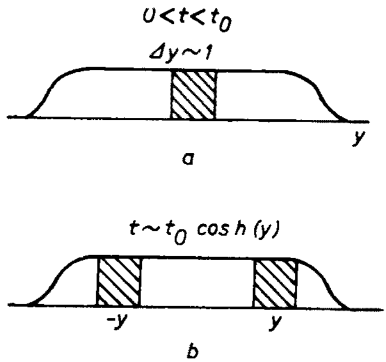

Fig. 2. The time evolution of a hadron collision: (a) at $0 < t < t_0$ where $t_0 \approx m_\pi^{-1}$, only the region around $y = 0$ is excited, (b) regions with rapidity $\pm y$ get excited at $t \approx t_0 \cosh(y)$

What actually happens during the excitation of a given rapidity region? Even if a detailed theory is missing, much can be guessed. From deep inelastic ep scattering it is known that in a free nucleon the density of $Q\bar{Q}$ pairs from the "sea" is about 0.6/RU. In multiparticle production about 3 particles (mostly mesons) are produced per RU. If they are to be obtained by the recombination of Q's and $\bar{Q}$'s, the density of $Q\bar{Q}$ pairs at the full excitation of a given rapidity region should be [13] about 3/RU. Another estimate [16] leads to the same number.

It is well-known that gluons (whatever they may be) carry about five times more momentum (in a free nucleon) than $Q\bar{Q}$ pairs from the "sea". It is reasonable to assume that the rapidity distributions of gluons and of "sea" are roughly the same. Assuming that during the excitation of a particular rapidity region gluons are first converted to $Q\bar{Q}$ pairs. one can express the density of $Q\bar{Q}$ pairs at the full excitation as

$$0.6\,\frac{\text{momentum of gluons}}{\text{momentum of "sea"}} = 0.6 \times 5 = 3/\text{RU}.$$

At the moment $t \approx t_0 \cosh(y)$ we have about 3 $Q\bar{Q}$ pairs within the region of volume $V(y) \approx V_0/\cosh(y)$ for time interval $t(y) \approx t_0 \cosh(y)$. The product $t(y)\ V(y)$ is independent of y and equal to $t_0 V_0$. If quarks move relatively freely within this volume we can calculate the number of annihilations in a way similar to that used in calculating the annihilation of positronium [18]. In order to perform this in detail one needs to know the distribution functions of Q's and $\bar{Q}$'s, in particular their y- and p_T-dependences.

Let $G_Q(y, \vec{p}_T)$ and $G_{\bar{Q}}(y, \vec{p}_T)$ denote the rapidity and p_T densities of Q's and $\bar{Q}$'s in the time when the region around the rapidity y is fully excited. These densities are just those which determine the amount of mesons created by the recombinations of Q's and $\bar{Q}$'s. Because of that we can take $G_Q(y, \vec{p}_T)$ and $G_{\bar{Q}}(y, \vec{p}_T)$ directly from our model on multiparticle production, i.e. calculate them in a straightforward way from the distribution given by Eq. (1). For the *spatial* density $\varrho_Q(y, \vec{p}_T)$ we have

$$\varrho_Q(y, \vec{p}_T) = \frac{G_Q(y, \vec{p}_T)}{V(y)}, \qquad V(y) = \frac{V_0}{\cosh(y)}. \tag{2}$$

If a quark within $dy_1 d^2p_{T1}$ is present in the excited region simultaneously with an anti-quark within $dy_2 d^2p_{T2}$ and if they are both in a region of volume V_1 whose excitation lasts for time t_1 then the expected number of annihilations $Q\bar{Q} \to \mu^+\mu^-$ is

$$dn = dy_1 dy_2 d^2p_{T1} d^2p_{T2} \varrho_Q(y_1, \vec{p}_{T1}) \varrho_{\bar{Q}}(y_2, \vec{p}_{T2}) |\vec{v}_Q - \vec{v}_{\bar{Q}}|$$
$$\times \sigma_A^Q(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2}) V_1 t_1,$$

where $\sigma_A^Q(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2})$ is the annihilation cross-section and other symbols are selfexplanatory. The expression is quite similar to the one used for the annihilation of a positronium [18]. The points is, however, that the regions around y_1 and y_2 are not excited simultaneously and as a consequence one has to multiply the r.h.s. by the function $w(y_1, y_2)$ which gives the overlap of the excitations of the two regions. The number of $Q\bar{Q}$ annihilations per hadronic collision is then given as

$$\Delta n = \sum_Q \int \ldots \int dy_1 dy_2 d^2p_{T1} d^2p_{T2} \varrho_Q(y_1, \vec{p}_{T1}) \varrho_Q(y_2, \vec{p}_{T2})$$
$$|\vec{v}_Q - \vec{v}_{\bar{Q}}| \sigma_A^Q(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2}) V_1 t_1 w(y_1, y_2). \tag{3}$$

Here V_1 is the volume by Q's with rapidity y_1 and t_1 is the time interval of the excitation of this region ($V_1 t_1 = V_0 t_0$), and

$$\sigma_A^Q = \frac{\alpha^2}{2E_Q 2E_{\bar{Q}} |\vec{v}_Q - \vec{v}_{\bar{Q}}|} \left(\frac{e_Q}{e}\right)^2 \frac{8\pi}{3} \sqrt{\frac{s - 4m_\mu^2}{s}} \left(1 + 2\frac{m_\mu^2}{s}\right)\left(1 + 2\frac{m_Q^2}{s}\right) \tag{4}$$

is the cross section for the annihilation $Q\bar{Q} \to \mu^+\mu^-$ which takes into account transverse momenta and finite effective masses of quarks. The dimuon mass squared is denoted as s. The other symbols are selfexplanatory.

The properties of the function $w(y_1, y_2)$ follow from the space-time evolution of the collision. The time excitation of the regions around y_1 and y_2 is shown in Fig. 3. In the rest frame of the region 1 we have $t_2' = t_1' \cosh(y_2 - y_1)$. The overlap of the two excitation curves is large if $y_1 = y_2$ and decreases fast with increasing $|y_1 - y_2|$. The overlap is expected to vanish if $t_2' \gtrsim 2t_1'$.

The function $w(y_1, y_2)$ depends thus only on $|y_1 - y_2|$, it is equal to 1 for $y_1 = y_2$ and decreases to zero at $|y_1 - y_2| \gtrsim 1$. In our calculations we have used the parametrization

$$w(y_1, y_2) = \exp[-A(y_1 - y_2)^2], \tag{5}$$

where the value of A is expected to be about 1.

Equation (3) gives the number of $Q\bar{Q}$ annihilations per inelastic hadron collision. In order to obtain the cross section for the production of dimuons in hadronic collisions we have to multiply Eq. (3) by corresponding inelastic cross section $\sigma_{\mathrm{inel}}^{\mathrm{hadr}}$. For practical calculations it is useful to express $\varrho_Q(y_1, \vec{p}_{T1})$ and $\varrho_{\bar{Q}}(y_2, \vec{p}_{T2})$ in terms of $G_Q(y_1, \vec{p}_{T1})$ and $G_Q(y_2, \vec{p}_{T2})$, and to replace the product $G_Q G_{\bar{Q}}$ by the corresponding joint probability distribution function

$$G_Q(y_1, \vec{p}_{T1}) G_{\bar{Q}}(y_2, \vec{p}_{T2}) \rightarrow G_Q(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2}). \tag{6}$$

Putting all this together we obtain

$$\sigma_{\mu\mu} = \frac{t_0}{V_0} \sigma_{\mathrm{inel}}^{\mathrm{hadr}} \sum_Q \int \cdots \int dy_1 dy_2 d^2 p_{T1} d^2 p_{T2} |\vec{v}_Q - \vec{v}_{\bar{Q}}|$$

$$G_Q(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2}) \sigma_A^Q(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2}) \cosh(y_1) \cosh(y_2) w(y_1, y_2). \tag{7}$$

This is our *final formula* for the production of dimuons by the annihilations of Q's and $\bar{Q}$'s created during the hadron collision. The two constants A (see (5)) and t_0/V_0 (Eq. (7)) are not quite free since they are given, up to a factor 2 or so, by the short range of interactions in rapidity and by the dimensions of hadrons.

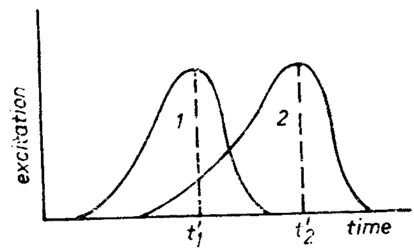

Fig. 3. Time dependence of excitation of regions "1" around y_1 and "2" around y_2

The joint probability distribution function $G_Q(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2})$ generated by a Monte Carlo technique by using the probability distribution Eq. (1) for the exclusive configurations of Q's and $\bar{Q}$'s. Note that $G_Q(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2})$ contains no free parameters since G, R and λ (see Eq. (1)) are fixed by the data on multiple production.

Only the effective masses of quarks are not fixed by the multiparticle production since they are smaller than or comparable to average transverse momenta. In dilepton production the situation is more favourable since the shape of $d\sigma/dM$ at low dilepton mass M is rather sensitive to values of quark masses. Anticipating the results quoted in Section 5 we can say that the dimuon data definitely require rather low quark masses (advocated already in Ref. [21]). In our calculations we have used $m_u = m_d = 10$ MeV/c^2 and $m_s = 160$ MeV/c^2. The value of m_s is not very important since the production of strange quarks is suppressed by phenomenological factors [16, 17]. In addition, strange quarks are, due to their charge of 1/3, less efficient in dilepton production than up quarks. In order to determine the effective masses of quarks, data on e^+e^- production and the deeper understanding of virtual Bremsstrahlung would be extremely helpful.

We shall now make a comment on, perhaps a bit surprising, occurrence of the factor $\sigma_{\rm inel}^{\rm hadr}$ in the r.h. side of Eq. (7). If the strong interactions were switched off, there would be no hadronic interactions (and no space-time evolution) and, consequently, no production of the LM component. The presence of $\sigma_{\rm inel}^{\rm hadr}$ just reflects these facts. In contradistinction to that, the DY component is influenced by strong interactions only via distribution functions of partons in free hadrons. The relation between multiparticle and dilepton productions carried by functions $G_Q(y_1, p_{T1}, y_2, p_{T2})$ is independent of whether the colour is taken into account or not, since a $Q\bar{Q}$ pair which can recombine to a colourless meson can also annihilate to a dilepton. Because of that in calculating the LM component we have not taken colour into account.

4. *The contribution from the Drell-Yan process*

The Drell-Yan process describes the annihilation of a quark from one of the incoming hadrons with an antiquark from the other one. This process occurs at the very beginning of the space-time evolution of the collision when quark and antiquark distribution functions still correspond to those of free hadrons. The process is of a long range in rapidity and yields the substantial contribution to the production of dileptons with large masses (above 2 or 3 GeV/c^2).

The process is well known and we shall mention here only a few items required for introducing transverse momenta and finite effective masses of quarks. The related questions of gauge invariance are discussed e.g. by Gunion [22].

Let $f_Q(x_1, p_{T1})$ gives the probability distribution of finding a Q with longitudinal momentum $p_{L1} = x_1 P$ and transverse momentum p_{T1} in the initial hadron and $f_{\bar{Q}}(x_2, p_{T2})$ denotes the similar quantity for $\bar{Q}$. If the spatial densities of colliding hadrons are normalized to $2E_{Hi}$ (E_{Hi}, $i = 1, 2$ are hadron energies) the spatial densities of Q and $\bar{Q}$ are

$$\varrho_Q^i(x_1, p_{T1}) = 2E_{Hi} f_Q(x_1, p_{T1}), \tag{9}$$

$$\varrho_{\bar{Q}}^i(x_2, p_{T2}) = 2E_{Hi} f_{\bar{Q}}(x_2, p_{T2}), \tag{10}$$

and the number of annihilations $Q\bar{Q} \to \mu^+\mu^-$ (per unit volume and unit time interval in beams of colliding hadrons) becomes

$$\Delta n = \sum_{\substack{Q \\ i \neq j}} \int dx_1 dx_2 d^2 p_{T1} d^2 p_{T2} \varrho_Q^i(x_1, p_{T1}) \varrho_{\bar{Q}}^j(x_2, p_{T2}) \, |\vec{v}_Q - \vec{v}_{\bar{Q}}| \sigma_A^Q. \tag{11}$$

To obtain the cross section we have to divide Δn by the flux factor of incoming hadrons $2E_{H1} 2E_{H2} \, |\vec{v}_{H1} - \vec{v}_{H2}|$. In this way we get

$$\sigma_{\mu\mu}^{DY} = \frac{1}{|\vec{v}_{H1} - \vec{v}_{H2}|} \sum_Q \int dx_1 dx_2 d^2 p_{T1} d^2 p_{T2} f_Q(x_1, \vec{p}_{T1}) f_{\bar{Q}}(x_2, \vec{p}_{T2})$$

$$\frac{2\pi\alpha^2}{3E_1 E_2} \left(\frac{e_Q}{e}\right)^2 \sqrt{\frac{s - 4m_\mu^2}{s}} \left(1 + 2\frac{m_\mu^2}{s}\right)\left(1 + 2\frac{m_Q^2}{s}\right). \tag{12}$$

In connection with introducing transverse momenta masses of quarks we have modified the Field and Feynman distribution functions [23] in the following way: we have multiplied them by factors $\exp(-p_T^2/R_{DY}^2)$ and replaced the usual factors $1/x$ by $(x^2+m_T^2/P^2)^{-1/2}$ where m_T is the transverse mass of the parton and P is the c.m. momentum of the incoming hadron.

More explicitly we have put

$$\begin{aligned} u_V(x, \vec{p}_T) &= C_1 u_V^{(FF)}(x)x(x^2+m_T^2/P^2)^{-1/2}\exp(-p_T^2/R_{DY}^2), \\ d_V(x, \vec{p}_T) &= C_2 d_V^{(FF)}(x)x(x^2+m_T^2/P^2)^{-1/2}\exp(-p_T^2/R_{DY}^2), \\ u_s(x, \vec{p}_T) &= C_3 u_s^{(FF)}(x)x(x^2+m_T^2/P^2)^{-1/2}\exp(-p_T^2/R_{DY}^2), \\ &\vdots \\ \bar{s}_s(x, \vec{p}_T) &= C_8 \bar{s}_s^{(FF)}(x)x(x^2+m_T^2/P^2)^{-1/2}\exp(-p_T^2/R_{DY}^2), \end{aligned} \tag{13}$$

where the symbol FF refers to Field and Feynman distribution functions. The normalization constants C_1 and C_2 of valence quarks in proton were determined by standard conditions

$$\int u_V(x, \vec{p}_T)dxd^2p_T = 2, \quad \int d_V(x, \vec{p}_T)dxd^2p_T = 1.$$

The distribution functions of "sea" quarks were normalized by requiring that the longitudinal momentum carried by them is the same as that given by the original Field and Feynman [23] distribution functions. In general, masses of u, d, and s quarks are different and as a consequence the normalization constants $C_3, C_4, \ldots C_8$ are not the same.

In the parametrization (13) the only correlation between p_T and x is that introduced by the factor $(x^2+m_T^2/P^2)^{-1/2}$. The same procedure can be also used when R_{DY} depends on x. In the present treatment we do not try such subtleties.

As it has been already pointed out, the DY and the LM components are of different dynamical origins and occur at different stages of the evolution of the collision. Because of that there are no compelling reasons for putting $R = R_{DY}$. In fact, as shown below, the data require R_{DY} significantly larger than R.

Our calculations were performed by using Eq. (12) with the r.h. side multiplied by 1/3 to take care of colour.

The total production of the dimuon continuum is given as a sum of the LM and DY components. This does not contain the contributions from the production and dimuon decay of vector mesons ϱ, ω, ϕ and J/ψ. Their inclusion further enhances the total dimuon production rate but in experiments with a good mass resolution the continuum and the vector meson contributions can be separated.

In a model like our one, where the vector mesons are produced directly by the recombination and dimuons by the annihilation of Q's and $\overline{\text{Q}}$'s, the p_T and x_F spectra of dimuons from the LM continuum and from vector meson decays should be approximately the same.

5. *Comparison of results with data*

The available data on dimuon production in hadronic collisions come from interactions of high energy hadrons (mostly protons) with nuclei. Nuclear effects are understood only in a rough way. This excludes attempts at very detailed comparisons of calculations with the data. Still, the cross sections vary by several orders of magnitude within the regions of $M_{\mu\mu}$ and x_F studied and even the comparison limited by (our) ignorance of nuclear effects is instructive.

In this section we shall compare our results with

(*i*) rather complete and detailed Chicago-Princeton II data [5] on $p+Be \to \mu^+\mu^- + X$ at 150 GeV/*c*, and with

(*ii*) Columbia-Fermilab-Stony Brook data [8] on $d\sigma/(dM_{\mu\mu}dy)$ in $p+Be \to e^+e^- + X$ at 400 GeV/*c*, which is consistent with the results on dimuon production of the same group [9, 10] and of the Chicago-Princeton group [7].

Our calculations were performed for p-nucleon collisions and they are compared with the p-nucleus data recalculated per nucleon by using a phenomenological A-dependence

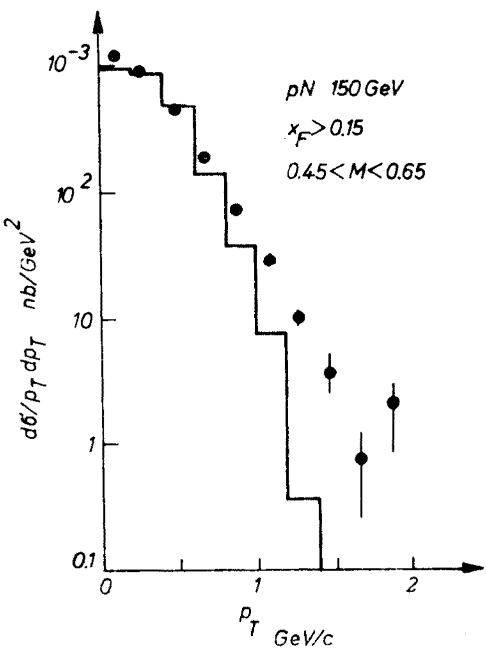

Fig. 4. The calculated p_T-distribution (histogram) of dimuons in the mass bin $0.45 < M_{\mu\mu} < 0.65$ GeV/c^2, $x_F > 0.15$ compared with the data [5] recalculated per nucleon

(the same as in [5]) of cross sections. Our results were obtained by Monte Carlo procedures and because of that they appear as histograms. Results given below correspond to the following choice of constants: $A = 0.8$ and $t_0/V_0 = (1.5\,m_\pi)^2$ — see Eqs (5) and (7).

We start with a piece of data which directly tests the close relationship between multi-particle and dimuon productions, namely with the p_T distributions of low mass

($0.45 < M_{\mu\mu} < 0.65$ GeV/c^2) dimuons. As we shall show below, the dimuon production in this mass bin is dominated by the LM component. The shape of calculated $d\sigma/(p_T dp_T)$ (see Fig. 4) is given by the p_T-dependence of functions $G_Q(y_1, p_{T1}, y_2, p_{T2})$ which was fixed [17] using the multiparticle production data. The data for $p_T > 1$ GeV/c are systematically higher than our results. This is not surprising since the hard parton-parton scattering which was not included in the present model is becoming important at large transverse momenta.

Fig. 5a shows the sum of DY and LM components. The shape of $d\sigma/dM_{\mu\mu}$ is well reproduced. The peak in data at about 0.8 GeV/c^2 corresponds to the rho-meson production followed by its dimuon decay and it is not, of course, reproduced by our calculations. As

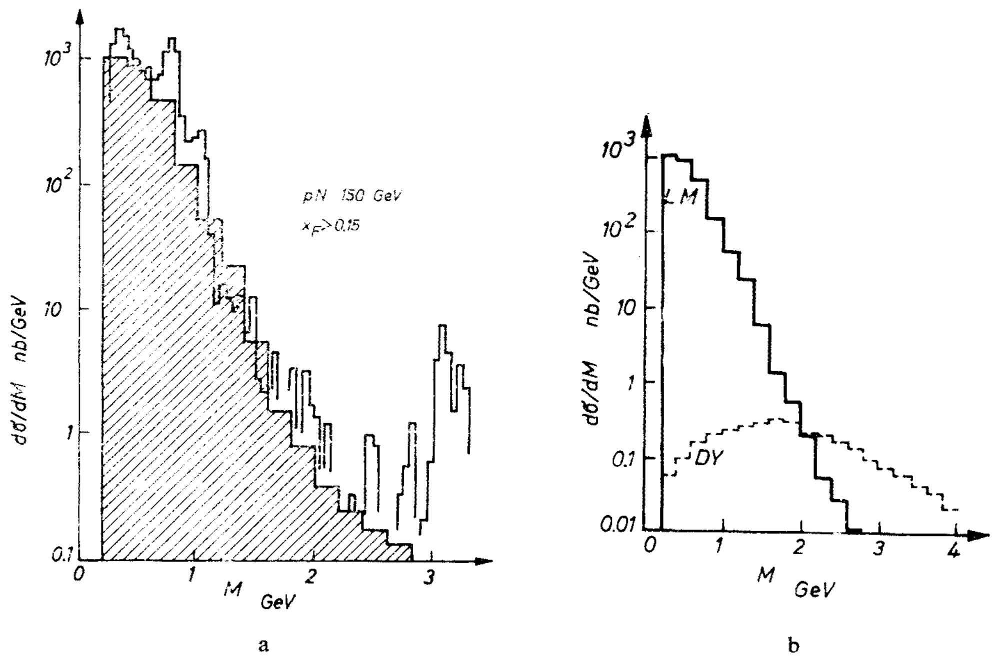

Fig. 5a. $d\sigma/dM_{\mu\mu}$ — the sum of LM and DY components (cross-hatched histogram) compared with the Chicago-Princeton data [5] recalculated per nucleon

Fig. 5b. $d\sigma/dM_{\mu\mu}$ — LM and DY components shown separately

seen from Fig. 5b, the LM component dominates for $M_{\mu\mu} < 2$ GeV/c^2. At higher masses the situation is reversed, the DY component takes over and the LM one can be neglected.

Another piece of data which checks our understanding of the LM component is provided by the x_F dependence of dimuon production in the mass bin $0.45 < M_{\mu\mu} < 0.65$ GeV/c^2. As seen from Fig. 6 the results are in good agreement with data. This is not a trivial fact since the x_F dependence of low mass dimuons is again regulated by the parameter-free functions $G_Q(y_1, p_{T1}, y_2, p_{T2})$.

The dimuon production shows a rather surprising rise of mean p_T of dimuons with their mass. In our two component model such rise can be explained if one admits that the average transverse momentum of partons within free nucleons (regulated by the constant R_{DY}) is larger than that of Q's and $\bar{Q}$'s created during the collision (given by the constant R which is fixed by multiparticle production data). In fact the analysis of deep inelastic lepton-nucleon scattering [24] (see also [25]) within the framework of the simple quark-

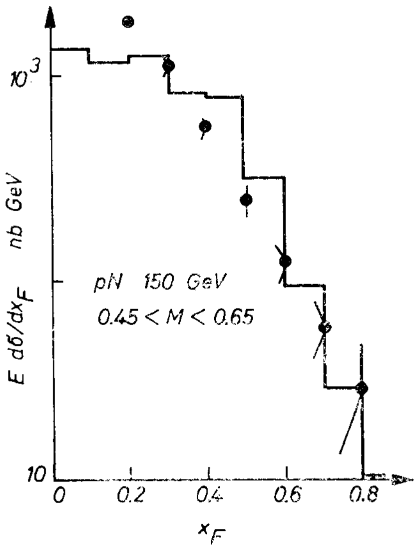

Fig. 6. $Ed\sigma/dx_F$ for the mass bin $0.45 < M_{\mu\mu} < 0.65$ GeV/c^2 compared with the data [5] recalculated per nucleon

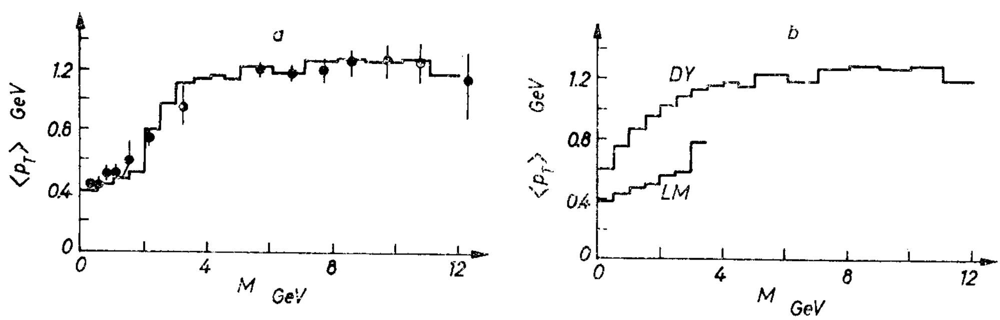

Fig. 7. The dependence of $\langle p_T \rangle$ of dimuons on $M_{\mu\mu}$: (a) calculated in the present model ($R^2 = 0.20$, $R^2_{DY} = 1.03$ GeV$^2/c^2$). The data are from Ref. [26] as quoted in Ref. [24]. Our results obtained at $p_{lab} = 400$ GeV/c, (b) LM and DY components plotted separately

-parton model indicates that the average transverse momentum of quarks within nucleons is about 0.9 GeV/c what corresponds to $R^2_{DY} = 1.03$. In Fig. 7a we compare the results of our calculations of the dependence of $\langle p_T \rangle$ of dimuons on $M_{\mu\mu}$. The observed trend of the data is reproduced quite well. The central point here is the assumed two component origin of the dimuon continuum, the LM part being dominated by annihilations of Q's and $\bar{Q}$'s created during the collision and large mass dimuons being produced predominantly by the DY mechanism.

We have to note, however, that in approaches based on QCD and taking into account also the "Drell-Yan type" annihilation $Q\bar{Q} \to \mu^+\mu^- +$ gluon [27–30] one can also explain a part of the rise of $\langle p_T \rangle$ of dimuons with $M_{\mu\mu}$. The whole issue will be probably clarified only later on when the lepton production in hadronic collisions will be understood more profoundly.

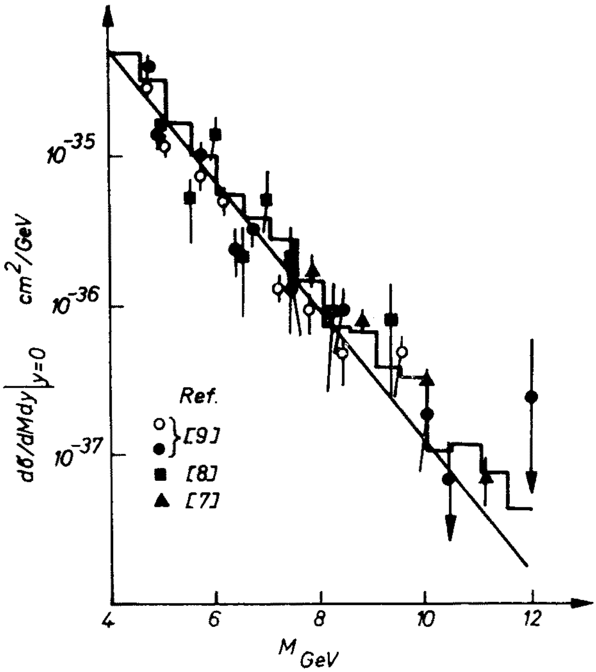

Fig. 8. Our results (histogram) for the large mass dimuon production compared with the data [7–10]. The solid line represents a single exponential fit to the continuum dimuons as measured in the experiment of Ref. [10]

It is amusing to note that $\langle p_T \rangle$ of dimuons rises in our model with $M_{\mu\mu}$ also within the LM and DY components taken separately (Fig. 7b). This is probably due to the fact that the cut off on $|y_1 - y_2|$ in the LM component forces the Q and the $\bar{Q}$ annihilating to a relatively heavier LM dimuon to have larger transverse momenta. A similar mechanism works also in the DY component.

The data [7–10] on the production of high mass dileptons are compared with our calculations in Fig. 8. The agreement is not surprising. At large $M_{\mu\mu}$ the DY component dominates and already from the previous work [7–10] it is known that the DY model gives a fair description of the data. The inclusion of transverse momenta and finite masses of quarks has only a little effect in this kinematical region.

The mass bin $0.65 < M_{\mu\mu} < 1.13$ GeV/c^2 contains the contribution from dimuon decays of ϱ, ω and ϕ, as well as the contribution from the continuum. Chicago-Princeton group [5] (see in particular Table I in their first paper) give separately the two contributions to the cross sections with $x_F > 0.15$. Assuming that both contributions have the same x_F- and p_T-dependence (see the discussion in Section 6) we can extract from the data the part which is to be compared with our model. Consulting Table I of Ref. [5] we see that in this mass bin vector meson decays give 1087 nb/nucleus whereas the continuum yields 438 nb/nucleus. In order to obtain x_F- and p_T-distribution of continuum dileptons we have to multiply the published data by $438/(1087+438) = 0.29$. In Figs. 9a, b we com-

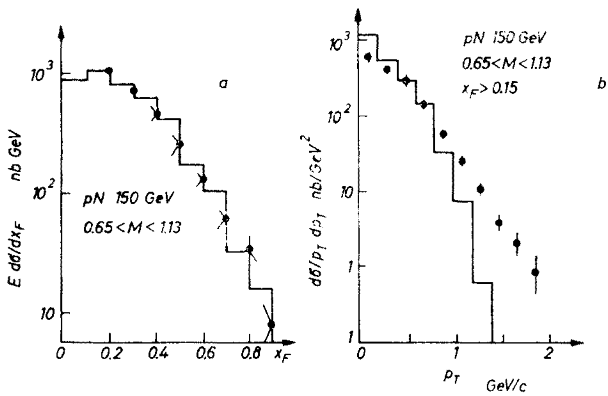

Fig. 9. Production of dilepton continuum for the mass bin $0.65 < M_{\mu\mu} < 1.13$ GeV/c^2 in our model (histogram) compared with data [5] recalculated as described in the text: (a) $Ed\sigma/dx_F$, (b) $d\sigma/(p_T dp_T)$

pare our results for the mass bin $0.65 < M_{\mu\mu} < 1.13$ GeV/c^2 and for $x_F > 0.15$ with the data [5] multiplied by 0.29 and recalculated per nucleon. The discrepancy in $d\sigma/(p_T dp_T)$ is not surprising. First, our model describes only the ordinary "soft" hadron collisions initiated by wee partons. For $p_T > 1$–2 GeV/c one can expect that hard processes are already dominating. Secondly, our parametrization for p_T distribution of quarks was the simplest possible. It is quite conceivable that other parametrizations like e.g. $\exp(-ap_T^2 - bp_T)$ might be more adequate, say, up to 2 GeV/c where hard processes take anyway over.

6. *Comments and conclusions*

In this paper we have proposed a model for the production of dimuons with low masses in hadronic collisions. The model is based on the Bjorken-Gribov picture of the space--time evolution of hadronic collisions and on the assumption that the annihilations of Q's and $\overline{\text{Q}}$'s produced during the collision are responsible for a sizable part of dimuon production.

The model leads to some qualitative predictions which, as it seems, may be tested in the near future. In our opinion, such tests are even more informative than comparisons with the available data. We shall now list these predictions:

(*i*) *The production of direct diphotons in hadronic collisions should be similar to that* *of dileptons*. If dileptons are produced by the annihilations of $Q\overline{Q}$ pairs existing for a short period in the rapidity region of size $\Delta y \approx 1$, then these pairs have also a chance to annihilate to diphotons with distributions in x_F, p_T and $M_{\gamma\gamma}$ similar to those of dileptons.

(*ii*) *Inclusive distributions of low mass dileptons produced in hadronic collisions should be similar to those of directly produced neutral mesons.* In fact formers come from annihilations, the latters from the recombinations of $Q\overline{Q}$ pairs separated by small rapidity gaps. Some evidence pointing in this direction comes from the Chicago-Princeton data [5]. Dimuons with masses within the region $0.65 < M_{\mu\mu} < 1.13$ GeV/c^2 come mostly from the production of neutral vector mesons followed by their dimuon decays. The way of obtaining Figs. 9a, b together with figures themselves clearly shows that the x_F- and p_T-distributions of dimuons in this region have the same shape as those of dimuon continuum in our approach. The point is discussed in more detail in Ref. [31].

(*iii*) *The production of low mass dileptons should be present with relative rate of about 10^{-4} in deep inelastic νp, ep and μp collisions.* The multiparticle production in these reactions seems to be rather similar [32] to that in hadronic collisions. From this fact one can guess that the mechanism of parton recombination into final state hadrons is not basically different from that which is operative in hadron-hadron collisions. One can therefore expect that quarks and antiquarks created during the lepton-hadron collision can annihilate to dimuons similarly to the LM component in hadronic collisions. Dimuon production in νp and μp collisions would be seen as trimuon events. Trimuons have already been found in both processes [33]. Trimuons produced by this mechanism are supposed to be distinguished from those of different dynamical origin by the predominantly low mass of "extra" dimuons. We hope that the increased statistics will facilitate a more detailed discussion of these problems soon. For more details on this point see Ref. [34].

(*iv*) *Dilepton production should be seen in e^+e^- annihilation and in large p_T processes.* In both cases one can assume that the production of final state particles goes via parton jets which evolve in space-time in a way which is quite similar to the evolution of the compound parton system in hadron-hadron collisions. In large p_T processes both members of a dilepton pair should have the same direction because of the low mass of the pair. The items (*iii*) and (*iv*) can be simply summarized by the scheme: parton jets → space-time evolution → low mass dileptons.

(*v*) *The two-component origin of the dimuon production leads to very specific predictions on the pattern of scaling violations.* As shown in Ref. [35] the available data including both the low and large mass dimuon production are consistent with this picture.

It has finally to be stressed that a complete picture of lepton production in hadronic collisions cannot be reached without understanding also the single lepton production at both low and large p_T. Preliminary calculations which we have recently performed indicate that the observed number of single direct leptons cannot be obtained in our model what means that additional contributions are still needed.

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